Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 59
Concept of Optical Soliton

Welcome student to the course of Physics of Linear and Non-linear Optical Waveguides. Today we have lecture number 59 and today we will going to understand the Concept of Optical Soliton. So, optical soliton we already defined in the last class qualitatively. So, today we try to understand the concept quantitatively by doing a direct solution and try to understand the concept.

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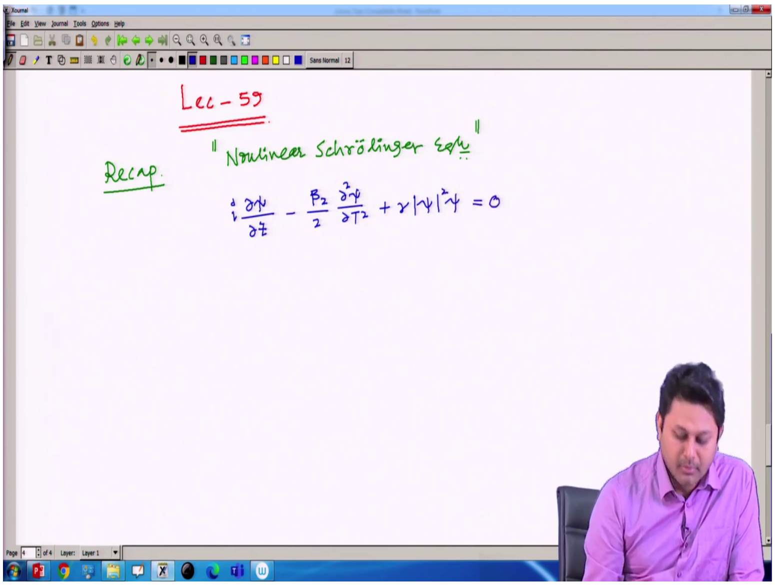

So, in the last class, a very brief recap. So, we had the non-linear Schrodinger equation and this non-linear Schrodinger equation was having this form, it was i del psi del z minus.

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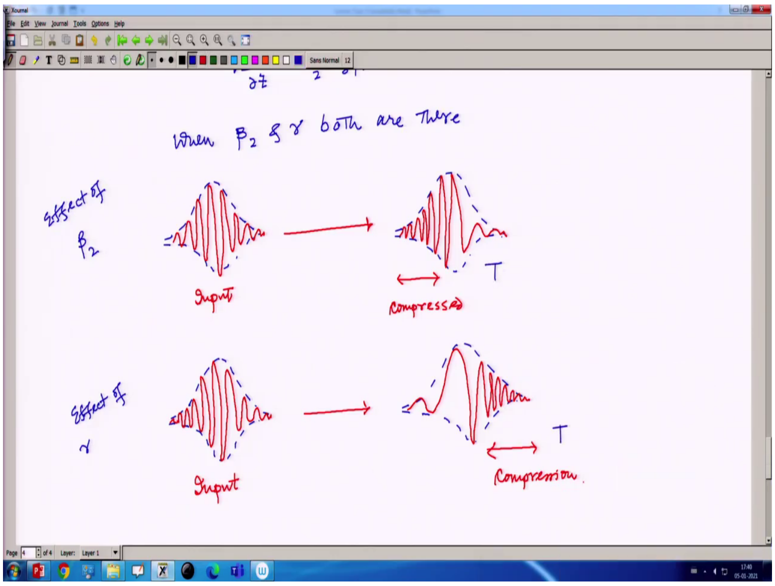

And we mention that when beta 2 and gamma both are there, both are there the waveguide then what happened? So, there is a. So, let me add this effect two effect together. So, first effect of beta 2. This is the input, in the output we have distortion and distortion was like this here we have a very closely spaced frequencies and then it rarified this side.

