Physics of Linear and Non-Linear Optical Waveguides

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Module - 05
Nonlinear Fiber Optics
Lecture - 58
Pulse Propagation in Nonlinear Dispersive Waveguide (Contd.)

Hello student, to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 58 and today we will going to study the Pulse Propagation in Non-Linear Dispersive Waveguide which we started in the last class. So, this class also we will going to continue the calculation and try to understand how the wave will going to behave when it is propagating a non-linear medium which is also dispersive at the same time.

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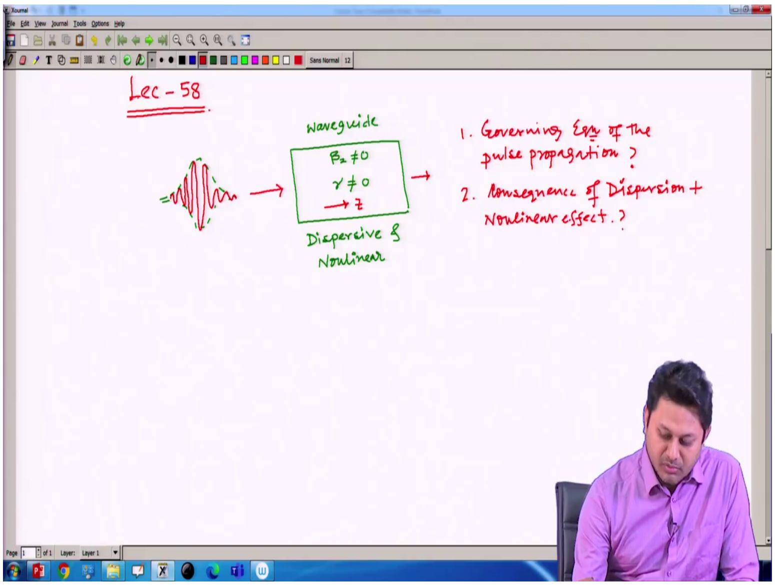

Let us recap what we have done so far, because we did few calculations. So, the original problem is, we have a waveguide like this and this is dispersive beta 2 not equal to 0 and non-linear at the same point gamma is not equal to 0. Now, I launch an waves, a wave is launched in the system. This is a wave packet that is launched, so that is going to propagate in this direction, which is a z direction. And the question is what should happen here? I mean first we want to find out, what is the governing equation?

So, first we find what is the governing equation of the pulse propagation this pulse will going to propagate inside this system, this waveguide, may be optical fiber, may be silicon based waveguide, whatever may be. So, governing equation is important thing that we first derive and then from this governing equation we try to understand, what are the consequences when the dispersion and non-linear both term are there in the equation.

So, consequence of dispersion plus non-linear effect. When both are there what are the consequences? So, these two we try to find out today.

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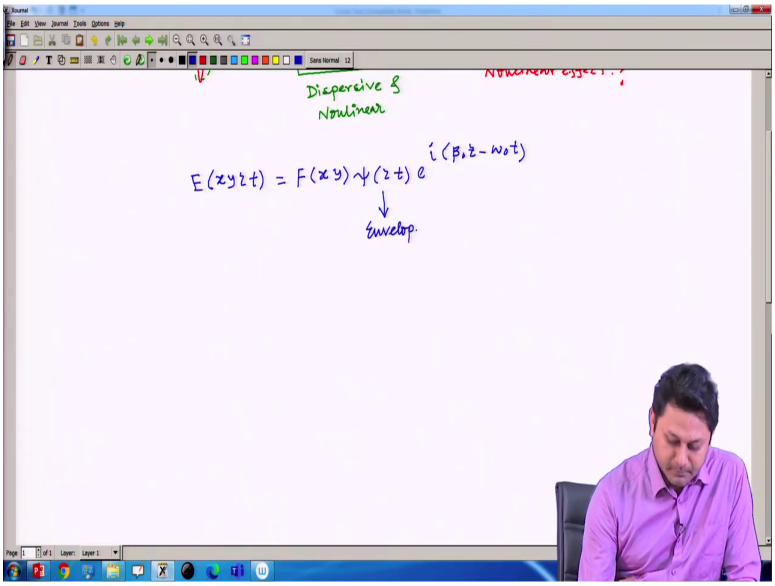

So, the total electric field we consider, whatever the electric field launch here, total electric field which is a function of x y z t, that we consider with this form, F x y. This is a transverse distribution. And then we have psi z t, which is basically the envelope which is changing with respect to time and also there is a z dependency, and the usual propagation term like this. Now, this is envelope.

