Physics of Linear and Non-Linear Optical Waveguides
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Module - 02
Basic Fiber Optics
Lecture - 09
Dispersion, Ray Path Constant

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture 9. And today we will going to cover the Detailed Calculation of the Dispersion, and a new concept called Ray Path Constant.

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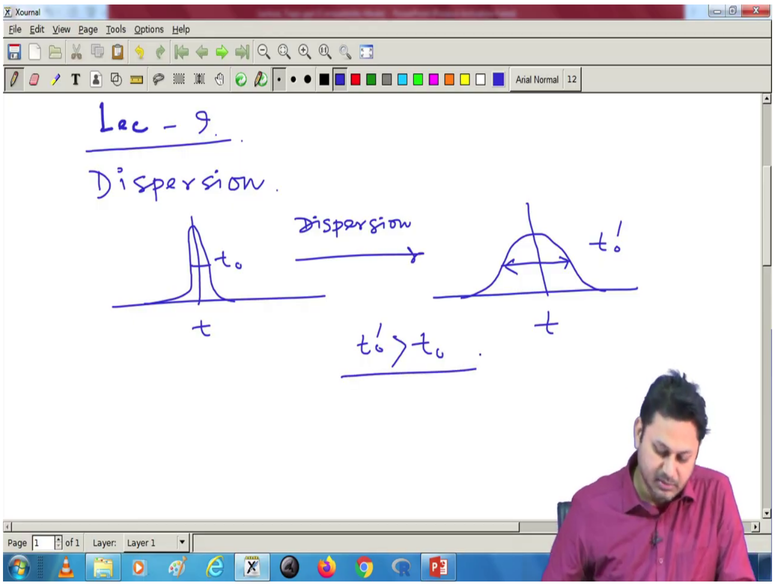

So, let me write it down, today we have lecture number 9. So, we were discussing about dispersion in the last class, and mentioned that when an optical pulse is propagating through a dispersion medium, it encounter a dispersion and as a result there is a temporal broadening. So, this is t; this is t. So, if I have t 0 as a width here, this width becomes say t 0 prime, where t 0 prime is greater than t 0. And in the communication, this is a problem, these things is a problem. So, we will going to discuss that as well.

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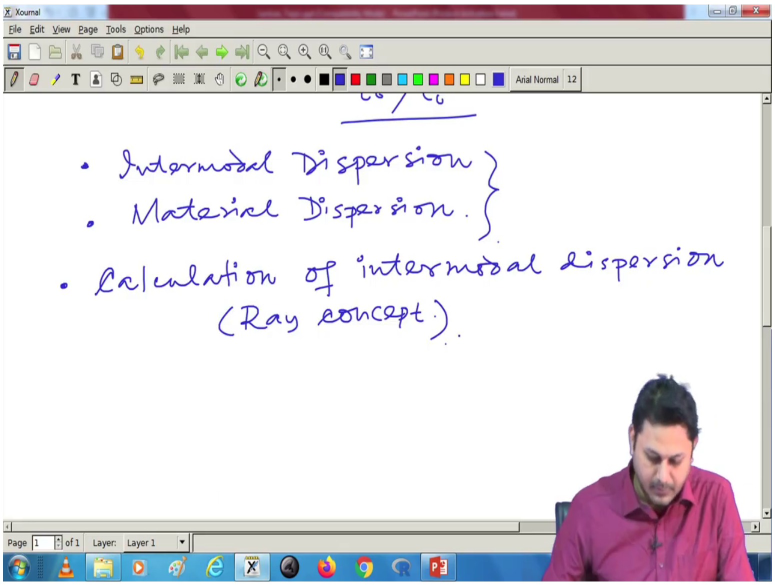

But today we will going to calculate the dispersion in terms of so there are two kind of dispersion we discuss; one is intermodal dispersion, and another was material dispersion. So, we can calculate the intermodal dispersion using the Ray concept. So, let us do that today first. Calculation of intermodal dispersion, and we will going to use the Ray concept.

