Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 57
Pulse Propagation in Nonlinear Dispersive Waveguide

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 57. In the last class, we tried to understand what happened on the pulse is propagating in a non-linear waveguide. So, there was no dispersion. Now, today we will going to understand what happen when the Pulse is Propagating in a Non-Linear and Dispersive Waveguide.

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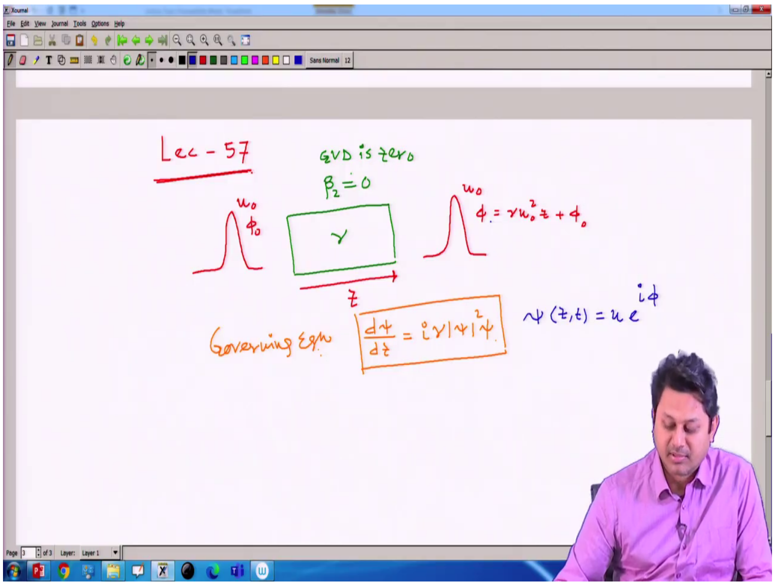

So, let us start with the concept. So, if you remember that in the last class, we tried to understand that if we have a waveguide defined by gamma and also I need to mention that there was no dispersion. So, group velocity dispersion is not there. So, beta 2 which is the defined which is basically, defined the group velocity dispersion is 0. So, group velocity dispersion is 0.

Under that such condition when we launch an optical pulse, it will experience this non-linearity. This is the distance over Z it is propagating. And we can have a pulse like this with a modification of phase. So, amplitude will not going to modify, but the phase phi will be if there is a initial phase we have phi, as gamma u 0 square, and then Z and then phi 0. So, there is a linear shift of phase when the pulse is moving in this kind of systems.

Now, this is the thing we have done in the previous class. So, this class and also we had a governing equation for that. So, let me also write down the governing equation for that. So, the governing equation was d psi dZ is equal to i of gamma of psi square psi, that was the governing equation.

And when we solve this governing equation with the consideration that my psi should be of the form psi Z was considered as this form, which is u e to the power of i phi, where u and phi both are function of Z and t.

Then, that was the information we have, that there will be no change in the at the envelope part and there will be a change in the phase there will be linear change.

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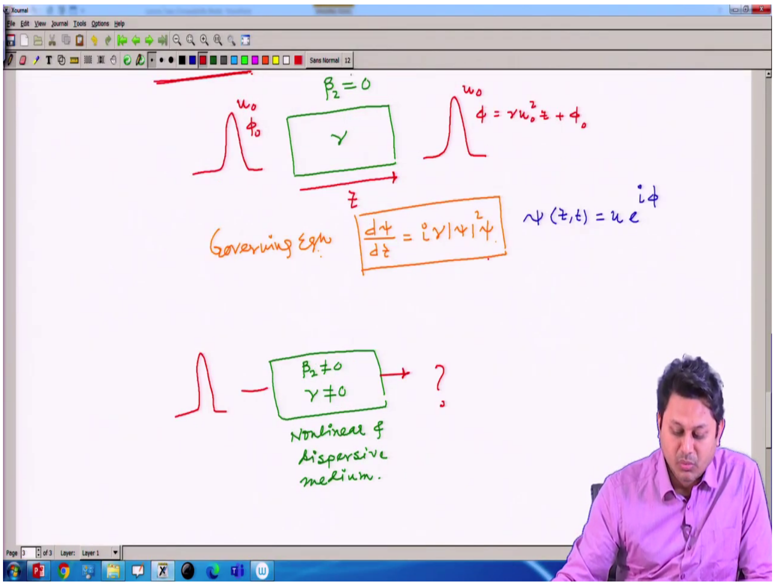

Now, today, in today's class let us modify this problem. The modification is I now I am having a waveguide, where beta 2 is not equal to 0 and gamma is as usual not equal to 0. So, the medium is non-linear and dispersive. So, now, beta 2 is not equal to 0. So, group velocity dispersion is no more 0.

