Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 54
Self Phase Modulation (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguide. So, today, we will going to continue the same topic which is self-phase modulation. We calculate few things in the earlier classes. So, today we will try to understand how the frequency will going to change under non-linearity.

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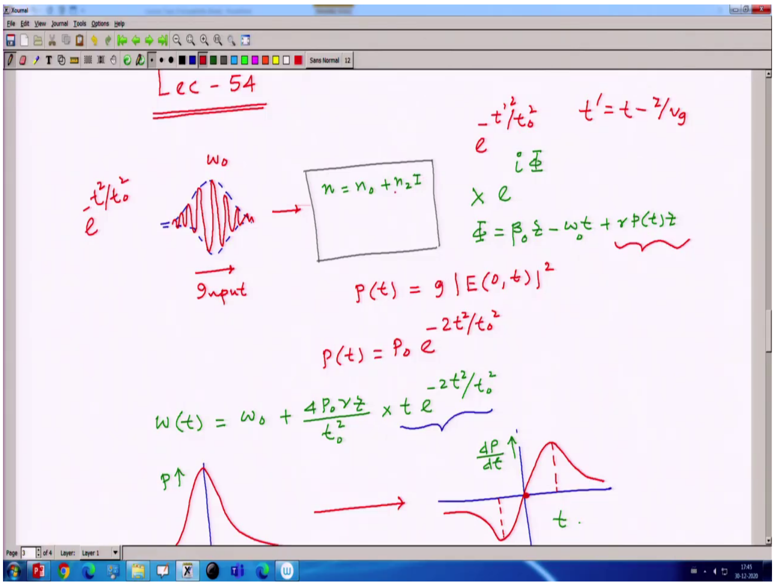

So, if you remember, in the last class that let me draw the basic diagram that we used. So, this is a waveguide and this waveguide is characterized by this refractive index. I launched an envelope here, a Gaussian envelop with frequency distribution omega 0 that was the input and we have a frequency omega 0 and the envelope was Gaussian.

So, it will be e to the power of t square divided by t 0 square; that was the envelope of the pulse and this pulse was moving inside this non-linear medium. During the motion after moving this part, we find that the form of the waveguide is e to the power the envelope, when it is moving through this non-linear wave guide is something like this.

The envelop is defined where t prime was t minus Z by V g, so that means, the wave is moving and if I move with the same group velocity that of the wave, then I will not going to see any kind of change in its envelope. However, there is a change in its frequency in phase. So, the phase there is some additional phase. So, e to the power of i phi, some additional phase was there and inside this phase we had. So, let me go back to the previous class.

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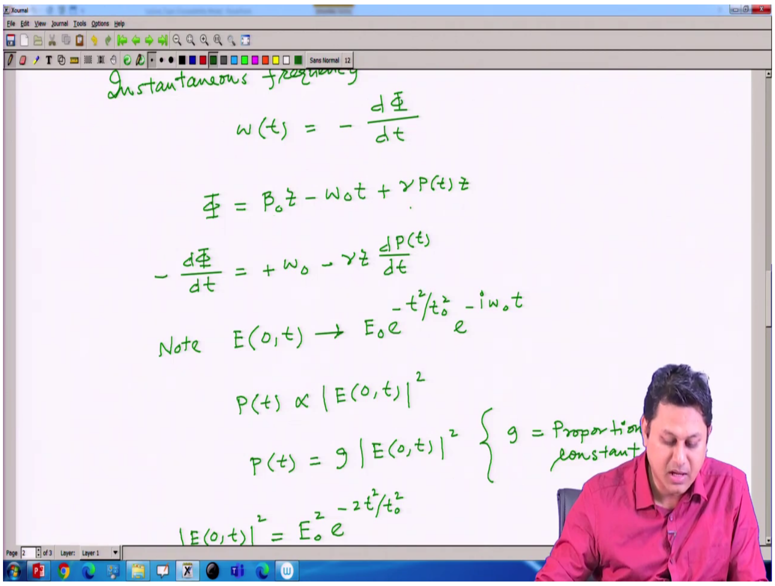

So, inside the phase we have beta 0 z minus omega t plus gamma P t z. So, phi was beta 0 z minus omega t omega 0 t and then it was gamma P as a function of t and z.