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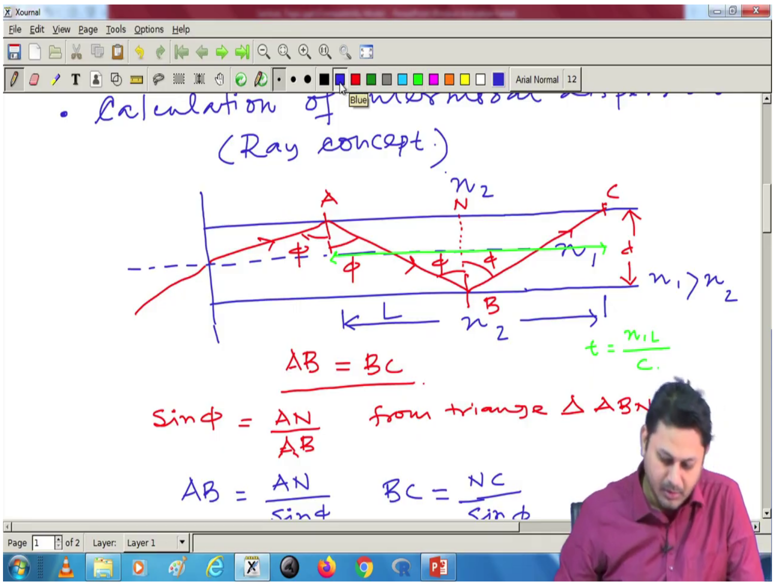

So, let us first draw the structure of the waveguide like this. This is the axis of the fiber core. This is n 1 – refractive index; this is n 2, this is n 2 – refractive index, where n 1 refractive index is greater than n 2. And if I launch a light here, let us make a different color, you will experience a total internal refraction like this, and hit here. Say this point is A, this point is B, this is C. This point says N, because I am going to use this point.

So, what essentially we will going to calculate here? The Ray a concept, this is the Ray that is passing by. This length is d. So, the time that we will going to take the ray a A, B, C and some other ray that we going to calculate. And we find there is a time lag between these two ray paths.

And this time lag is essentially the value of the dispersion in Ray concept. So, we will going to find first the time lag of these two rays. First we calculate the time taken by a arbitrary ray coming from point A to C with this path ABC. So, this angle is phi suppose this angle has to be phi, because this is a total internal reflection is taking place at that point A.

Similarly, this is phi as well. So, now, I can calculate few things. So, first thing, we can see here from this AB C this structure, this is a symmetric kind of structure that AB – the length AB is equal to BC. This is the first condition I can find from the simple geometrical structure. This angle is also phi.

Now, what is sin phi? Sin phi is AN divided by AB from triangle ABN. So, this information I have beforehand. And now we calculate AB is now AN [music] divided by sin phi. In the similar way, I can write BC is equal to NC divided by sin phi. This from, this is from another triangle NCB; from NCB I can write the similar thing.

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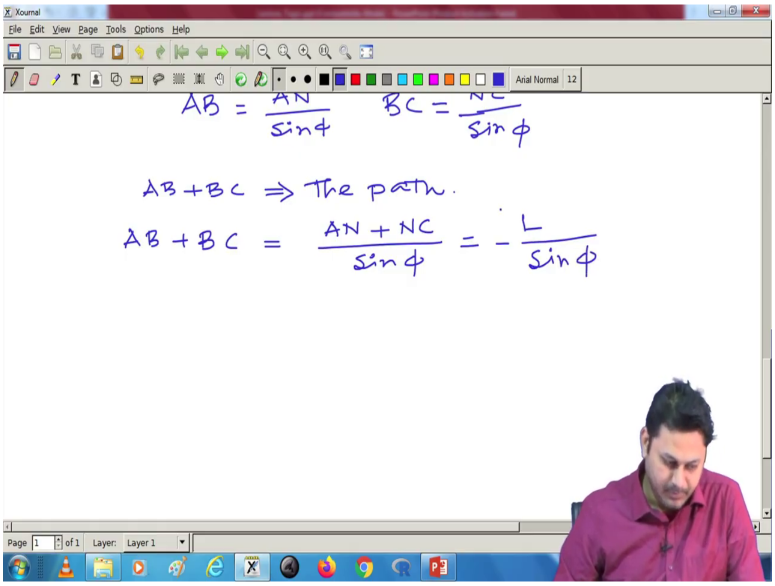

Now, what is AB plus BC? This is the path that is travelling by the light. So, this is the path that is travelling by this ray. Now, I can write this AB plus BC in terms of sin phi because I need to find out the relationship with this angle, whatever the angle it is making phi with the distance. So, AB plus BC is equal to AN plus NC divided by sin phi.