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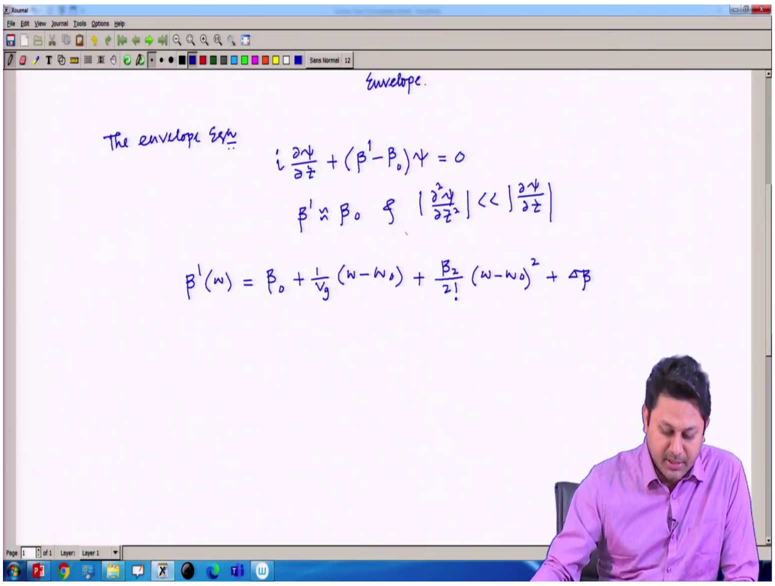

And from the Maxwell's equation, we derive the envelope equation. And, the envelope equation when we derive is come in now becomes like this become; the envelope equation becomes like this, under the consideration when the beta is nearly equal to beta 0. And also the slowly varying approximation was there.

That is why we retain the first order derivative term and omit the second order derivative term, because it is varying in a very slow fashion compared to the first order derivative term. With these two approximation we got this. Then after that what we did, we expand the beta prime as a function of omega using the Taylor series that was beta 0 plus 1 by v g omega minus omega 0 plus beta 2 divided by factorial 2 omega minus omega 0 square and the modification due to non-linearity delta beta.

That was the expansion of beta prime term. and then we put this beta prime term here in this equation.

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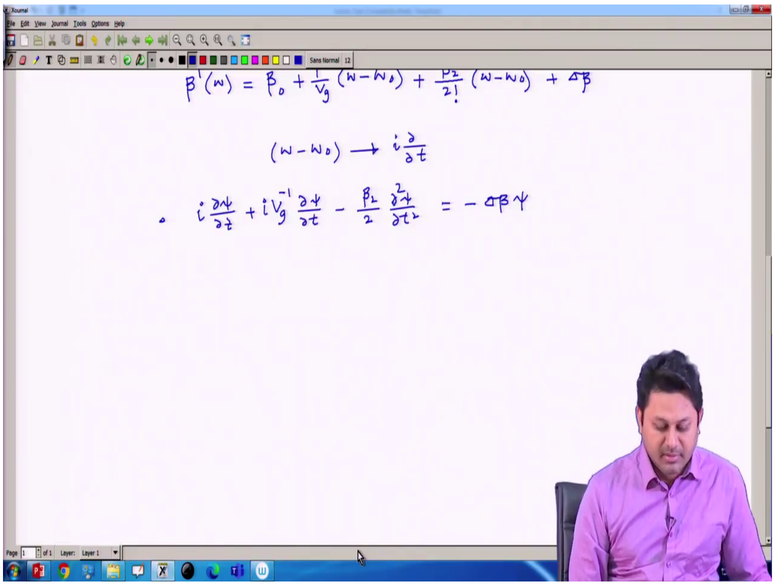

And after putting that we get something, but at the same time we make a very important we use a very important relation between the frequency and time with this form, and we just change this omega minus omega 0 to a operator form, i del del t that we can do. And, after doing that we got something like, 1 by v g or v g inverse, that was the expression.