So, this portion is compressed. Now, if I write effect of nonlinearity gamma then we have a similar looking structure, this is input and when it is propagating under beta 2 I have. So, same figure I am drawing which I draw in the last class, but this is important that is why I am doing it again conceptually. So, we have some rarification here and then the compression in this side. So, both these things are distributed over time. So, it is T, it is T and now if I add these two effects.

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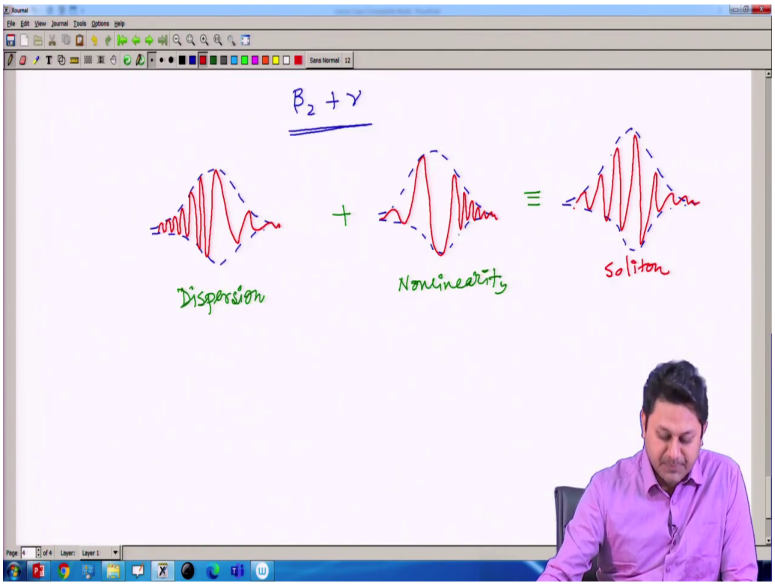

So, beta 2 plus gamma. So, I am adding these two effect dispersion and nonlinearity. So, for dispersion I had this figure. So, I like to reproduce this figure here. This is for dispersion plus for nonlinearity we have this curve. So, this is for dispersion and this is for nonlinearity.

And when we add up we find that these things are counter balancing each other. So, in dispersion we have the compression in the left hand side, on the other hand for nonlinearity we have compression on the right hand side and if I add these two in a correct way, then there is a possibility that we have a very very stable structure and there is no distortion at all in time domain as well as frequency domain and that kind of structure is called optical solitons which is very very important.

So; that means, I am launching a pulse which will not going to be distorted anymore even though there is a the dispersion and nonlinearity is present in the system. They balance each other in a nice way so, that there will be no distortion of this pulse there will be a stable structure which we call soliton this is a qualitative understanding. So, there is no change. So, this is called the soliton.

So, eventually I launch a pulse without any distortion means, even though the pulse is moving through a medium as if it is not experiencing anything disturbing from the medium, it is just moving and without any change.

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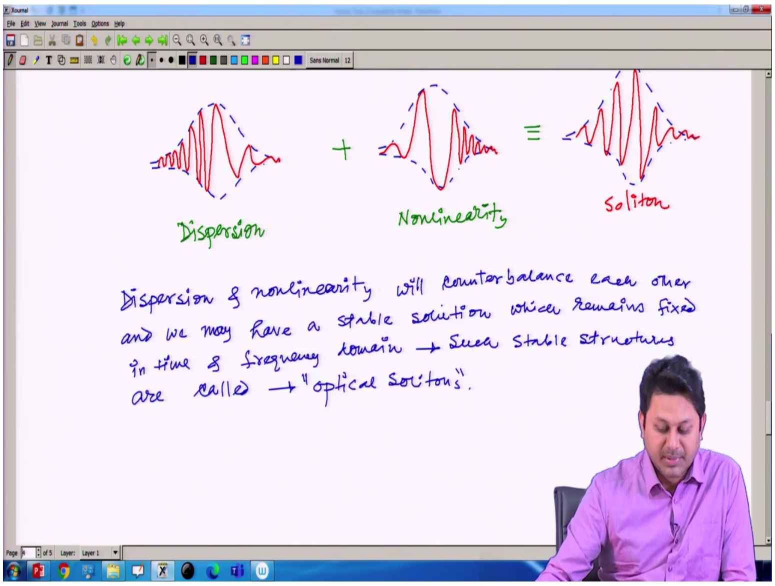

So, dispersion; so, dispersion and nonlinearity will counter balance each other and we may have and as a consequence and as a consequence we may have a stable solution or structure which remain which remains fixed in time and frequency domain such solutions or such stable structures are called optical solitons.