What is AN plus NC? This is the length of the fiber. Whatever the length it is travelling this is a length of that amount and this length suppose it is L. So, this is this length the straight line length for the point AC. However, the ray is travelling through this path with a reflection.

So, if it goes to a straight line, then it will going to take a smaller amount of time. It is quite obvious. But since the ray is passing through this, this internal reflection, it will take a little bit lot I mean longer time because it is travelling a larger amount of path. So, AN plus NC as I mentioned it is the length of the fiber. So, I can write L divided by L divided by sin phi. Now, I need to find out what is the time it is it with this fellow, there is no minus sign.

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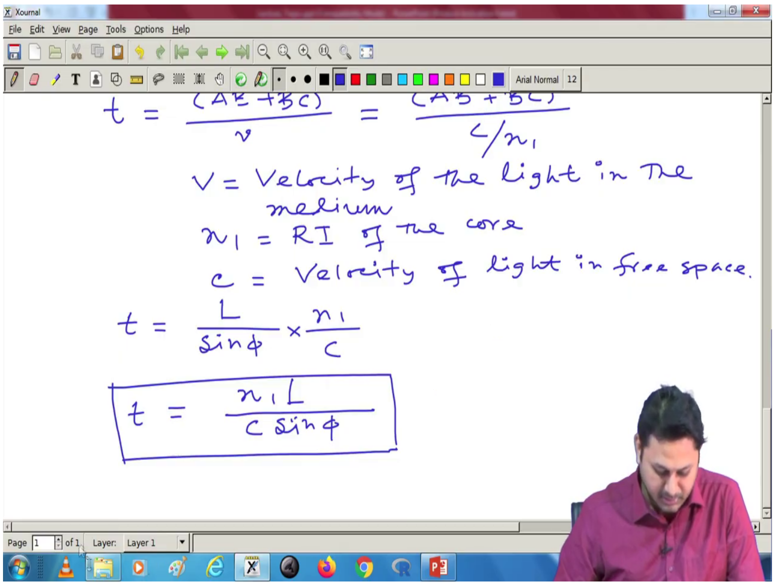

So, t that is the time that should take the path from go to A to B point through this A to C point through this reflection should be AB plus BC whole divided by the velocity. And we know the velocity inside a medium of refractive index n can be written as C divided by n 1, because n 1 is a refractive index and C is a velocity of light.

So, here V is a velocity of the ray of the in velocity of the light rather in the medium; n 1 is the refractive index of the core; and C here is the velocity of light in free space in free space. See eventually my t becomes if I now replace AB plus BC which I already calculated here AB plus BC is equal to L divided by sin phi, it should be simply L divided by sin phi multiplied by n 1 divided by c.

So, n 1 L divided by C sin phi. This is expression a general expression of the time I have when the ray is propagating as a length L of the fiber, and then having an angle phi. So, this ray having an angle phi here is moving from A to C with a reflection at the point B. And if I calculate the amount of time it will going to take is n 1 L divided by C sin phi. So, if now I change sin phi, so obviously this time will going to change and that I need to calculate next.

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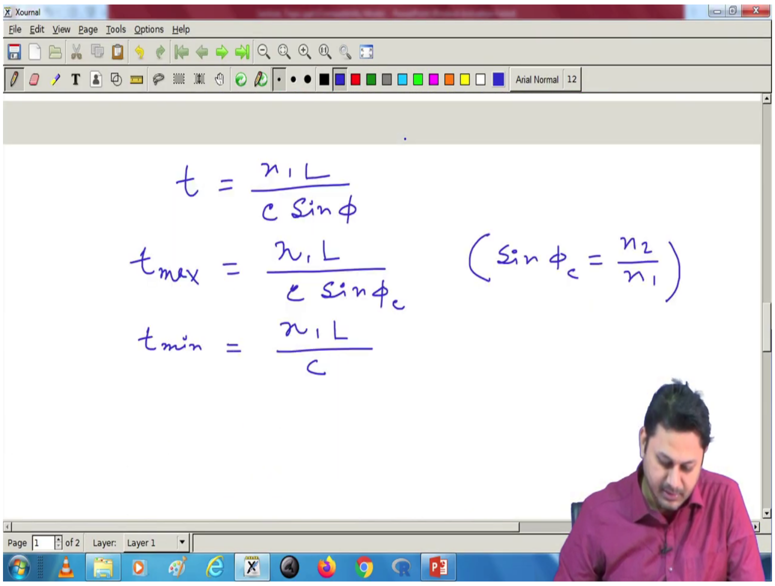

So, we have the general time the path going to take is n 1 L divided by C of sin phi. Now, I can calculate the maximum amount of time a ray can take. And it will going to take if you look carefully to the figure here, the maximum if theta, if phi increases, if phi increases, then if phi increases, then what happened?