After having that we make a rescaling or make a transformation, and that transformation was important.

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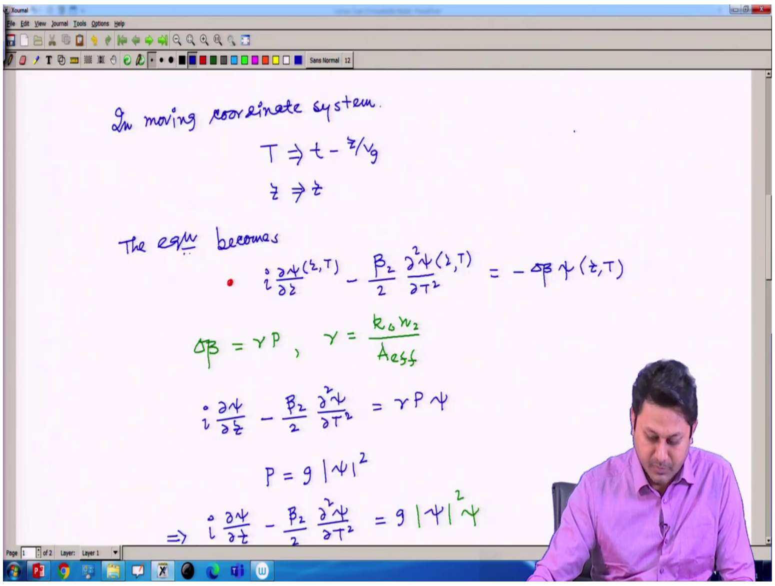

In moving coordinate system, we make a transformation that big T is now t minus z by v g and z. So these two transformation of the coordinates we have. And after this having this two transformation; finally, the equation becomes, this psi is now function of z and big T. So that was the form. In the last class we did up to this.

Now today, we continue with this point. So, what is delta b beta, that we already defined. This is nothing but gamma P. And what is gamma? Gamma is equal to k 0 into divided by effective area that was my gamma. So, putting this beta here, whatever the expression we have for beta, we can write this equation as.

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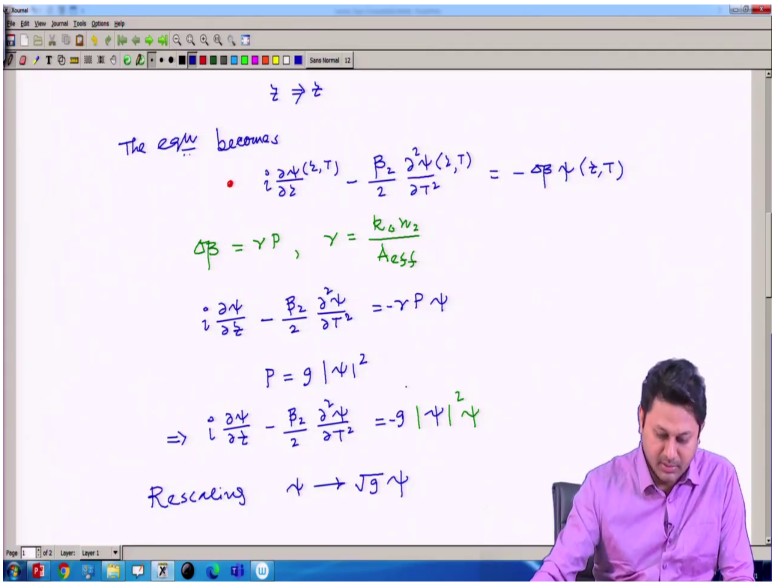

Now this power I can write in terms of psi. So, power is proportional to the mod of, so this proportionality constant I I can have like g, this we use this earlier also. This concept of the relation between P and mod of psi square. Well so, the equation then the equation becomes simply.

