Physics of Linear and Non-Linear Optical Waveguides
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Module - 02
Basic Fiber Optics
Lecture - 06
Light Guidance in SIF (Skew Ray), V-Parameter, Discrete Ray

Hello student, for the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number 6 and today we will going to discuss the light Guidance in step index fiber for a ray, which we called skew ray and then the V-Parameter and then the concept of Discrete Rays.

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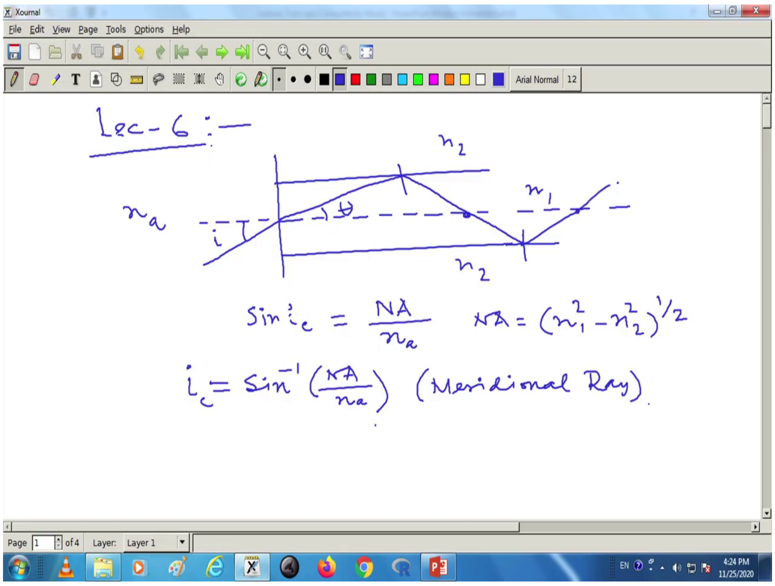

So, today lecture number 6. So, in the previous class if you remember we calculate the for meridional ray this was the structure. So, let me write it once again n 1 is the refractive index of the core and n 2 is the refractive index of the cladding. And, we calculate the acceptance angle this is i this was theta.

And, we calculated the acceptance angle. And, it was sin i c equal to NA divided by small n a where small n a is a refractive index of this medium, where the waveguide or the fiber is placed. And, NA we call the numerical aperture, whose value was n 1 square minus n 2 square whole to the power half.

And, then we find that i c is sin inverse of NA divided by small na and that is for meridional ray. Please remember meridional ray is the ray which can passes through the axis. So, here for example, it is passing through the axis and when it again make a reflection, then it will again crosses will cross the axis here at this point and so on. So, today we will going to learn how to calculate this quantity i c, which is the acceptance angle.

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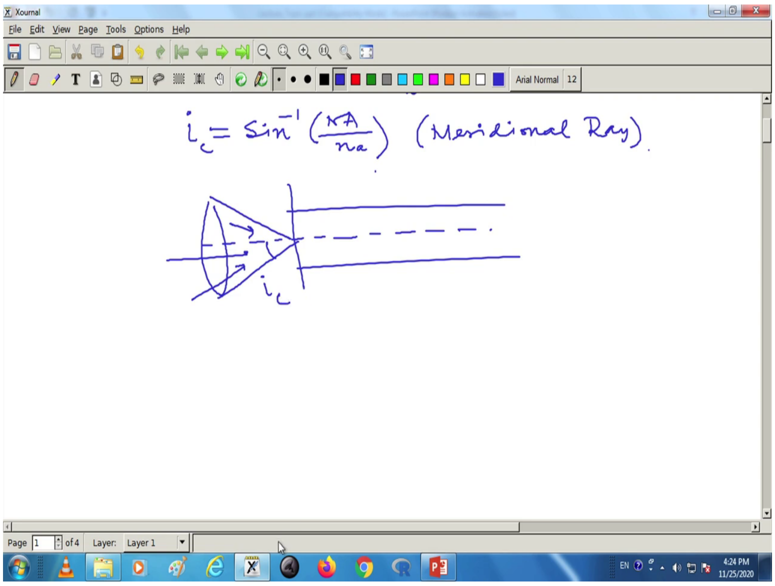

So, also we draw this that basically there is a region, there is a solid angle and all the rays, that is falling in this solid angle in principle should pass through this, this angle was i c and that is for meridional ray. Now, we will do we will going to calculate the same thing for skew rays.

