Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 05
Nonlinear Fiber Optics
Lecture - 56
Pulse Propagation in Nonlinear Waveguide (Cont.)

Hello students to the course of Physics of Linear and Non-linear Optical Waveguides. So, today we have lecture number 56 and in the last class we started with the concept of Pulse Propagation in Nonlinear Waveguide tried to derive few equations and today also we will continue with that topic.

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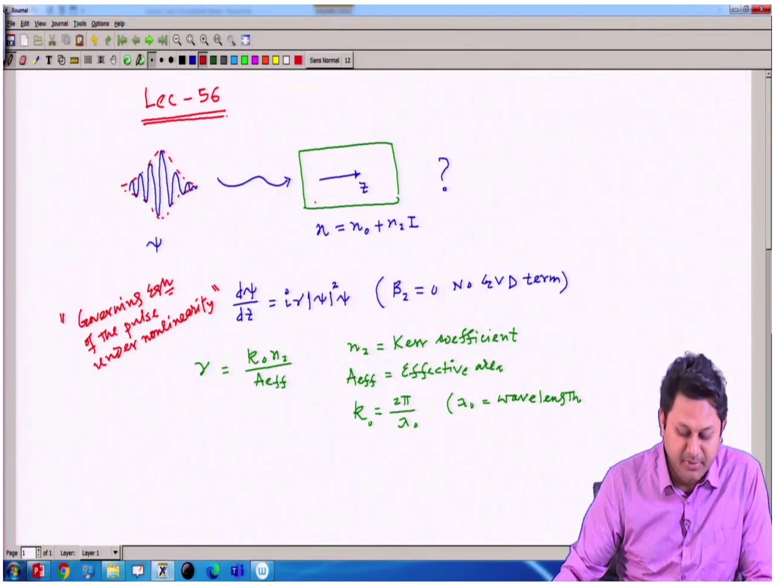

In the last class we tried to understand if we have a non-linear waveguide. Suppose, this is a box which is a non-linear waveguide, where the non-linearity is defined by this equation which is refractive index n is equal to n 0 plus n 2 I, where n 2 is a Kerr coefficient and when the wave is coming through this with a structure with a certain structure say this is the envelope of the wave and it should have some frequency distribution inside this envelope.

So, this is the wave packet that is moving and when it is passing through this non-linear waveguide what should happen here in the output? So, we need to calculate the propagation equation which basically govern this pulse dynamics. And we derive that we derive an equation, this equation was if psi is envelope then the evolution of the psi with respect to Z which is the propagation direction along this. This evolution one can write this as i gamma mod of psi square psi with no group velocity dispersion. So, here beta 2 is 0, no group velocity dispersion term.

So, the effect is only due to the non-linearity and this nonlinearity is coming through this parameter called gamma, gamma is equal to k 0 n 2 divided by A effective where effective A effective is the effective area, n 2 is the Kerr coefficient, A effective is a effective area and k 0 is related to the input wavelength, it is 2 pi by lambda 0 where lambda 0 is the wavelength of the light. In free space the light that is passing through this wave guide.

So, there are few important terms here for example, A effective and we discussed that that A effective if somebody reduced A effective then there is a possibility one can increase gamma this non-linear coefficient if somebody increase the Kerr coefficient then also one can increase this gamma.

So, in this equation if somebody increase the gamma. So, the effect of this quantity in the right hand side will going to increase and due to that what happened to the pulse that we are going to understand in today's class. So, we already have the equation in our hand which is the phase map which is the governing equation of the pulse under non-linearity. So, this is essentially the governing equation. So, this is essentially the governing equation. So, how the pulse will behave that today we are going to understand.