Now, we can rescale this equation. So this equation we can rescale. So that also we did earlier; so rescaling. So, how you rescale? Psi, I will write a new psi which is root over of g multiplied by psi. So, I multiply the entire equation, whatever the equation we have with root over g and then root over g multiplied by psi whatever we have, whenever we have root over g multiplied by psi I just replace this to psi. This is a straight forward rescaling.

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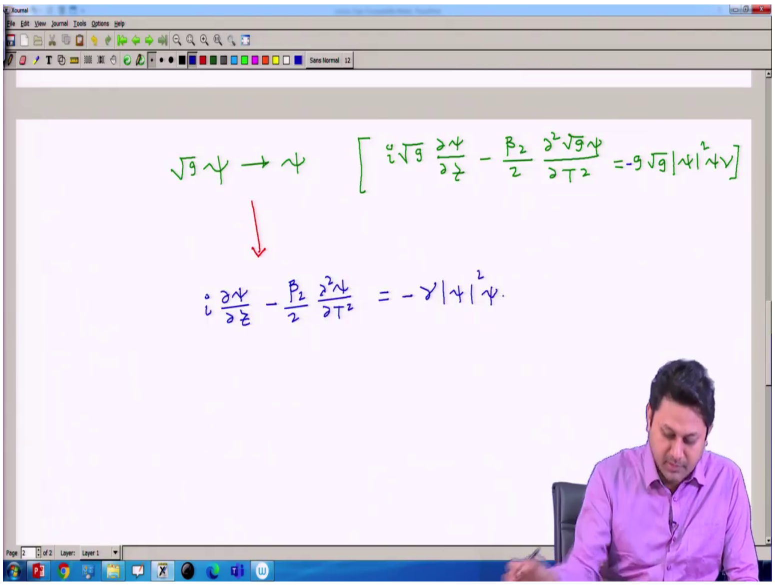

And when we make this rescaling, that root over of g psi goes to psi, and I multiply every this equation with all. So, let me write down what we basically have. So, I write down the equation here. So, I multiply with root over g, all the equations, so it should be like this.

This is the proportionality constant. So I can put in inside this derivative as well. So, I can have this, and here we have g root over of g and then P I can write this P as mod of psi square and psi, I make a small mistake here, I need to change it. So, when I write this equation, this P should be mod of psi square. So, let me write properly.

So, this equation will be g then mod of psi square and psi. So, that is the equation I am having here. And now, I make it this rescaling and after which this rescaling we have the equation which is i let me write in different colour, i del psi del z minus beta 2 by 2 del 2 psi del t square, and this part, this part I have is equal to I am. So that there is a minus sign I am missing. So I should put a minus here, because this is minus delta b. So, a minus sign should be here, a minus sign should be here, and a minus sign should be here.

So, this is minus of gamma. So, also gamma, so here I also make some mistake here. So gamma should be here, a non-linear coefficient gamma, also some gamma here. So, let me erase this for part and put a gamma here. Well, now it looks ok. And now after making the rescaling, I have, let me change the colour this. So, we put everything in the one side.

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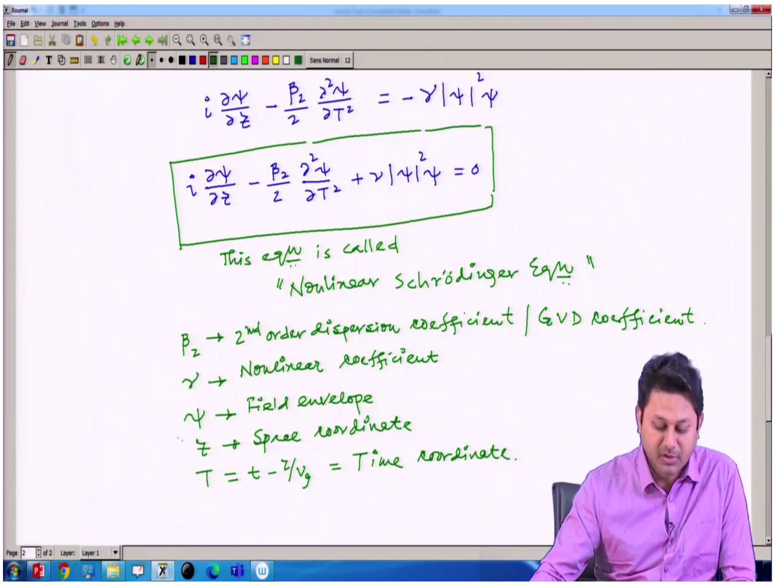