At some point, it should be it should be 90 degree, and the ray will simply pass with a linear distance L without having any kind of any kind of reflection. But if theta decreases, then I can see sin theta will going to decrease. And when sin theta is decreasing, the t will increase.

So, what is the, what is the minimum value we can have for sin theta should give us the maximum value of the t – the path? So, this minimum value of sin theta for total internal reflection we all know that it is the critical angle that it will have, so sin phi c. When we have sin phi c, the left hand side t will be a maximum value. And in terms of refractive index, I can also write sin phi C as sin phi C equal to n 2 divided by n 1 using the Snell's law.

So, that I know. And what is t minimum? t minimum is straight forward. It is n 1 L divided by C when sin phi is phi C phi is pi by 2 as I mentioned. And if I calculate very simple way that the ray is travelling in a straight line path, the ray which is so let me draw which ray you will going to take this time, it will be clear.

So, I make it another color. So, the ray that is passing through this simply this path following this path, we will going to take a time, here I can write t for this ray – this green ray will be n 1 L divided by c. So, these two ray is having a t max and t mins, obviously, there is some time difference between these two ray.

So, when we start from one single point, so one ray will going to take this time – t max, and another ray will take t min. So, there is a maximum difference between these two rays which is basically the measurement of the dispersion in this problem.

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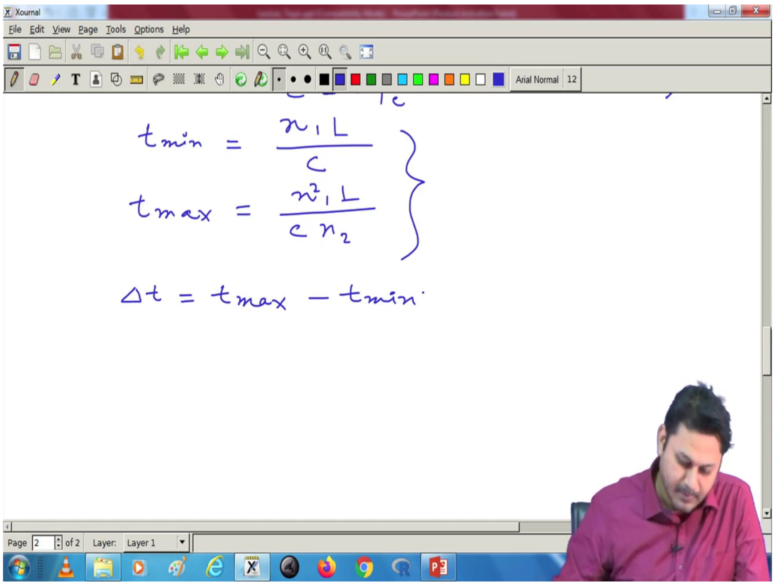

So, let me write t max once again in terms of refractive indexes. I just want to remove this sin phi. So, n 1 divided by L C sin phi is n 2 and I have n 1 square. So, this is my t max and this is my t min, these two are final expression. So, what is the delta t? So, delta t is the time lag between these two. So, I have t max, it should be t max minus t min.

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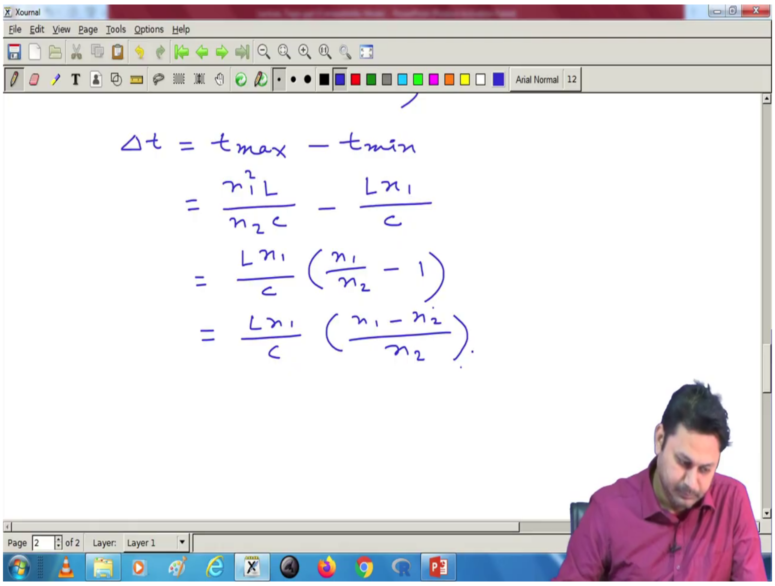