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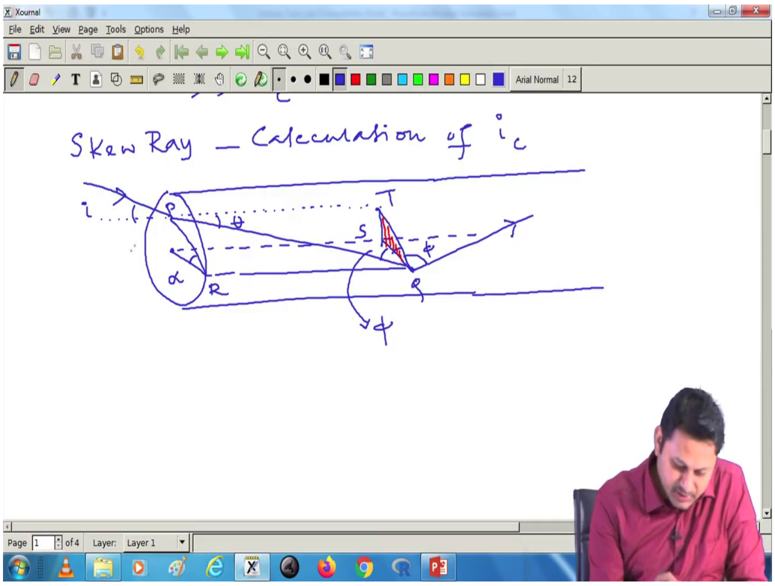

Calculation of acceptance angle i c, well this is a bit tricky; because the geometry is not straight forward first we need to understand the geometry. So, let me first draw the geometry very carefully. So, this is the fiber structure, three dimensional fiber structure we have a axis like this. And, at some point here say P this is also some point P, the light is falling like this. And, then refracted with this interface, let me extend this part.

So, this is the fiber wall. So, it is refracted in the fiber wall. So, this angle is i. So, this angle is our theta. And, now it is at this point Q, which is over the fiber surface it is reflecting and it goes in this way. So, if I join this point. So, I will have like this a plane like this here and also a parallel line, if I draw I should have something like this.

So, let me call this point T, this point S, and this is the angle alpha this alpha angle is very very important, because we find at the end of the day that i c which is the acceptance angle will be related to this alpha. This angle then this angle; obviously, will be alpha and suppose this angle is phi this angle is phi and it is again reflected.

So, this angle should be phi as well. Let us put the name here. So, this is my P, this is say R, this point is S T and I have a triangle, piece of triangle here ok. So, now, try to understand how to correlate this theta with i with this phi. So, my goal is to correlate the incident angle i with the phi to find out what is the acceptance angle.

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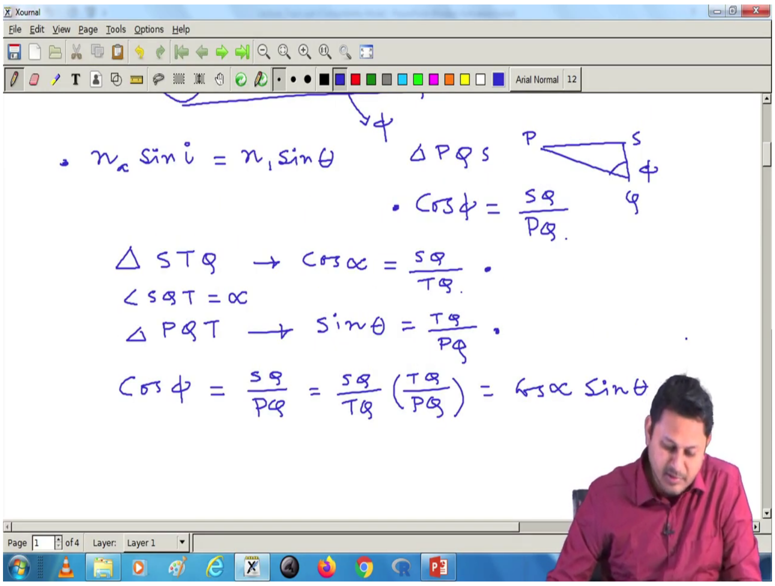

So, let us start with the equation which directly tells us how i and theta is correlated. So, first we have n a sin i is equal to n 1 sin theta Snell's Law, I apply just the Snell's Law. And, mind it this is the core of the fiber. So, refractive index of the fiber is n 1. Then, let us consider angle there are many triangles are there.

So, let me first consider PQS; PQ and S this angle is phi. So, it is associated with phi. So, if I draw in a simpler manner. So, that we can understand which angle is phi it should be phi, this is the point P, this is the point Q, and this is the point S. Well, from here I can have cos phi, because mind it I need to correlate with I and phi.

So, whatever the geometry I have from that I need to correlate with I and phi. So, that precisely my goal and I will like to do I will try to do that with whatever the ray geometry I am having. So, these are the rays so, let us put the ray sign. So, cos phi should be S Q divided by P Q ok; so, this information I have.

So, let us consider another angle, another triangle. So, triangle say, this triangle which is now shaded. So, S T Q, from the triangle STQ I can have cos alpha which is equal to SQ divided by TQ. Mind it angle SQT is equal to alpha here. So, this angle is alpha this angle is alpha ok.