Then I have, and this equation is a very well known equation in non-linear pulse propagation problems and this equation is called the non-linear Schroedinger equation. There is a specific name for this equation, this equation called non-linear Schrodinger equation.

So, this non-linear Schrodinger equation is a governing equation of the pulse. That, if we have beta 2 which is a dispersion coefficient so let me write down one so one by one, what are the terms here. So, beta 2 here is the 2nd order dispersion coefficient or GVD coefficient; group velocity dispersion coefficient.

Gamma is a non-linear coefficient. Then psi which is the field envelope, then z is a space coordinate and big T which is t minus z by v g in a moving frame is a time coordinate. So, these are the components in the non-linear Schrodinger equation. And now, we try to understand the importance of the non-linear Schrodinger equation. And we basically, calculate the dispersion and a effect of nonlinearity individually in our previous classes. So, let me go back to that part once again, what we did there.

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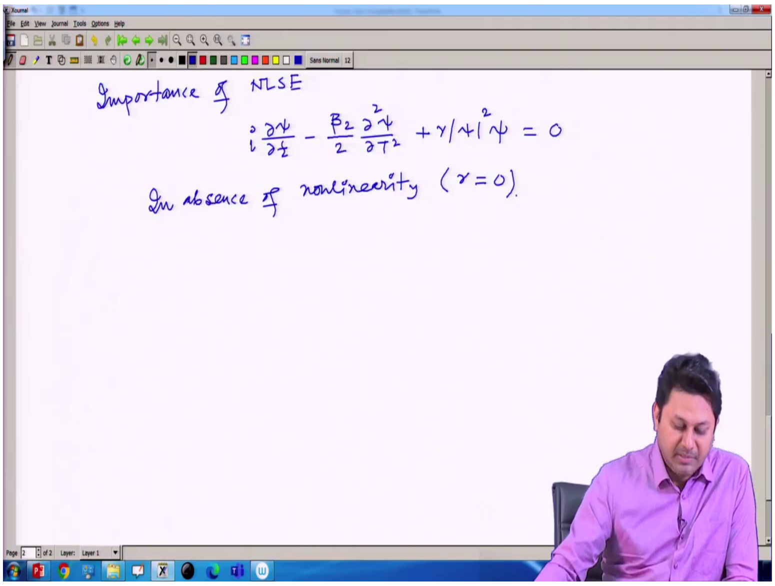

So, the importance non-linear Schrodinger equation in short. So, this I can write in short as non-linear Schrodinger NLSE. So, the importance of non-linear Schrodinger equation is, let me first try to understand what happened if, so the equation let me write once again.

I am writing this equation several time so that the student can use to with this notations and this, because this is in general a non-linear differential equation and very important equation. So, I write several times so that it becomes a bit easy for the student to understand all the terms and student can use to this expressions, whatever the expressions we find finally.

So, in absence of non-linearity; that means, when gamma is equal to 0, I simply have the equation.