I have t max as n 1 square L divided by n 2 C minus L n 1 divided by c. So, I can have L n 1 divided by C common, and then I can have something like n 1 divided by n 2 minus of 1, or L n 1 divided by C is equal to n 1 minus n 2 divided by n 2.

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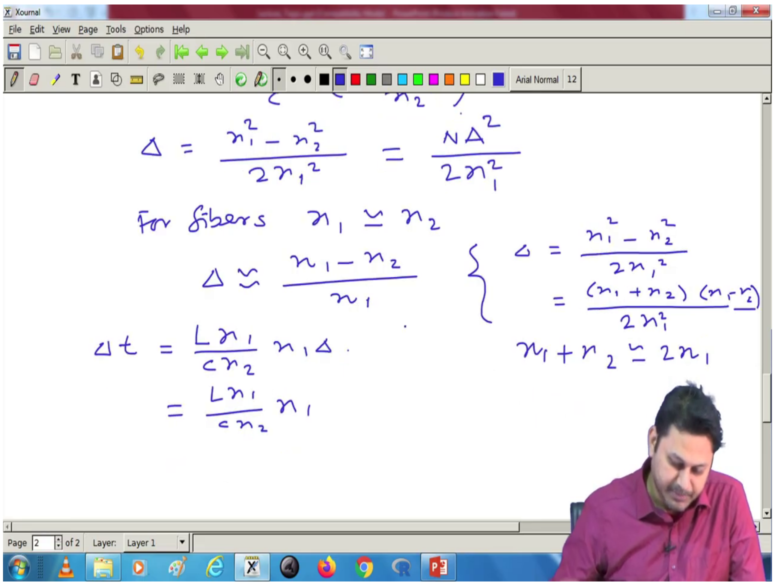

Now, I introduce a new parameter called delta which I write as n 1 square minus n 2 square whole divided by 2 of n 1 square. This is the definition. And if I look carefully n 1 square minus n 2 square is nothing but my NA square. So, NA square divided by 2 n 1 square. Now, normally what happened for fibers? n 1 is very nearly equal to n 2.

The refractive index difference is normally small. In that case, delta can be near delta can be written in terms of nearly equal to n 1 minus n 2 divided by n 1. Why it is that? Because originally delta is n 1 square minus n 2 square whole divided by 2 n 1 square which is n 1 plus n 2, and n 1 minus n 2 divided by 2 of n 1.

This n 1 plus n 2, this n 1 plus n 2, I can approximate as here is a square as 2 of n 1. So, if I make an approximation that since n 1 is very close to n 2, so n 1 plus n 2 is very nearly equal to 2 n 1, then delta can be simplified as n 1 minus n 2 divided by n 1, which is just to write this stuff in this way.

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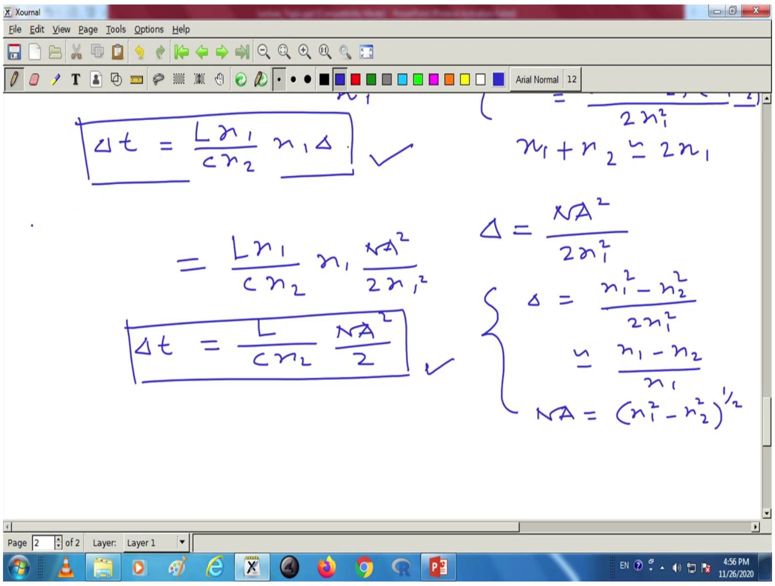