So, dispersion and nonlinearity we find they counter balance each other and there is a possibility that they counter balance each other in a correct way so, that at the output we get some very stable pulse which we called the optical solitons. Well now, we like to solve this this is a qualitative understanding how the optical soliton is formed now quantitatively we try to understand by solving the non-linear Schrodinger equation directly.

Since, it is coming from this non-linear Schrodinger equation so; that means, we have a stable solution for that. So, we need to solve this differential equation and when we have a solution for that then we understand that what should be the shape of this envelope and that is basically the shape of the optical soliton.

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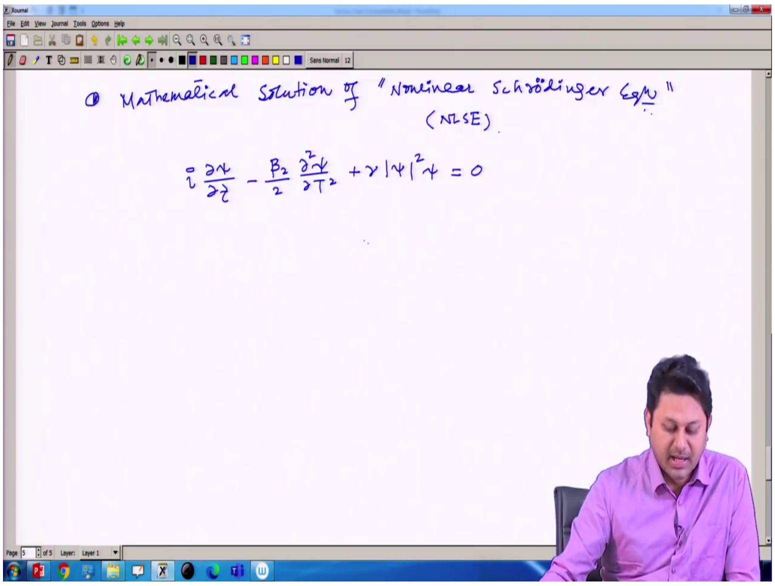

So, next what we do? We should solve this. So, the mathematical solution of non-linear equation. So, the equation let me write once again, this is the equation we have. So, the solution exist a stable solution exist when this value of beta 2 the numeric sign of beta 2 is negative; that means, in anomalous dispersion then only this compensation is meaningful otherwise not.

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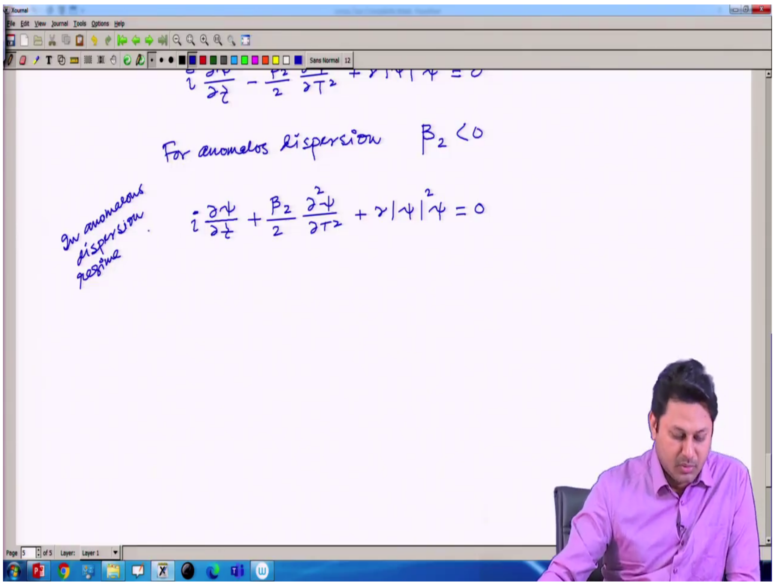