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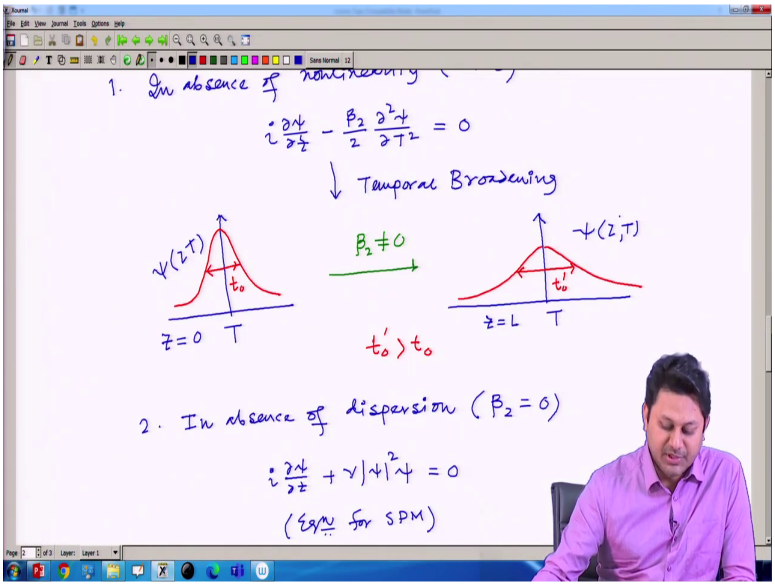

The governing equation of the pulse; that means there is no non-linearity, only dispersion is there. I am having this effects which is only the effect of dispersion. This equation basically tells us how the pulse will change under dispersion, and that we already know. And, as a consequence of this we have temporal broadening.

As a consequence we have temporal broadening. So, if we have an envelope like this having certain width, where the pulse is distributed over t and then when it is passing through some dispersive medium; that means, beta 2 is not equal to 0, a medium having beta 2 group velocity dispersion.

So, there is a change in temporal distribution of the pulse that we know, rather or envelop and we have a broadening of the pulse. This is called the temporal broadening. So, initially we have say t 0 pulse width, after certain distance I one can have a width t 0 prime. This is basically the time coordinate.

Where, t 0 prime is greater than t 0. So this basically tells at temporal broadening, the temporal broadening is happening. So, psi here is a function of z and t, and psi here also a function of z and t, and at initial point when z equal to 0, so this, basically at z equal to 0 and this is at some z equal to L, so this time one can replace because the pulse is moving.

So, the time also be replaced by big T, because now I am calculating everything in a moving reference frame. So, it should be big T here, big T here. So, this is the phenomena we have in the time domain, when beta 2 not equal to 0 ok, in absence of non-linearity that is important one.

What happened in absence of dispersion? 2, in absence of dispersion, that means, now beta 2 is equal to 0. The non-linear Schrodinger equation, what we derive simply going to have a form like this, which we did in the last class in fact. So, this is nothing but the equation of self phase modulation; self phase modulation.

So, under self phase modulation what we find?

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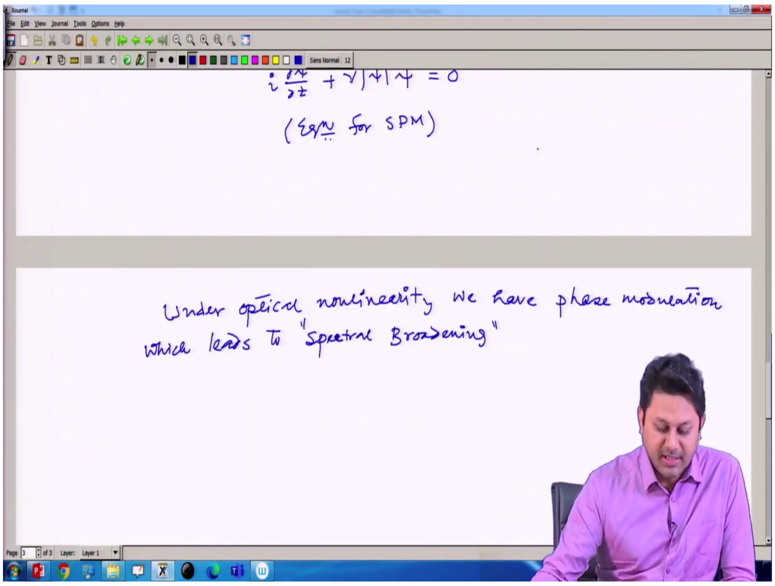

Under optical non-linearity we have phase modulation which leads to spectral broadening. So, the previous case when there was no non-linearity, only dispersion is there, we find there is a broadening and this broadening is; broadening was a temporal broadening.