So, next I will consider another triangle and this triangle should be P Q and T. So, angle P Q and T. From P Q and T, I can have another expression which is sin theta is equal to; that means, this is my theta. So, sin theta should be equal to TQ divided by this is TQ divided by PQ.

So, I have three quantity in my hand, one is cos theta, one is cos alpha and another is sin theta. So, with these three I can correlate the angle theta and phi, and the relationship with the theta and I is already here in this equation. So, I can eventually correlate i and phi that is my goal.

So, let us just write cos phi whatever the cos phi I calculate cos phi is equal to SQ divided by PQ and I write it as SQ divided by TQ multiplied by TQ divided by PQ, because TQ TQ will cancel out. So, finally, I have SQ divided by PQ if I do then I have simply cos phi is equal to cos alpha sin theta.

So, now I will going to use this expression here in this Snell's Law. So, let me write it once again the Snell's Law under critical condition; that means, it is a when try to note it when i increases, then theta decreases, when theta decreases, then phi will going to increase.

And, then at some point there will be an critical angle for which this phi become 90 degree. And, no light will be allowed after that because then the total internal reflection is not going to happen. So, that is precisely the condition we will go to find.

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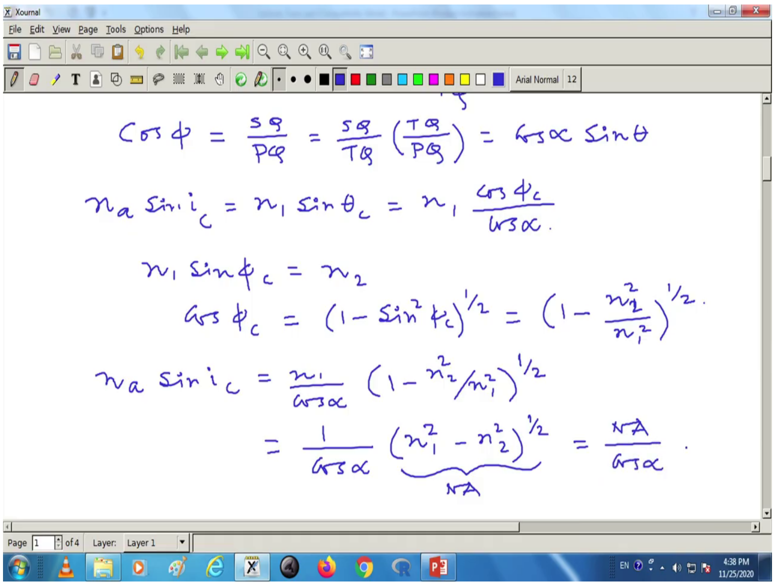

So, at critical condition I can write n a sin i c is equal to n 1 sin theta c at critical condition I just replace i to i c and theta to theta c, it is n 1 because this relation now I have. I can replace this sin theta in terms of cos. So, it should be cos of phi c divided by cos of alpha.

Now, n 1 sin c since it is a critical condition can be written as can be written as n 2, because since it is a critical condition. So, I do not here we should have a sin n 2 sin another angle and that angle has to be 90 degree, because it is a critical condition.

So, then I can have sorry. So, this angle phi c will be the critical condition. So, this angle, so, I may need to this is not theta c, because I am considering the critical condition for. So, this angle will be phi c. And, try to find out the critical condition for this angle, which is phi. These angle and it will be refracted to the cladding part and this refraction will happen, when it be this angle phi will be greater than phi c.

So, at critical condition this is the condition. At critical condition this is the expression we have. So, from here I can write cos phi c is equal to 1 minus sin square phi c whole to the power half, because cos phi c is sitting here so, that is why I try to find out cos phi c in terms of n 1 and n 2. So, it should be 1 minus n 2 square divided by n 1 square whole to the power half.

So, finally, I have n a sin i c is equal to n 1 divided by cos of alpha. Then, I have cos of phi c which I calculate here, it should be 1 minus n 2 square divided by n 1 square whole to the power half. If I put n 1 inside, then I have 1 divided by cos of alpha. Then, n 1 square minus n 2 square whole to the power half, please note it this thing is numerical aperture. So, I have eventually NA divided by cos of alpha.

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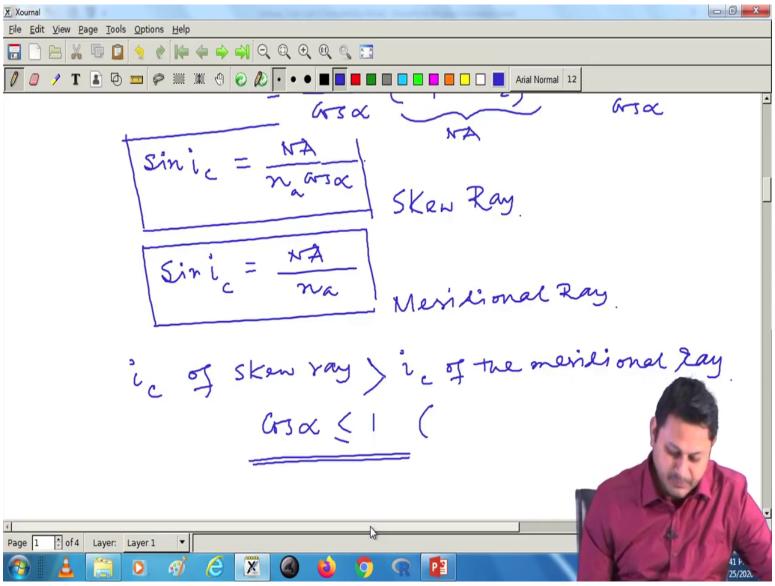