So, for anomalous dispersion we have beta 2 less than 0. So, I put this concept here the beta 2 less than 0 means, I put the value of beta 2 and then this sign. So, it becomes simply i del psi plus beta 2 divided by 2 this is this equation is for anomalous dispersion. So, when the optical pulse is propagating in a dispersive medium not only that this dispersion medium should be anomalous in nature. So, equation becomes. So, this in anomalous dispersion regime, this equation we have.

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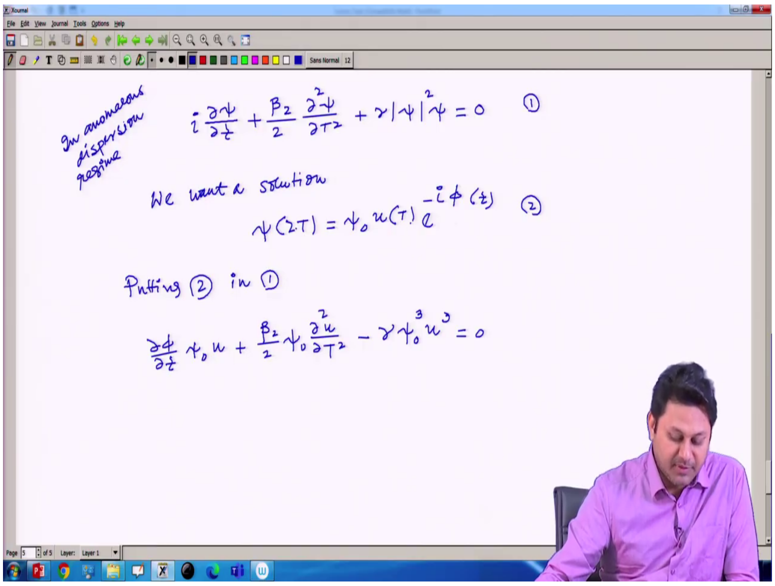

Now, we work for a solution, we want a solution of the form which is psi in this this way which is a function of z T that should be psi 0 one amplitude term and then we should have one envelope and then we have a phase like this. So, we have want to have a solution in this form and when we try to unders when we try to find this kind of form then every time we can put this into the equation, that we did in the last class also to find out the self phase solution of the self phase modulation equation.

Here we are doing the same thing. So, we have a form like this want a solution like this, we have amplitude and then and then we have a envelope and then a phase and I put this to this equations. So, let us put some name of the equation 1 and this is 2. So, putting 2 in 1 we have this. So, this I put there and make all the derivatives.

So, we have del phi, del z psi 0 u plus beta 2 2 psi 0 del u del T square because u is a function explicit function of T, gamma psi 0 cube u cube is equal to 0. So, here you can see phi is a function of z and u is a function of T. So, these two functions are some sort of separation of variable method I am using and I when I put, I am getting this one.