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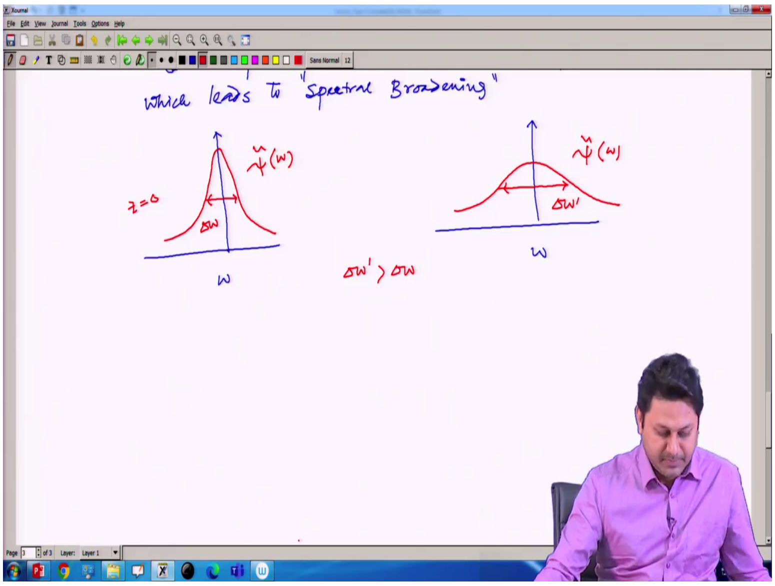

And now, here we find under non-linearity the pulse will going to experience a spectral broadening. That means, if I go to the Fourier domain, so let me draw these coordinates, so this is omega and this is omega. And if this is a input pulse which is Gaussian in shape, so if I make a Fourier transform it has to be a Gaussian.

So, I should have some spectral band width here, this is at say z equal to 0 point and this is a function of frequency, tilde means it is in Fourier domain. When it is propagating, it will now going to broaden, like this the envelope is going to broaden. So, if initially it is say delta omega, this is delta omega prime, so delta omega prime is now greater than delta omega. The spectral, so that basically tells that there is a broadening in spectra this. So, it broadens in frequency domain.

So, there is a there is something which is important here. So, one case we find that there is a temporal broadening and in other case we have a spectral broadening. Now if somebody add then what happened?

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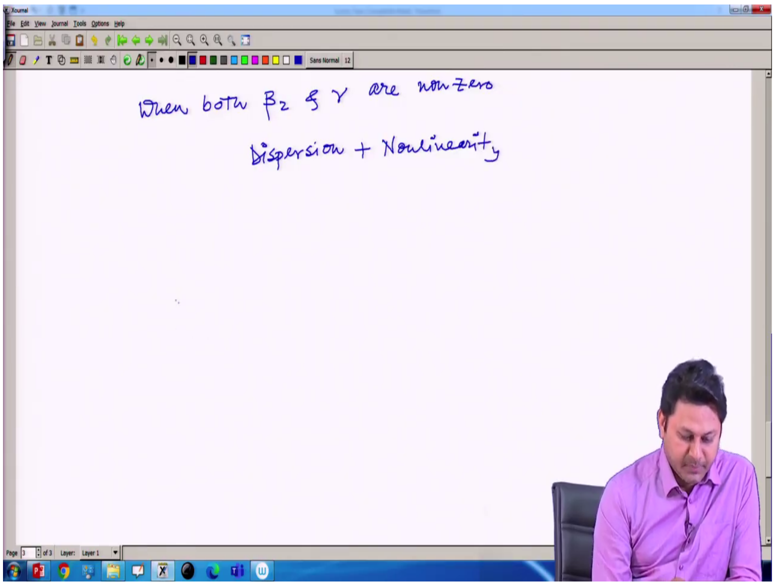

So, when both beta 2 and gamma are non zero, what happened? That means, dispersion plus non-linearity both are there, so both effects are there. So, when both effects are there.