Well, then sin i c should be equal to 1 divided by NA divided by small n a and then cos alpha. This is the expression I find for skew ray; obviously, the i c should depends on alpha, because this alpha is the angle related to where it is where the light is launched. So, if the alpha is 0; that means, the light is launched exactly at the axis point, then whatever the equation I have this equation should merge with the equation, that we derived for meridional ray.

So, this is for skew ray and if I write the same expression for so, these two expression suggest that, the i c of skew ray. That means, the acceptance angle of the skew ray should be greater than, the acceptance angle i c of the meridional ray. Because, why it is this because cos alpha is greater than equal to 1; because of this cos alpha as there is a restriction of the cos alpha, so, i c will always be i c for the skew ray will be always be greater than the ic of the meridional ray ok.

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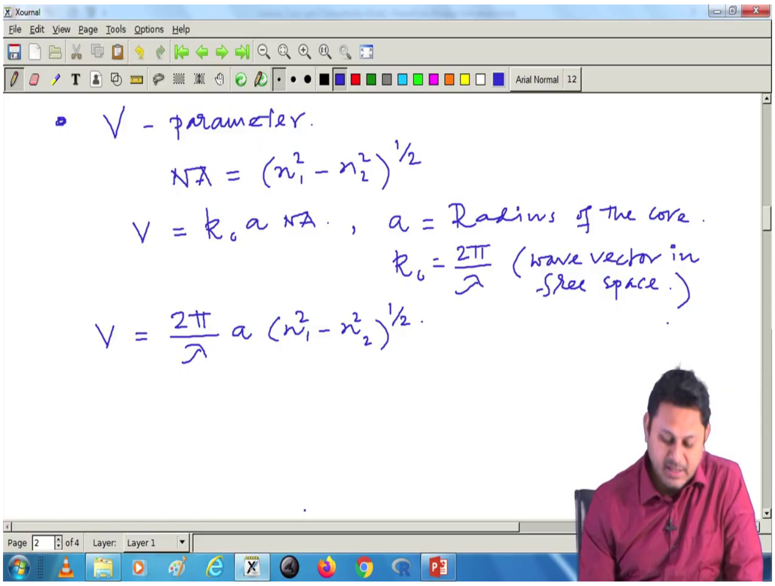

So, after that we will now learn another important concept called V parameter, V parameter. So, we already have an expression of numerical aperture we called NA as n 1 square minus n 2 square, whole to the power half, this is my numerical aperture. So, by definition V parameter is something like k 0 a multiplied by NA, for fiber a is radius of the core.

So, whatever the core I will going to use, it should have some kind of radius typically few 10 around 10 micron. And, this a is basically that value. And, k 0 is the wave vector in free space. So, it should be 2 divided by lambda this is wave vector in free space, wave vector in free space.

Well then the, V parameter is defined by definition it is 2 divided by lambda a, n a if I write in terms of refractive index it should be whole to the power half. Now, what is the importance of the V parameter? Please note that, the V parameter is a dimensionless quantity, there is no dimension a is length and lambda is also length. So, there should not be any kind of dimension of V so, it is a dimensionless parameter first of all.

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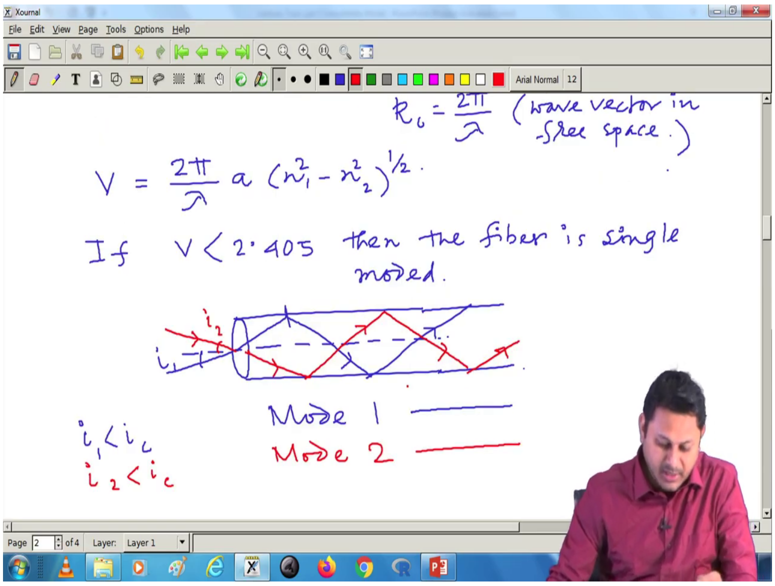

And, then if V is less than certain value, this value I am writing 2.405 there is a critical value 2 point why is it this value is 2.405 we will discuss in our future classes. Then, what happened the fiber is single mode, behave like a single moded fiber, single moded. So, what is the meaning of mode that also we will going to learn in this course.