Then my delta t will be simply L n 1, I can write it as L n 1Cdivided by n 2 then n 1 delta. So, this is my original value of delta t. I just put n 2 outside and put an n 1 here. So, that delta is coming and I multiplied n 1, so that I can have this value. And also in terms of numerical aperture, if I look carefully in terms of numerical aperture, my delta is.

So this I can also write as delta was originally in terms of numerical aperture in this. So, we can write L n 1 divided by C n 2 then n 1 delta was originally NA square divided by 2 n 1 square. So, I can make use of that; so, I can make use of that and I can write it as delta let me write it delta equal to NA square divided by 2 of n 1 square.

So, I will put it here. NA square divided by 2 of n 1 square. So, it becomes simply LC into NA square divided by 2, this is also a way to write this. So, either I can write in this way or I can also write in this way in terms of delta. In many cases, the value of delta is given to a fiber.

If the value of delta is given to a fiber, then which should be the value of the dispersion, and this should be the value of the dispersion if the NA is given to the fiber. Let me erase this now; erase this part it looks ugly, yeah. So, and also you can see the definition of the delta and n NA is slightly different.

So, delta I defined as n 1 square minus n 2 square divided by 2 of n 1 square which is nearly equal to n 1 minus n 2 divided by n 1. On the other hand, NA is n 1 square minus n 2 square whole to the power half. If NA is given, then this delta t is my amount of dispersion. And if delta is given, then this is the amount of dispersion.

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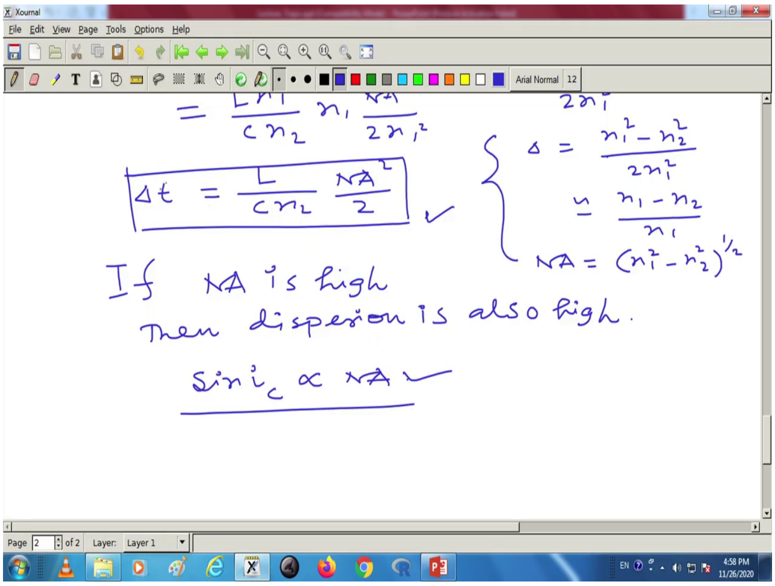

Now, you should note that there is a in delta t, we are having NA. So, if this is the important thing that if NA is high, then dispersion is also high.

If NA is high, the dispersion is also high. Previously, we have calculated that sin i c was proportional to NA that means, if I increase we want to increase the acceptance angle one can increase that by increasing NA. But here in this calculation we find that if I increase NA, the delta t has to be increased also, that means, delta t will going to increase if I increase NA.

So, if I want to have more and more light in the fiber, then I want to increase NA, so that i c can be increased. But at the same point, in that kind of cases, my dispersion also be too high, so that is why it is not judicious to increase the NA because the dispersion will be high in this case.