And again, we try to understand what happen when the optical pulse is moving in such mediums in the combine effect of dispersion and nonlinearity which is very very interesting by the way. So, it is moving like that, and when it is moving what should we have in the output that is something we need to figure out.

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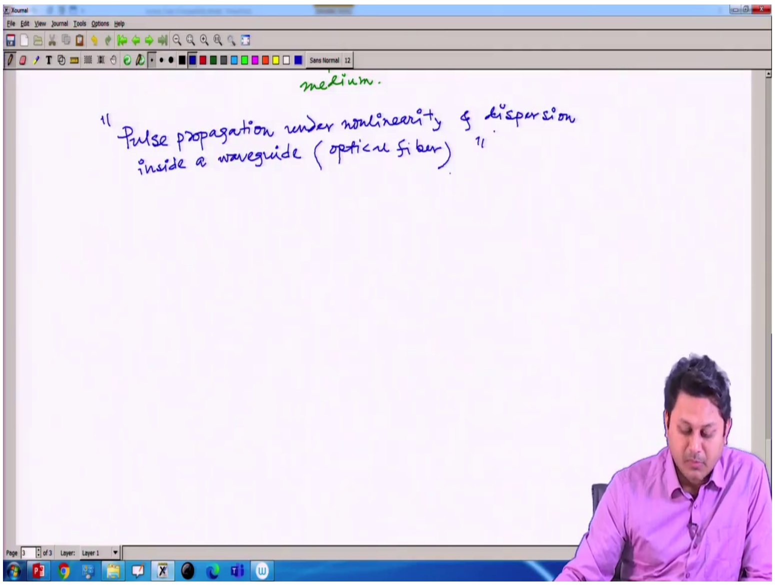

Well, let us try, let us start with the; so, our topic is pulse propagation under non-linearity and dispersion inside a waveguide which may be optical fiber. So, this is the topic today we will going to, we will going to study that what should be the governing equation first and how to solve it.

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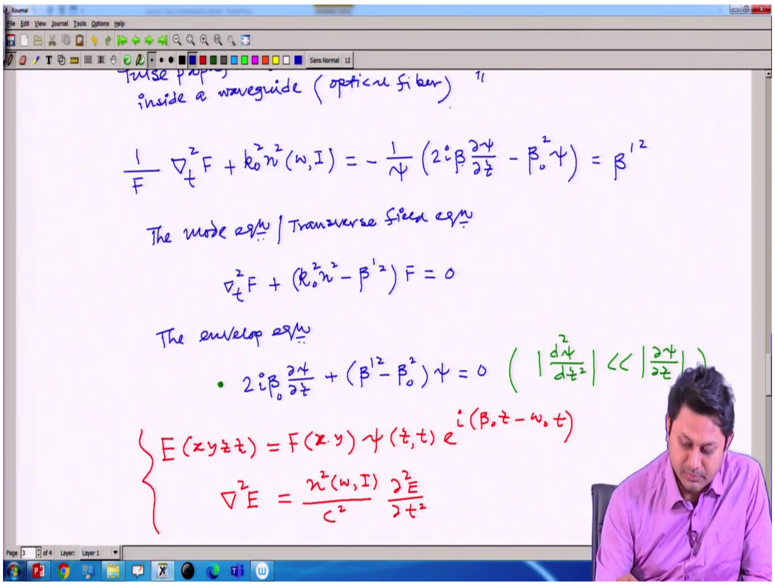

So, if you remember in the earlier classes. So, I am not going to derive the entire thing which is pretty same. We got an equation like this, the field equation and the envelope equation. And after making a separation of variable, we find from non-linear from the Maxwell's equations, we got this.

This derivation was done when we try to derive the governing equation of the pulse under non-linearity only non-linearity. So, I am writing down the same thing once again. So, that was after doing the separation of variable that was the expression. I suggest you to go back to your class note and check. I will not going to derive, because the derivation is identical; this. So, this part is the field equation.

So, the field equation or the mode equation, the mode equation or the transverse field equation whatever is this from which we can have the idea how the mode will going to be distributed of an optical field. When the optical field is made of the, ok; let me write down the optical field. So, let me first write it. And this is the modification of the propagation constant under non-linearity. So, that was the field equation.

An envelope equation, I can write down as this. What was the electric field, total electric field, form of the total electric field? It was something like E which was a function of x, y, z, t, one can divide this electric field as the transverse component which we call the mode distributed like this.