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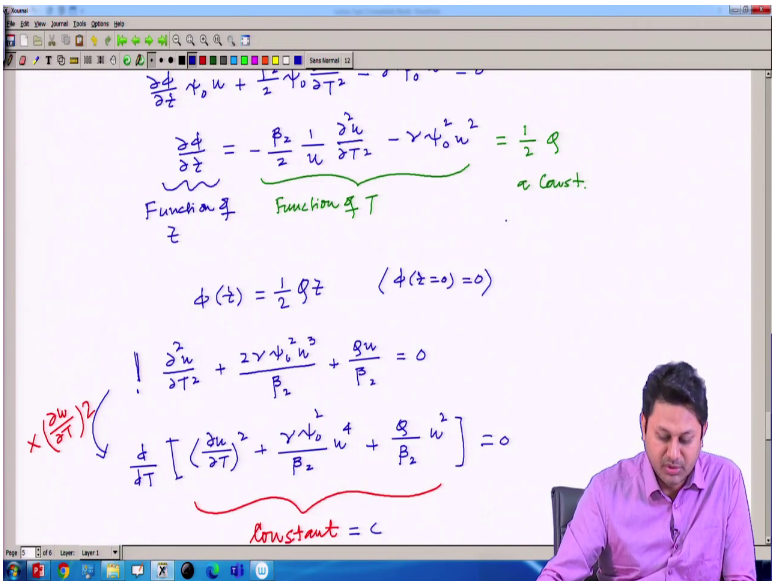

Now, I can have this equation in this way. Using the concept of separation of variable, I gather all the terms related to u on one side all the term related to phi in other side, this is a function of z and this is function of T and when these the entire term rather these entire term. And that should be equal to certain constant because I put this constant like this half Q a constant. I put half because of some reason we will find this reason soon.

Then the phase equation becomes phi z, it will be simply half of Q ok let me define it let me write it properly. It is Q multiplied by z I make a integration of that and I make half Q z assuming that. So, there is a boundary condition assuming that this equal to 0 then I have a straight forward solution for phi

What about the other term? I have this one because this is beta 2. So, I divide everything with respect to beta 2 and multiply u. So, it should be minus of. So, every term will be plus if I put this thing here and take the negative sign out of the picture, then it should be simply d 2 u, d T square equal to ah plus 2 of gamma of psi 0 u cube divided by beta 2 and then plus Q multiplied by u divided by 2 of ah this 2 will cancel out because of these two.

So, we will going to have this one as my envelope equation. So, this is the phase equation I have, here we have the envelope equation this equation by the way I can write in this this fashion. So, it should be del del T and then if I write this as del u del T square of that plus gamma square divided by beta 2 multiplied by e to the power 4 and then I have plus Q divided by beta 2 and then u square equal to 0.

I just multiply the entire term with. So, I multiply this entire term with del u del T and if I multiply this del u del T, then this derivative I can write this term in this way. So, if you make a derivative with respect to T it should be 2 of del u del t. So, 1 1 2 term will be here. So, multiply 2 of this term.

So, one term you will get 2 of del u del T del 2 u del T square multiplied by del u del T and so, on the rest of the term like that. So, here we have a multiplication. So, I will going to get these simplify this differential equation to this one just multiplying 2 del u del T. This is a trick we always use actually to solving the differential second order differential equation.

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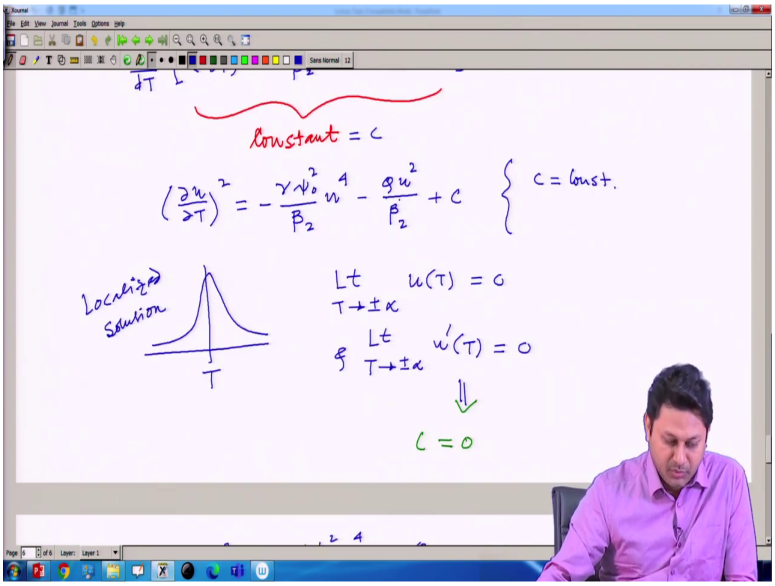