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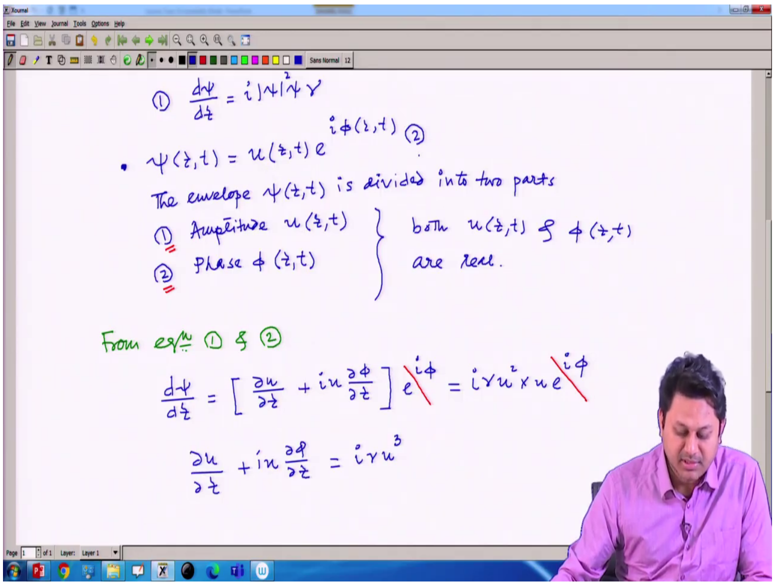

So, let me write down the equation once again, it is d psi d Z is equal to i mod of psi square psi and the coefficient gamma. This is a non-linear kind of differential equation. So, we will going to solve in this way. So, let us assume the solution in this form we assume the solution in this form psi which has a amplitude or envelope kind of term like these and a phase term like this.

So, u z, t to the power i phi, I just consider this psi entire wave function in this particular form which I have which is having one amplitude term and one phase term. So, the envelope psi z, t is divided into two parts, one is amplitude which is also a function of Z and t, I call this as and second one is the phase phi which is also function of Z and t we assume that. Now in this particular form psi is complex, but u and phi both are real.

So, u. So, both u z, t and phi z, t are real; however, psi is overall a complex term. So, now, if I put this solution whatever the solution we assume to this equation, then let us let us put some equation number here. So, this is equation number 1 and this is equation number 2, this is equation number and this one and these two just I mentioned here is amplitude and phase.

So, from equation 1 and equation 2 from equation 1 and 2 I can have so; that means, I just put this equation 2 into equation 1 and try to extract the information about this amplitude and phase if I able to let us see. So, then what happen? d psi d Z will be simply del u del Z plus i u del phi del Z e to the power of i phi this is the left hand side and right hand side it should be i gamma mod of psi square means u square multiplied by psi means u e to the power of i phi and e to the power of i phi should cancel from both the side.

So, this term and this term going to cancel out and eventually we have del u del Z plus i u del phi del Z is equal to i psi u cube. So, this is the equation we have after putting the solution which we assume in this way which is written in equation 2 and put it in equation 1.

After putting that I am having a differential equation this differential equation now slightly modified in terms of u and phi and I am having this equation say equation 3. Now you should note that this equation 3 we have u and phi both are real. So, I can. So, left hand side is a complex term, right hand side is also a complex term. So, when two complex things are equal to each other then the real and imaginary part of this complex equation should be equal to each other.

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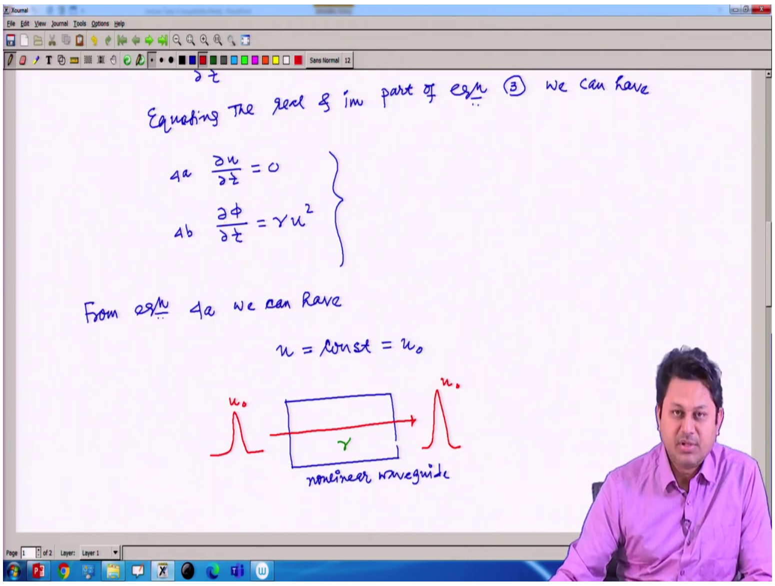

So; that means, equating the real and imaginary part of equation 3, we can have real and imaginary part if I equate then we should have an equation like del u del Z is equal to 0, this is the real part of this side in the left hand side and the right hand side the real part is 0.