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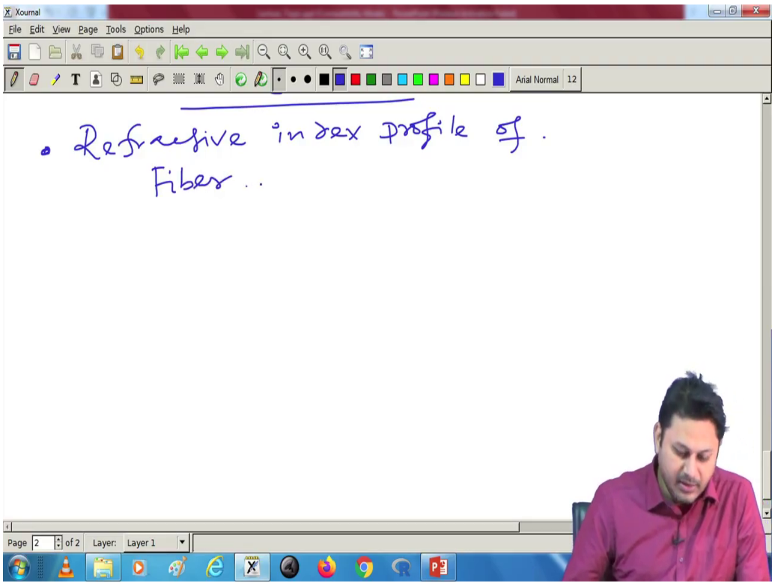

Next we quickly discuss about the refractive index profile of fibers.

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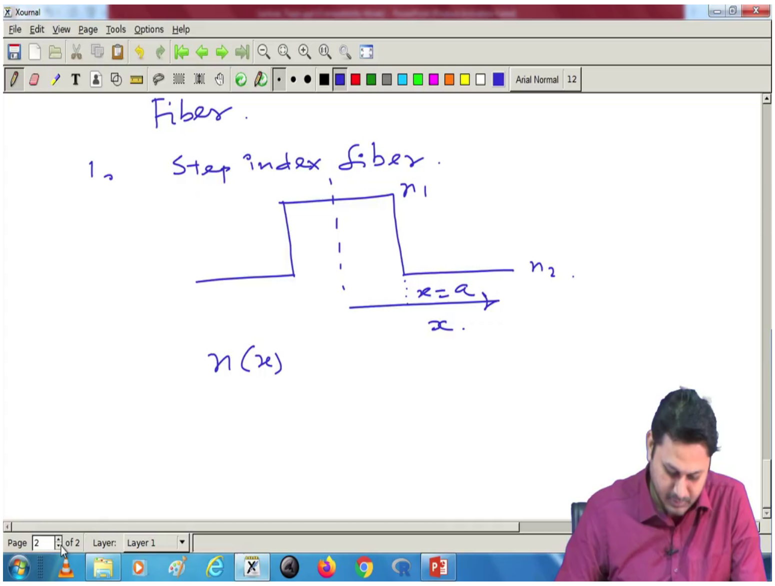

So, two kind of fiber refractive index profile I will going to mention today. And next class we will going to calculate something based on that. One is we already mentioned this one is step index fiber. For step index fiber, we simply have a step like structure like this. This is n 1; this is n 2. And along this we have say x; and this point is x equal to a.

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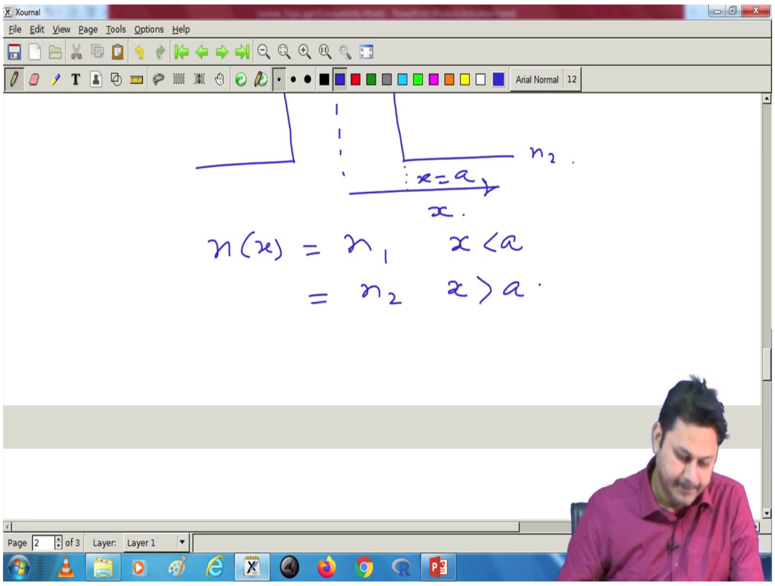

So, n, which is a function of x can be written as n 1 for x less than a, and equal to n 2 for x greater than a.