And a envelope which is changing with respect to Z and t, the envelope part, temporal envelope and then E to the power of i the propagation constant beta 0 Z minus omega 0 square t that was sorry; omega 0 t. So, that was the field that was the field. And then that field should follow the Maxwell's equation, and it was simply n square that was the structure. And from that we can find out what was the x; what was the equation for the transverse field distribution which is F and what is the envelope equation.

So, our interest is here, the envelope equation. Try to find out how the pulse envelope is going to move under the non-linearity and dispersion, and that is why this envelope equation is our interest here. And here also I need to mention one thing that the slowly varying approximation.

When we have the first order derivative we remove the second order derivative, we neglect the second order derivative, because the second order derivative of psi with respect to the propagation Z is much much less than this quantity, mod of that. So, this is called the slowly varying envelope approximation. That means, the envelope is varying slowly.

So, that is why, the second order variation is rather very slow, so we can neglect. So, almost this is not varying, but the first order variation is important. So, that we include in the equation and we will not going to take the second order derivative and we just omit that.

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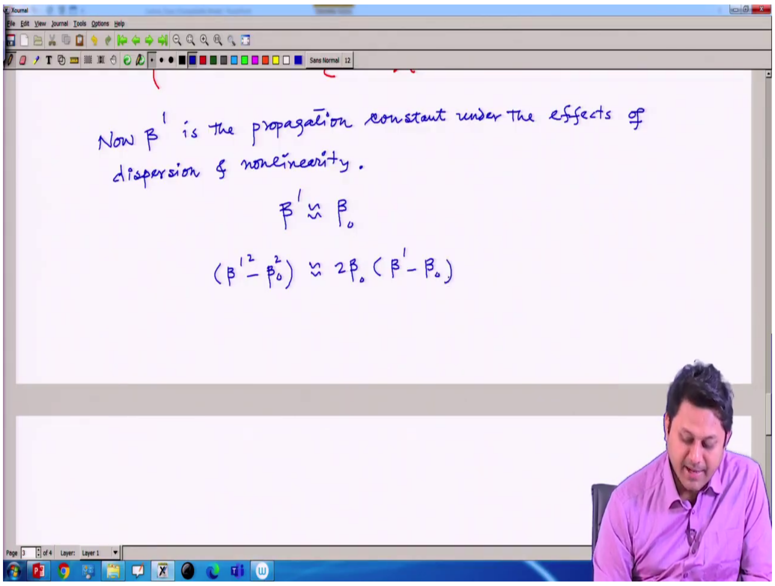

Well, that was the structure we have. And here we should mention that this beta prime, so, now, this beta prime is a propagation constant that contain. So, rather this is the propagation constant under the effect of dispersion and non-linearity under the effect of, under the effects of dispersion and non-linearity, under the effect of dispersion and non-linearity.

So, the effect of dispersion and non-linearity, if the dispersion and non-linearity effects are relatively small, then we can approximate that there is a change of propagation constant, but this change is small. And with this consideration we can have this, this beta 1 square minus beta square simply as; so, this approximation we also took in the previous calculations.

So, beta 1 square minus beta 0 square can be written nearly equal to as 2, 2. So, here we have a beta 0. So, 2 beta 0, 2 beta 0 multiplied by beta prime minus beta 0, that approximation we can take.

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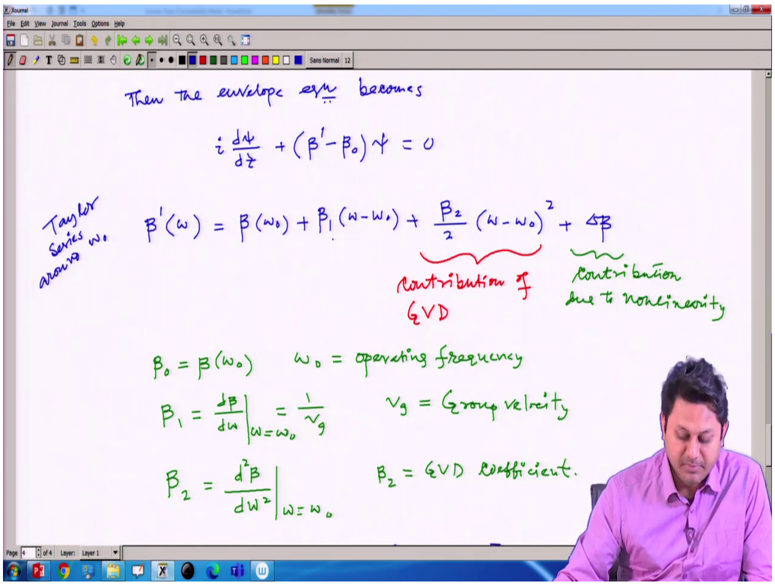

And after putting this approximation this envelope equation, the envelope equation, then the envelope equation leads to becomes i del Z plus beta prime minus beta 0 psi equal to 0. This is again a not a new equation, this equation already we derive. Only thing now, we need to concentrate about what should be the form of this beta prime. Previously, beta prime was considered only the non-linear effect. Now, beta prime is considered the dispersion effect and non-linear effect as well.