So, it is simply 0. On the other hand, the imaginary part is u del psi del Z. So, that should be equal to the right hand side of the imaginary part that is i, then gamma, then u cube. So, if I equate this it should be simply. Del phi del Z is equal to one u will going to cancel out.

So, it should be equal to simply gamma u square. So, two differential equation now I obtained from the single differential equation, which is complex in nature just equating the real and imaginary part of these two ok. So, the first differential equation is straightforward because this is simply du. So, let me write down these two equations and let us put a name this is a 4 a and this is 4 b.

So, from equation 4 a, we can have for equation 4 a, we can have u equal to some constant which is say u 0. So, very very important outcome we have from this first equation which is very very simple that du dZ equal to 0; that means, under the non-linearity gamma, in principle there is no change in the amplitude part. So, the amplitude is constant is u 0.

So; that means, when the pulse is propagating. So, this is the waveguide I am talking about, this is the non-linear waveguide and this envelope is moving here having a amplitude like u 0 and these things when it is moving the non-linearity it will going to experience as gamma, but the temporal part the amplitude part will not go into effect.

So, it should remain u 0 here. So, there is no change in amplitude, but you can see in the phase we should have some kind of change because the differential equation of the phase suggests that it should be multiplied by gamma and u 0. So, this differential equation phi is not a constant anymore. So, from the from equation 4, a we can have a equation simple equation that u is a constant.

So, physically; that means, when the optical pulse is moving in a non-linear waveguide, then amplitude if I do not consider any kind of dispersion if I consider n loss; obviously, the loss is important thing, but for the timing let us considered there is no loss only considered what happened when the non-linear non-linearity is there, the optical nonlinearity is there what should be the fate of this optical pulses.

So, we find the amplitude will not going to change at all. So, this is the amplitude equation. So, what about the phase equation?

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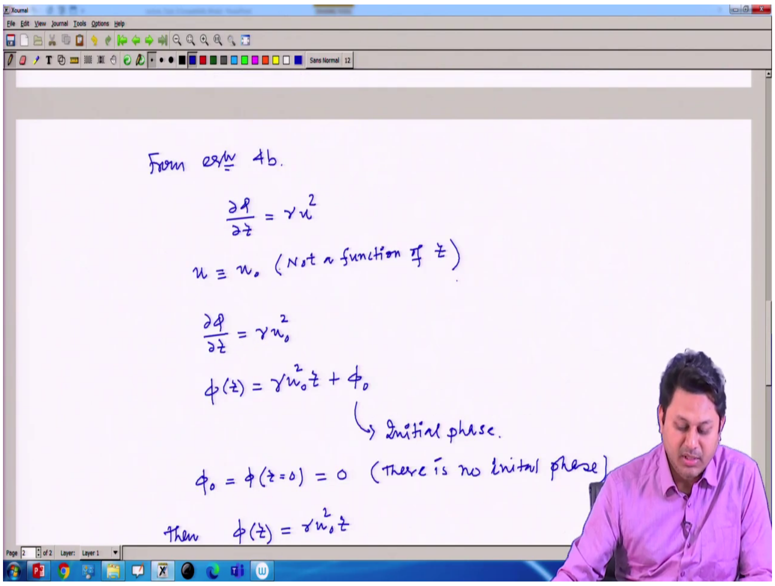

So, from equation 4 b what we have? From equation 4 b we have del phi del Z is equal to gamma and then u square sorry it should be u square. And u we know is a constant this is not a function of Z, that basically I mentioned here it is not a function of Z. So, u is simply u 0 which is constant not a function of Z. So, better to write not a function of Z, then in differential equation u 0 will be simply a constant.

So, I can have I can write del phi del Z is simply equal to gamma u 0 square and from this differential equation I can write phi Z in explicit form which is a function of z; obviously, is equal to gamma u 0 square Z plus integration constant say phi 0. I just need to integrate and when I integrate I need to put integration constant and this integration constant is nothing but the phi 0 which is the initial phase.