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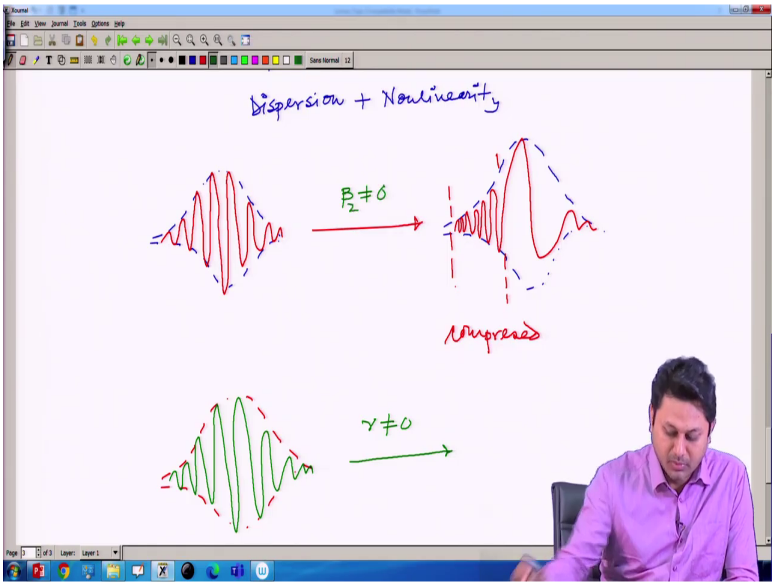

So, let me draw what happened, inside the envelope as well, that is interesting. So, we know, that when there is a dispersion that we done in the earlier classes, so this is the frequency distribution inside the pulse and we had a dispersion and due to dispersion what happened, there was a frequency distribution. So this portion was compressed and then this portion was rarefied. So, this portion was compressed under dispersion.

And what happened under non-linearity? And the non-linearity, so I am drawing the envelope first. So that was the distribution, and the frequency, and this is gamma not equal to 0, and this is beta 2 not equal to 0. And here what happened?

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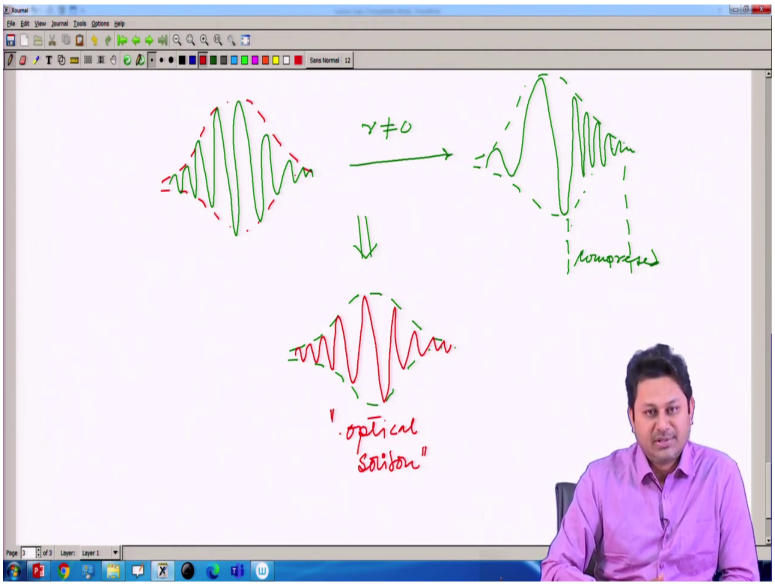

If I draw the envelope there is a frequency distribution, but in opposite way. Now, this portion is compressed. Now, if these two phenomena is adding up, if we add these two, because now beta 2 and non-linearity group velocity dispersion and non-linearity both are there then what happened? I should have, so if both are added then I should have a compensation and due to the compensation we have a structure like this which is called optical soliton.

There will be no change, there will be no change in time domain and frequency domain at all. So, in both time and frequency domain we have a stable structure, and this stable structure is called the optical soliton. So, till now we arrive the concept of optical soliton. So, today I will going to end this. In the next class also we start from this point and try to understand, what is the consequence of optical soliton why it is so important, etcetera. So, Thank you for your attention and see you in the next class.