But here, if I try to understand the mode in terms of rays, then how many rays, how many different kind of rays can propagate through the fiber can be loosely considered as modes.

Suppose, this is the fiber structure and there are one way that I have one ray that is passing through like this. And this angle whatever the angle I have i is less than i c say this is i 1. So, i 1 is less than i c.

So, one ray is passing. In the similar way I can also have another ray, which I write this say this angle is i 2, i 2 is also following the similar restriction that it should be less than i c. So, it can also propagate through the fiber like this. So, I can see that a fiber through the fiber, there are number of rays that can propagate.

Now, in optical fiber, if I have so, if all the rays are considered to be 1 Mode then for blue line, this is say mode 1, this is the blue line, this is mode 1, mode 2 this is red line and so on. Now, the V parameter basically tells us that how many rays can propagate through this fiber. So, here if I look very carefully to the expression of the V parameter, it suggests that V is equal to 2 pi divided by lambda a NA.

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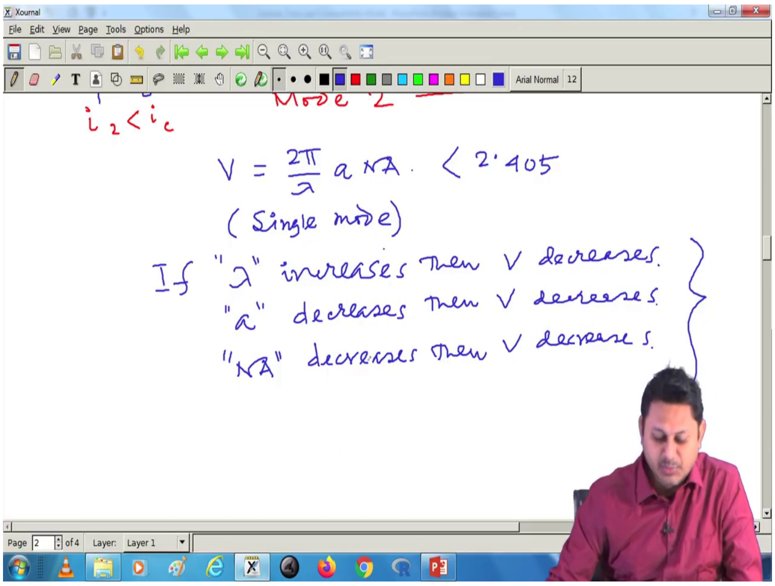

And, if this value is less than 2.405 roughly 405, then only one ray can propagate. And, that is why this fiber is called the single mode. That is why one mode is propagate and this fiber is called single mode. This is the condition of single mode, if I try to understand mode in terms of the ray picture.

Now, from this expression we can check that, if lambda increases then V can reduce there is one way that we can have the restriction over V, less than 2 point the value of V we can make less than 2.405 by just increasing lambda or you can decrease. So, if “lambda” increases then V decreases, if “a” decreases, then also V decreases. And, another parameter is still there “NA”, if NA decreases then also V decreases.

So, there are three different parameters using which we can reduce the value of the V. So, that whatever the wave guide we are using that wave guide can behave like a single moded waveguide. In many applications we required only one mode. So, that is why this definition is very very important.

And, once we have this definition, we can have the idea how the V parameter, which controls whether the wave guide should be a single moded waveguide or multi moded waveguide can be controlled by the external parameter. So, lambda is external parameter because you are launching the light with the wavelength of lambda. But a and NA are the geometry dependent parameter it depends on the geometry of the waveguide. So, since it depend on the geometry of the waveguide, we can prepare our waveguide with proper geometry.

So, that we can allow only few modes not many or even we can reduce this number of mode to 1. So, that our wave guide can only support a single ray which is useful in few application, we will discuss this in detail when we discuss about the modes of the fiber. But, beforehand it is handy to understand what is the value, what is the meaning of the V parameter and what is the definition rather of this V parameter at this stage ok.