So, now this thing is constant when it is 0; that means, this quantity again become a constant. So, I can have an equation like del u del T whole square is equal to minus of gamma square divided by beta 2, u to the power 4, then minus of Q u square divided by beta 2 and the constant term plus C.

So, this constant one can evaluate by putting certain boundary condition of A. So, C is a constant let us write. So, C is a constant. So, let us put some kind of boundary solution. So, we required a localized solution. So, localized solution means in this shape suppose I want this shape of the solution. So, this is called the localized solution.

So, in order to get the localized solution the boundary condition should be the limit because this is distributed over T, the limit T tends to plus minus infinity then u which is a function of big T has to be 0 because this portion if I go plus minus infinity, this trailing part of this pulse that has to be 0 and also the derivative should vanish.

So, that makes that this T tends to infinity 0 means C equal to 0. So, this condition readily gives us the value of C. So, the C will be 0 with this boundary condition and when we have C equal to 0 my differential equation becomes something like this.

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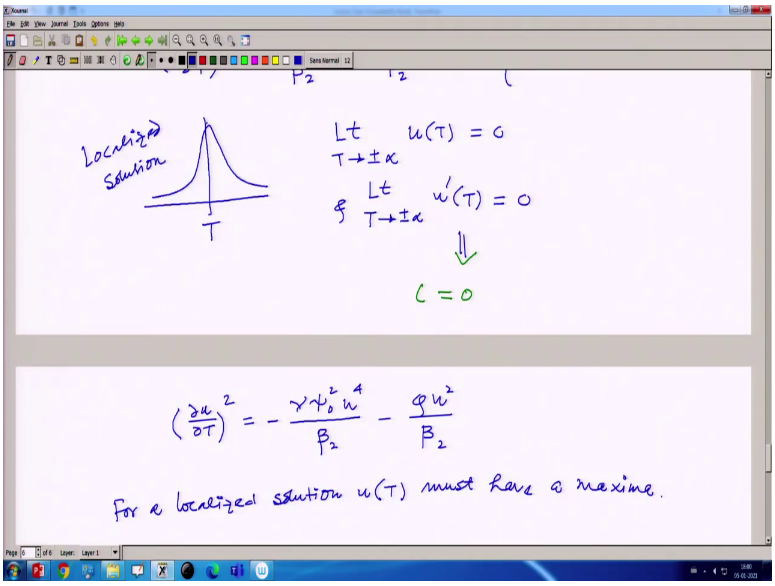

We have del u del T square which is equal to minus of psi u square u to the power of 4 divided by beta 2 minus Q of what was there? Q of u square divided by beta 2 C 0. So, now for a localized for a localized solution u T must have u T must have a maxima. So, that is a very important statement here.

And we already show the form of the solution we want and this form of the solution suggests that this pulse should be a bell shaped pulse and when it is a bell shaped pulse that means I should have a maxima somewhere and for this case it is having a maximum here.