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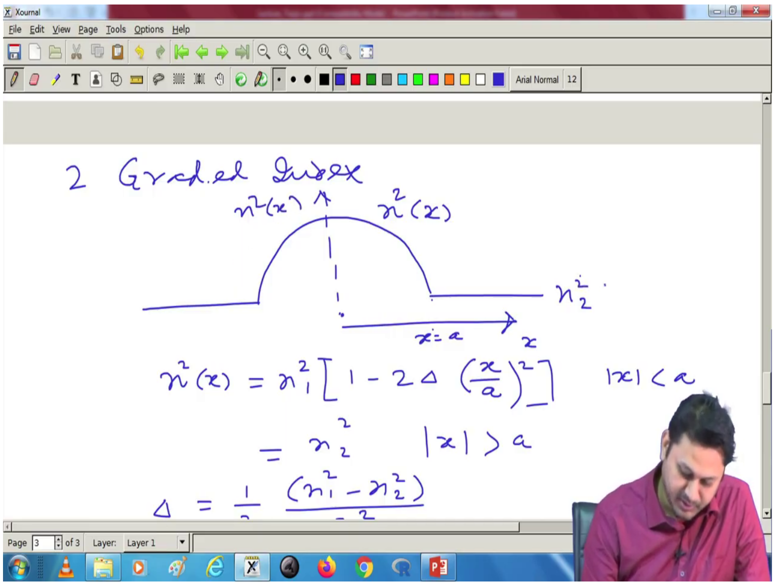

Another kind of refractive index quickly I like to mention which is graded index. In graded index, the refractive index profile has a variation like this – a parabolic kind of variation. This is the central part. This is along x.

So, this n is a function of x. So, I have to be careful in defining n. In this case, n square x will be defined as n 1 square 1 minus 2 delta x by a whole square. This happens when mod of x is less than a. And it should be n 2 square when mod of x is greater than a. This point is my x equal to a.

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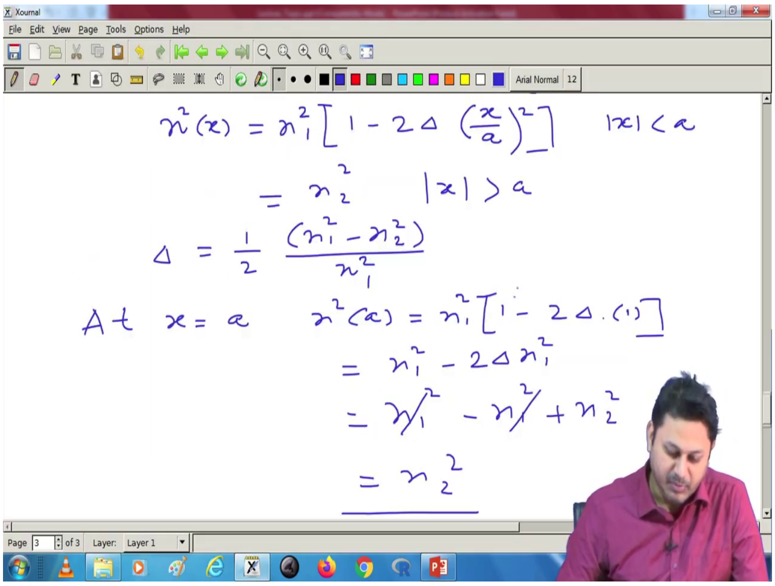

Using the delta, I can also verify that what happened at x point. So, delta is equal to half of n 1 square minus n 2 square divided by n 1 square that we know. And if I put this delta here in the expression, then I can verify that at x equal to a n square a is simply becomes n 1 square 1 minus 2 delta into 1 because I replace n 2, n 2 a here x 2 a here.

So, this is simply n 1 square minus 2 of delta n 1, and delta n 1 from here delta n 1 square. And 2 of delta n 1 square here is n 1 square minus n 1 square plus n 2 square; n 1 square n 1 square cancel out, so I will have this is n 2 square. So, at x equal to a, from the profile, we can see at this point, it is simply merges two value n 2. This is n 2 square. So, I plot it as n square. So, if I plot I should write it y-axis as n square as a function of x, so this value is square.

So, with this, I will like to conclude here. We do not have much time today. In the next class with these two profile, we are try to understand a general path equation, and how the ray will going to follow with this general path we calculate.

Thank you very much for your attention.