So, this is the initial phase. Let us considered this initial phase phi 0 which is equal to phi at Z equal to 0 is 0; that means, there is no initial phase. So, we assume there is no initial phase. So, that is why I can put this constant as 0, then phi Z will be simply as gamma u 0 Z this is the equation of the phase.

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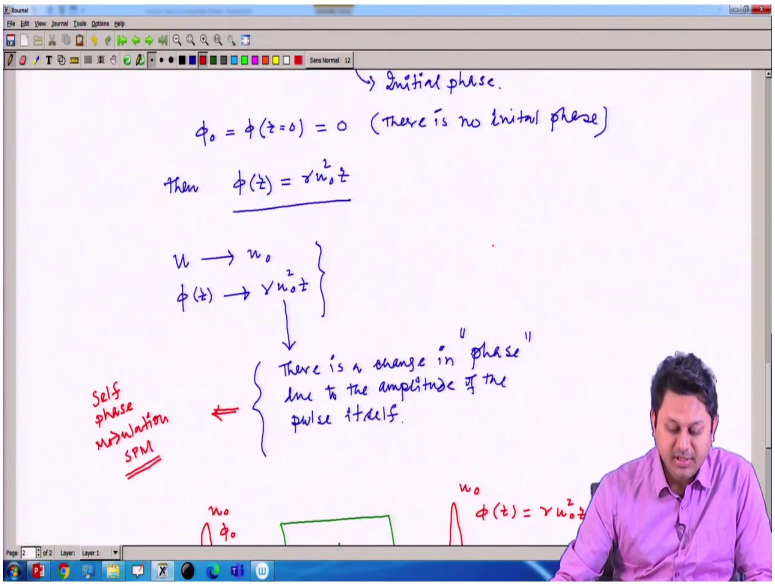

So, we have two very important information and that is u is u 0, there is no change and phi which is a phase we are going to change as gamma u 0 square and Z ok. I should put u 0 square here u 0 square Z so; that means, that there is a change in phase you can see there is a change in phase and this change is guided by the amplitude of the pulse itself.

So, there is a change in phase, there is a change in phase due to non-linearity because the gamma term is present here if gamma is equal to 0; that means, if there is no non-linearity you can see there is no change in phi at all whatever the initial phi in this case initial phi is 0.

So, that there is no dependency of Z at all of the phi; that means, the initial phase is phi. So, it will remain the initial phase is phi 0, and it will remain phi 0 throughout the preparation so; that means, there is no change in amplitude no change in phase. So, del psi del Z it will be simply 0 and which is obvious from this equation.

But here we find that if gamma is not equal to 0; that means, if there is certain non-linearity then obviously, there should be some change in the pulse and u 0 what we have is amplitude which is not going to change at all, but the phase is changing and there is a change in phase due to the amplitude of the pulse itself.

There is a change in phase, but this change in phase is due to the amplitude of the pulse itself this phenomena is called the self phase modulation. Already we discussed these things in earlier classes, again we find from the governing equation how the there is a modification of the pulse shape and we find the amplitude is not going to change and then non-linearity, but there is a change in phase not only that this change of phase is depending on the amplitude of the pulse itself.

So, that is why this phenomena is called the self phase modulation of simply SPM is a very very important phenomena in non-linear optics. Specially, the pulse propagation problems that how the self phase modulation is coming that when the pulse is moving. So, there is a change in phase.