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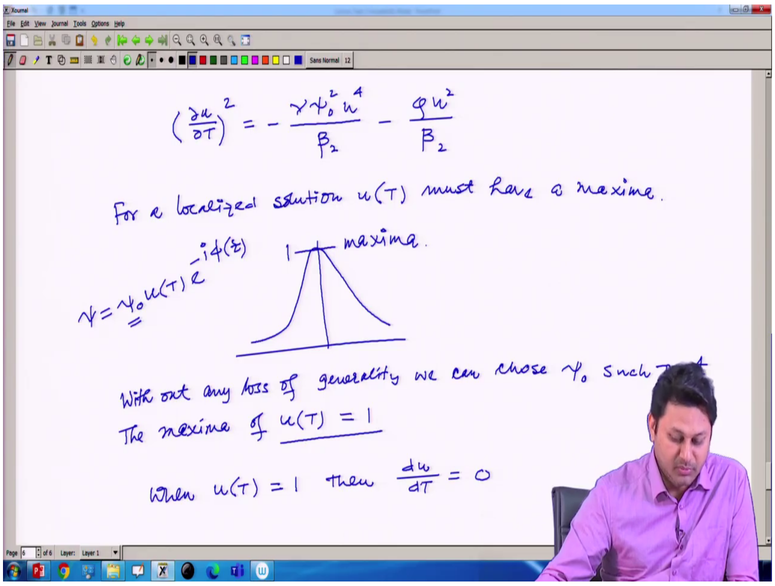

So, this is the point of maxima and without loss of generality, we can take psi 0 which is a maxima. So, in the solution in the state of the solution we have this. So, let me go back to this solution. So, what was the original form? So, psi we want to take as psi 0, u function of T e to the power of minus i phi z that was the form of the solution I wanted to take.

Now, here we had a maxima of the pulse and when we have a maxima of the pulse. So, I can fix my psi in such a way that this u T can be 1. So, the maxima of this pulse can be 1. So, we can write without any loss of generality we can choose psi 0 such that the maxima of u T is equal to 1. So, I choose these equal to 1.

So, here please note that Q is still unknown. So, our total emphasis should be to find Q otherwise I we cannot solve this properly so; that means, when u T is equal to 1, then we have d u d T equal to 0. So, from this equation at the maxima when this is equal to 1 we put this is 0 and when it is 0 we should have we should have 0 is equal to minus of psi this square divided by beta 2 minus Q divided by beta 2.

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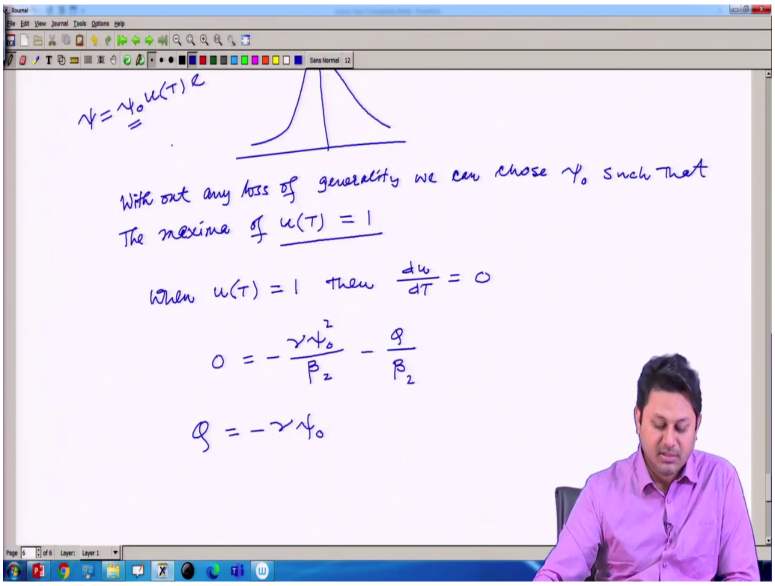

So, from here we can have the value of Q and my Q is minus of gamma psi 0. So, I find my Q as minus of gamma psi 0 and then I will going to put this here in this equation to find out what is the ultimate solution, because now my Q is known. Now my Q is known and this Q is in form of gamma and psi 0 which is known and then we try to find out what should be the solution of this differential equation this one.

So, today I do not have that much of time to complete this calculation. So, in the last in the next class which supposed to be the last class of this course, we will solve this differential equation and try to find out the form of optical soliton. So, thank you for your attention and see you in the next class for to understand what happened after that how the soliton will form. So, see you in the next class and.

Thank you.