Now, this beta prime which is a function of omega can be expand under Taylor series, and I can write this as beta at some frequency, around the frequency omega 0 if I make the Taylor series. So, the Taylor series around the frequency omega 0. So, plus beta 1 omega minus omega 0 that plus a very very important term. The higher order effect now I am adding, beta 2 divided by 2, omega minus omega 0 square and then the modification due to the non-linearity. This is the contribution of the non-linearity.

So, now, I am adding the contribution of the dispersion which is this one, this is the contribution of group velocity dispersion. If it is beta 2 is not equal to 0, then I we have to take this term and this one this green one is the contribution due to non-linearity. So, to contribution now, I am incorporating through this beta prime which is the modification of the propagation constant and the non-linearity dispersion, that is why I make a Taylor series expansion.

I am taking at least the first order effect of beta which is beta 2, which basically, the group velocity dispersion term. And delta beta which is the contribution of the non-linearity that is always there under non-linear effect. So, physically. So, what is beta 0 here? Which is beta at omega 0 that we know; that is the one that is one thing.

This is when omega 0 is operating wavelength, omega 0 is operating wavelength, operating frequency rather. Operating frequency means, at the frequency at which the optical pulse is launched. Then, beta 1, the term beta 1 is del beta del omega at omega equal to 0, which is nothing but the inverse of group velocity term.

Again, this is these things is not very new, we have used these things. The concept of inverse of group velocity in previous classes also. This is v g is, where v g is the group velocity and beta 2 which is a dispersion term which is d 2 beta d omega square at omega equal to, here I write omega equal to 0 it is not 2, it is omega equal to omega 0.

This is beta 2 is a group velocity coefficient, group velocity dispersion coefficient. So, inside the beta we have all. Now, these things I go to insert in this envelope equation, whatever the envelope equation I have.

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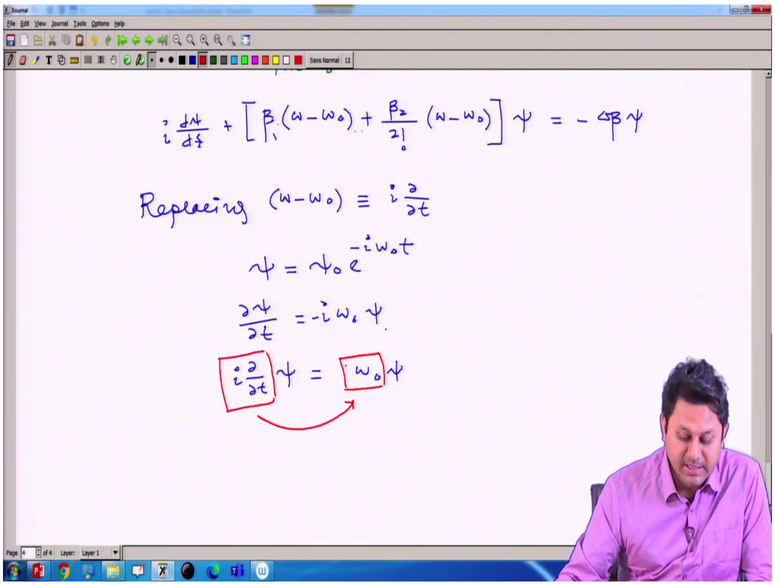

So, the envelope equation becomes i del psi del Z plus beta 1 omega minus omega 0 plus beta 2 divided by factorial 2, then omega minus omega 0 bracket close psi that is equal to minus of delta beta the term that we have and then psi. So, I just replace this the explicit form of beta prime here.

So, beta 0 will going to cancel out I have the first term beta 1, which is inverse group inverse group velocity beta 2, which is the group velocity dispersion coefficient and delta beta the contribution due to non-linearity. If we have this and this is a very very important now, thing now, I am going to do here is replacing this term omega minus omega 0 to an operator i del del t. So, that is the Fourier related to Fourier transform.

So, whatever we have whenever we have the term omega minus omega 0 e to the power i omega, we can always write equivalently at this in the operator form. So, this is an operator.

For example, I am just giving you an example that psi is say psi 0 e to the power of i omega 0 t.