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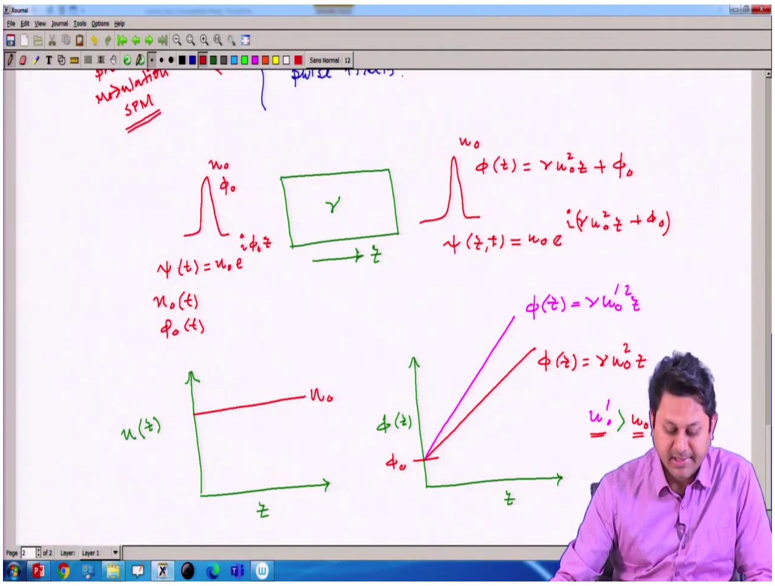

So; that means, if I have a waveguide like this, when a pulse is moving here with a amplitude let me draw the pulse in a different color that should be a different color to make it more attractive, it has initial amplitude phis an initial phase say phi 0. So, the pulse shape of the pulse is like u, which is a function of t Z is 0.

So, I should not write any Z. So, it is u 0 e to the power of i phi 0 Z. If there is initial phase if there is no initial phase it should not be. So, phi 0 and u 0 both are function of t that is always true. But when it is moving now in this equation governing equation we are dealing with the variation with respect to Z the space when it is moving what happened in the output? That we have the pulse here and this u 0 remain u 0.

But this phi which is now going to change with respect to Z and it will change like this. So, the form of the pulse will be simply u 0 e to the power i gamma u 0 square z; obviously, I need to add a phi 0 if I consider there is some initial phase plus phi 0. Well, after that I mean you can see there is a change in the phase. So, this phase; that means, if I launch with certain phase, this phase will going to add up there is a linear change in phase.

So, if I now plot these two functions as a function of z. Here I plot Z and here I plot whatever the u I launch assuming that is a function of z, but our result already suggests that it is not a function of the Z it is constant. So, I should have a curve like this, this is for amplitude there is no change. So, this value is u 0 is a constant there is no change with respect to Z of the amplitude on the pulse. On the other hand, if I plot with respect to Z how the phase is going to change? So, this is phi Z I will going to plot.

So, this value in general should be something like this. It is a straight line moving like this. So, this phi Z will be simply gamma u 0 square and Z and this value the initial phase which is simplify 0. So, it is linearly this, this portion is linearly changing you can see that this change of phi depend on u 0. So, u 0 will behave like a parameter. So, in the same wave guide where the gamma is fixed.

So, this wave guide is characterized by the parameter gamma. So, that is the important parameter of the waveguide if you know gamma then you can calculate what should be the phase and now these things will going to shift these things we are going to change with different u 0. If the u 0 value is higher then we have a different line like this.

So, this. So, where my phi Z will be something like phi Z is equal to gamma and I write u 0 prime square and z. So, u 0 prime which is the amplitude of the pulse is greater than my u 0 which is the amplitude I launched previously. So, if two pulses are launched, in one case the amplitude is higher than the previous one this non-linear effects suggests that the phase modulation will be higher for that which is quite obvious actually.

But from this we can say that the pulse itself modify its phase that is why this phenomena is called self phase modulation as already discussed. So, I will going to stop todays class here.

So, today we understand a very very important concept that how the pulse will behave when it is propagating in a system of which in the system when the system is non-linear in nature.

And we find that there is there should be the change of amplitude and there should be the change of the phase at in general, but the governing equation suggests that the amplitude will not going to change or the envelope will not going to change rather there should be linear change of the phase and this phase will be modified by the amplitude of the pulse itself that is why it is called a self phase modulation effect.

So, with this note I like to conclude today's class, in the next class we try to understand when the pulse is launched and when there is no dispersion I find this solutions. If there is certain dispersion here beta 2 is equal to 0. So, our primary condition was there was no dispersion.

Now we like to understand what happened when the pulse is experiencing the dispersion as well along with the non-linearity. So, the medium should be dispersive as well as non-linear, then what happened to the pulse shape that we are going to understand in the next class so.

Thank you for your attention and see you in the next class.