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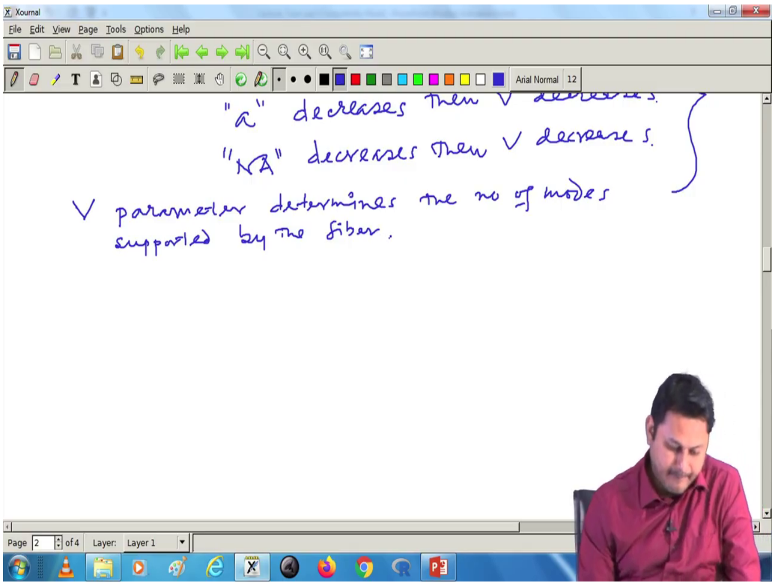

So, V parameter so, let me write it down what we just mentioned. So, V parameter determines the number of modes supported by the fiber ok.

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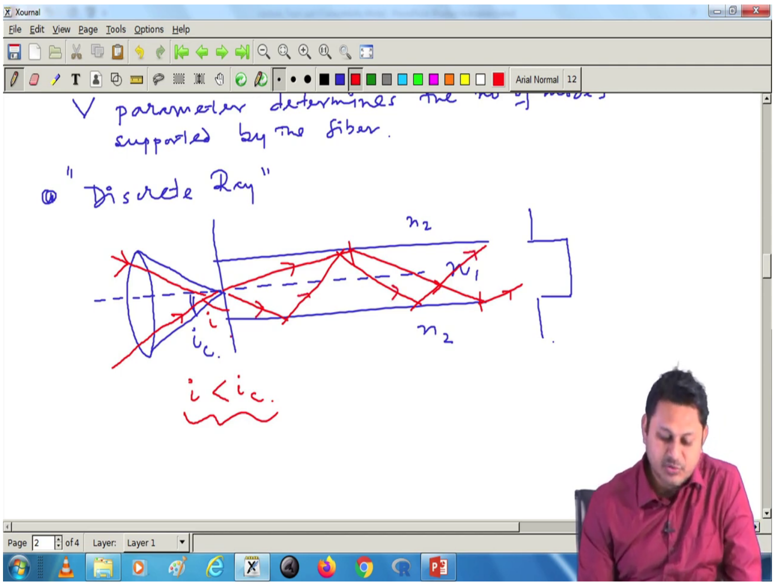

Next very important concept which is discrete ray; so, we understand so far that this is the geometry of the fiber. So, I have refractive index n 1 refractive index n 2 and n 2, if I draw the refractive index it should be like this. And, this is the axis of the fiber several time we draw this. And, we have a range of incidence angle here, this value is the i c.

And, in principle whatever the ray that falls here, if this angle i is less than i c, then there is no problem it appears to be very straightforward that, it can travel through the fiber. But, is it really the case, is it really the case that all the rays that falls with this angle i less than i c will pass will through the fiber like this?

This is another ray that is passing through the fiber. So, all the rays that should pass to the fiber is it true? The answer is not really. There are few selected rays that only pass through the fiber. So, in order to understand I will like to draw this first and then if time permits today then we will do the calculation as well.

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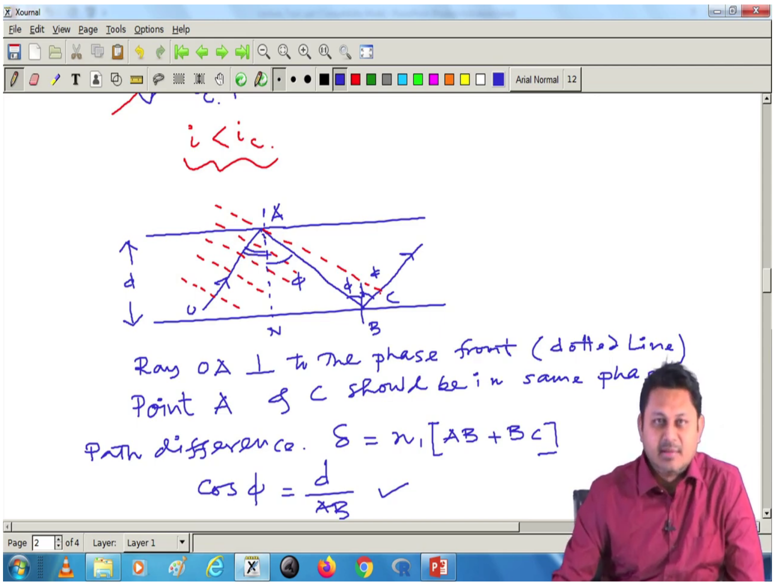

Otherwise, we will do the calculation in the next class. So, suppose we have a ok, I need to draw it carefully. So, let me. So, suppose this one ray it is inside the fiber I have a ray that is having experiencing total internal reflection at this point. So, this is A point A, say this is point B, and it is passing like this.