So, if I make both the side and the derivative del psi with respect to t like this, then we have I omega then the entire term, psi 0 e to the power i omega 0 t. So, i psi, now, if I multiply with if I now multiply with both the side with i, then it should be i and then; so, I should write a negative sign normally we write beta Z minus omega t. So, that is why. So, it should be minus of these things.

So, I can have i del del t that will operate on psi is equal to is equal to minus of i; sorry, if I multiply i this this things will not be there minus i. So, it should be simply omega 0 psi. So, like an eigenvalue equation. So, here the important thing is whenever I have these kind of term, in terms of omega, I can replace these things.

So, these two things are equivalent. So, here I am just using the same concept. But only thing to note that this omega minus omega 0 will come, because omega is a omega 0 is a constant, but this is now variable with the variation of frequency how these things are there.

But I can always use this operator, and for the timing let us consider that this is the operator and this operator will going to operate these, and when it is operating over the parameters say psi, I can have an equation and that is important.

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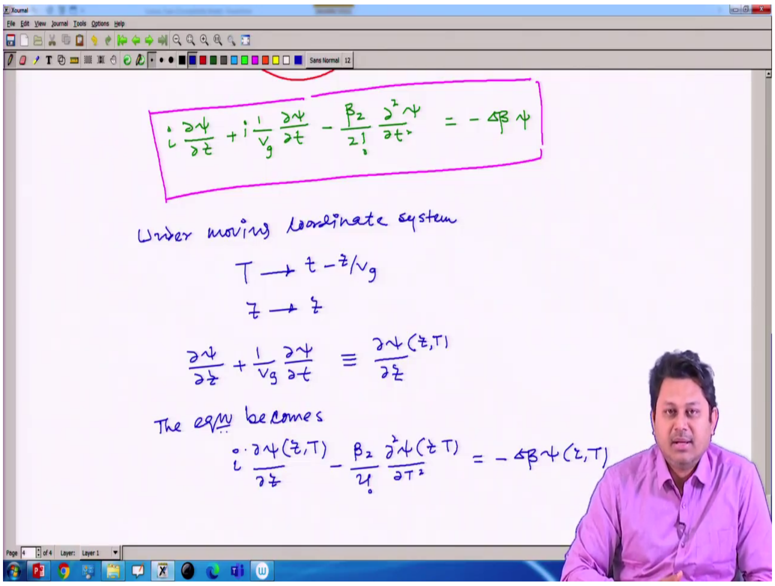

So, now, I have an equation after doing this operation I have i del psi del Z plus i 1 by vg then del psi del t. I just replace this omega minus omega 0 as an operator, i del l tau, then minus of beta 2 divided by factorial 2. Again, omega minus omega 0 square, I will going to replace as the second order derivative. So, it should be minus of this. And then psi and the right hand side I have delta beta psi.

So, this is roughly the equation we have, we will modify this equation in the later classes. But here in today's class we find that an equation the a form like this, which basically tells us how an optical pulse, what is the governing equation of the optical pulse under non-linearity and dispersion. The non-linearity is coming through the term beta 2, delta beta, and beta 2 is our dispersion.

So, I can now, also rescale this equation and there is a coordinate transformation and that we also done in the previous classes. So, under moving coordinate system so, let me just quickly write, under moving, under moving coordinate system I can have a rescaling like; T can be replaced at t Z by v g and Z the space coordinate will remain thing.

So, if we do this transformation, one can write this term del psi del Z plus 1 by v g del psi del t equivalent to del psi, which is a function of now, Z and big t. Now, I am going to the moving coordinate system which is the coordinate that is moving with respect to the pulse with a velocity v then I can absorb this v Z with this coordinate like that.

And now the equation becomes, the equation becomes i del psi dependency will now change to big T, because I am now moving with a reference frame which is moving with the pulse, minus whatever we have beta 2 divided by factorial 2 and del 2 psi. Again is a function of Z and t del t square is equal to minus of delta beta psi, which is a function of again Z and t.

Well, this is the equation now I am having with the moving coordinate system. So, I now, somehow able to derive the equation of motion of a pulse under dispersion and non-linearity. In the next class, we will start from this equation and try to understand what is the consequence of this equation. A

nd then we find that, because of the presence of non-linearity and the presence of dispersion together there should be a balancing act, and you find that the optical pulse that is moving under dispersion experiencing some sort of temporal broadening that we already derive in the previous classes.

Here due to the non-linearity there will be a back effect, there will be a compensation, and the pulse will no longer be expanded in time domain. So, there will be a stable structure called optical solid. And that should be our last topic. So, with this note let me conclude todays class. Thank you for your attention. And see you in the next class for further studies on these equations.

Thank you and good bye.