Now, when the ray is passing through this to the fiber, then it should be associated with the wave front, and this wave front should be perpendicular to the direction of the propagation, and I can draw this dotted line like this. So, these are the wave front that is passing like this way. So, when the wave front is passing like this. So, when the ray is coming here, then after bouncing I can have this wave front here.

So, let me call this point as point C. And, this angle which is equal to this angle, which is equal to this angle is called phi this is d. So, and this is say ray O. So, this is the geometry. So, Ray OA is perpendicular to the phase front, which is defined as a dotted line. Now, point if the ray is propagating, then in order to propagate one condition should follow that every time point A and C should be in same phase.

Then only we have a constructive interference and the ray will going to sustain. Otherwise, there will be destructive interference and it will not going to sustain over a long run. So, that is the important thing here and we are going to calculate that. So, what is the path difference?

So, path difference delta is equal to n 1, AB, plus BC, AB plus BC that is the. So, if I write in terms of phi, it should be cos phi, we know because this is d. So, this A say A n so, A n is d. So, I can write the cos phi in terms of d and this AB. So, it is AB next AB is here. So, next we need to find out what is the AC.

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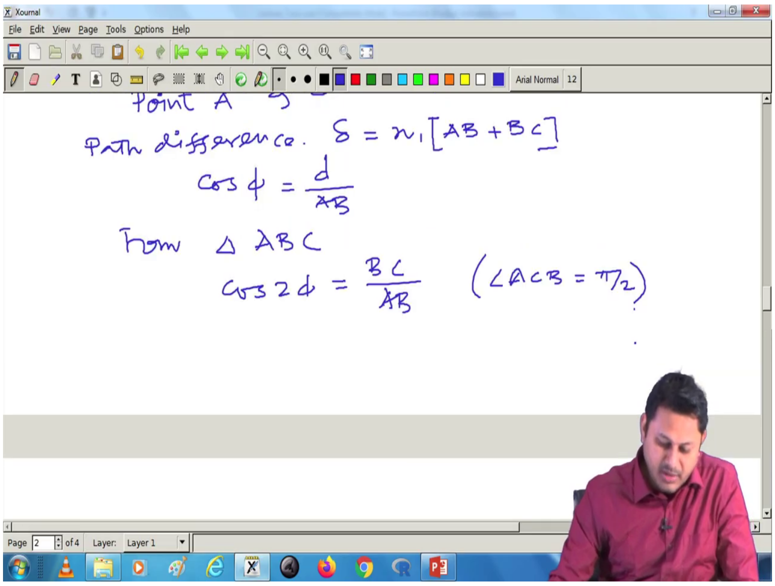

So, from triangle ABC whatever the triangle we have here ABC. I can have another expression which is cos of 2 phi is equal to BC, because this angle is 2 phi. So, it should be BC divided by AB. Because, angle ACB is 90 degree, divided by AB as angle ACB is pi by 2 ok.

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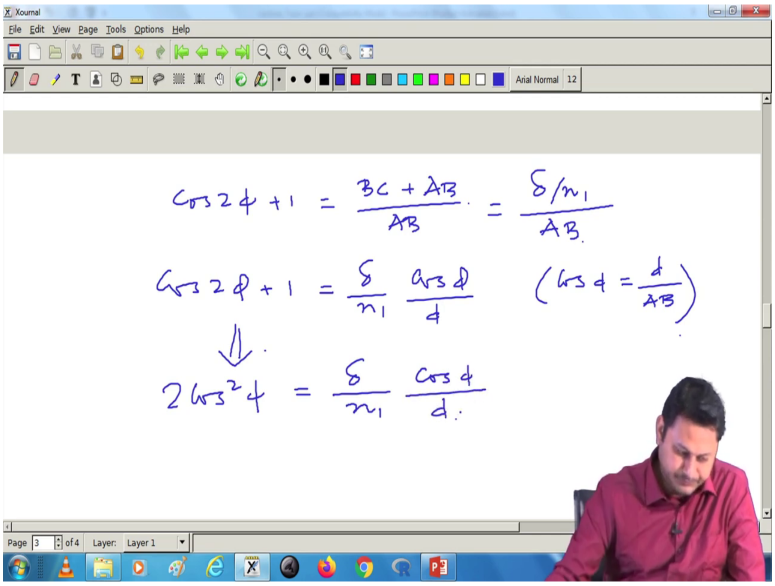

So, now I can also have AB and BC in this equation. So, I can write that cos of 2 phi plus 1 is equal to BC plus AB divided by AB just add 1. So, I have BC divided by AB and then I add 1. So, now, this BC by AB I can write in terms of path difference it should be delta divided by n 1 whole divided by AB.

So, cos of 2 phi plus 1 is eventually delta n 1 and AB, I already figure out here in terms of cos theta cos phi. So, I can write it as cos phi divided by d, because cos phi was d divided by AB ok. This quantity is nothing but cos square phi which 2 cos square phi rather, which is equal to delta of n 1 cos phi divided by d.

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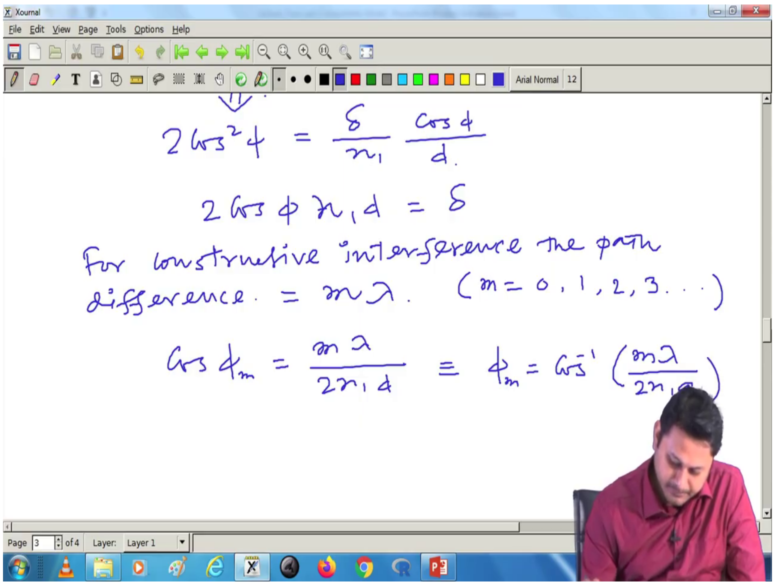

So, 1 cos phi will cancel out so, I have 2 of cos phi is 2 of cos phi n 1 d is equal to delta, the path difference 2 of cos phi n 1 d is equal to delta. Now for constructive interference as I mentioned earlier, the path difference has to be the integer multiple of lambda. So, here I have some discretization where m is 0, 1, 2, 3 and so on.

So, it is a discrete value. So, if that is the case I can have that cos of phi I need to put a discrete value m here is equal to m in place of delta, I just write m of lambda divided by 2 n 1 d; which gives me that phi of m which is the angle reflected by the fiber is cos inverse of m of lambda whole divided by 2 of n 1 a.

That means, the angle what is phi, phi is this angle, this angle is phi. So, this angle is now discretized in order to have the interference condition not all the angles are allowed.

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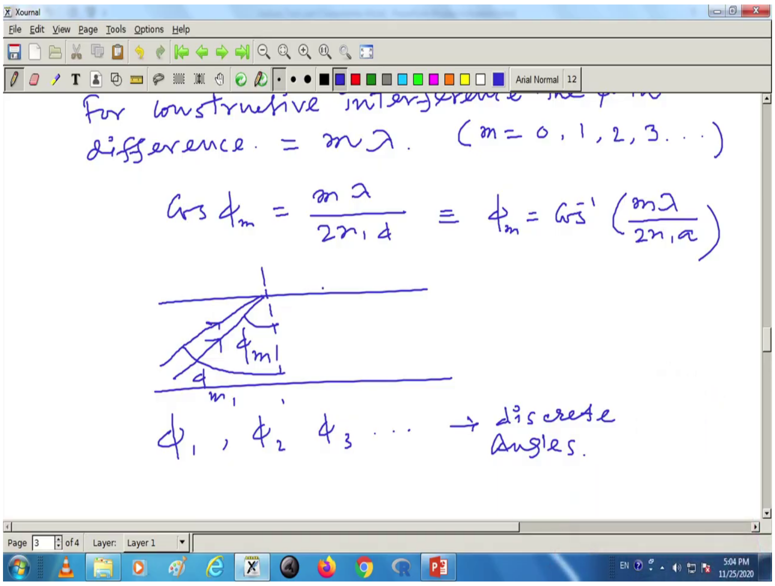

So; that means this angle phi m it depend on certain values. If this is allowed then continuously if I change it will not going to allow. So, again another phi m m 1 will allow. So, for example, this is phi m 1 this is so; that means, phi 1, phi 2, phi 3, these are few discrete angles. And, due to these discrete angles only few discrete rays that are allowed to pass through this system not all the rays.

Well, today I like to conclude my class here. So, today we covered two important concept one is the V parameter and also the physical concept that, how a discrete ray can propagate through the fibers. So, with this note I conclude. So, next class we will start with another concept called the cut off wavelength.

So, thank you for your attention see you in the next class.