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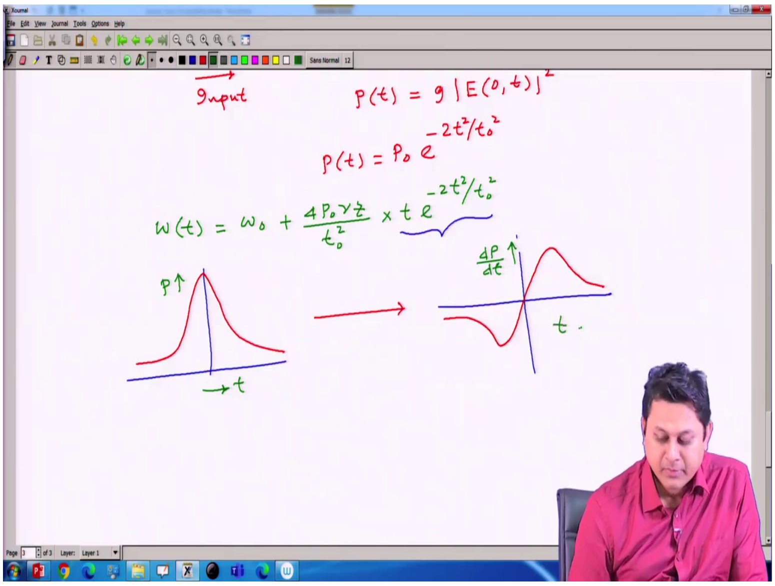

Since, these additional part is related to non-linear coefficient, the contribution is coming through the this part of the phase due to non-linearity. So, P which is so, P which is a function of t is related to the it is basically, related to the input of the incident pulse g was some constant and then, we figure out that P is essentially something like P 0 e to the power of minus 2 t square divided by t 0 square.

And when we calculate the instantaneous frequency, omega t which is omega 0 plus 4 P 0 gamma z divided by t 0 square and then multiplied by a function like t e to the power of minus 2 t square divided by t 0 square. So, that was the instantaneous frequency. And when we plot these instantaneous when we plot this power; so, the power was something like this Gaussian kind of distribution and for this particular power, one can have the instantaneous frequency and through this part.

So, this is a form if I plot this it will be this and this is coming due to the fact that I need to calculate a quantity which is del P del t, which is varying over. So, this is P along this direction which is a function of t and this is del P del t. In x axis, both the cases we had time. So, this was the calculation we derived in the last day.

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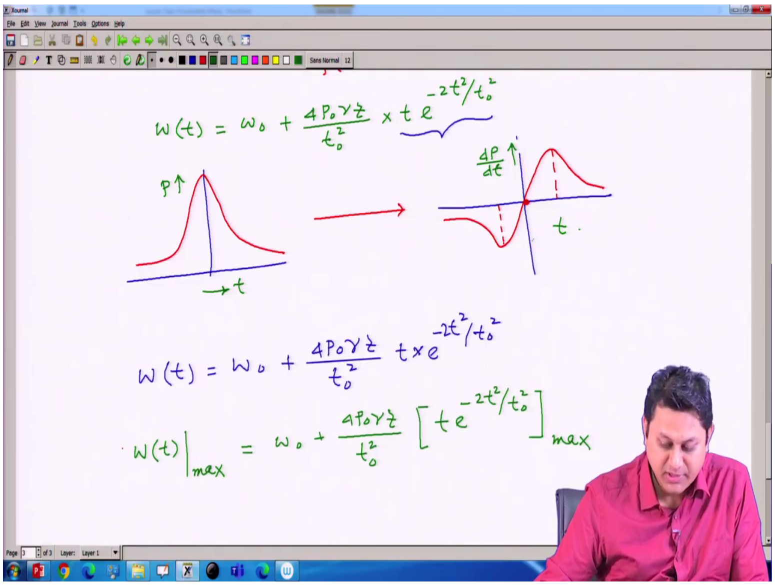

So, today we will going to extend this part, try to understand more on that. So, let me find out what is then. So, let me write the omega t once again. So, omega t is omega 0 plus 4 P 0 gamma z divided by t 0 square and then, t multiplied by e to the power of minus 2 t square divided by t 0 square; that was value of formality.

So, you can see that there is a point where this is the change and it will going to add with omega 0 at this point and this portion is negative, so that means, there will be a subtraction of the omega 0. So, omega will be omega 0 if it is the central frequency at this point at this point then at some point here, I have to add something with omega 0 and at some point here, I need to subtract something from omega 0; this curve is symmetric.

So, if I calculate what is the maximum omega? So, omega is gradually changing with respect to time, because of these varying part, which is coming through the derivative of this power term. Well, if I calculate the maximum value. So, what is I need to calculate omega t which is the instantaneous frequency it is changing and I took I need to calculate what is the maximum of this value. So, this value is omega 0 plus whatever the function we have here.

So, 4 P 0 gamma z divided by t 0 square; this portion is not depending on the value the time parameter, but this t e to the power of minus 2 t square divided by t 0 square this part will depend on t. So, if I want to find out the maximum of this function then, I can find out what is the maximum change of omega, which is continuously changing.

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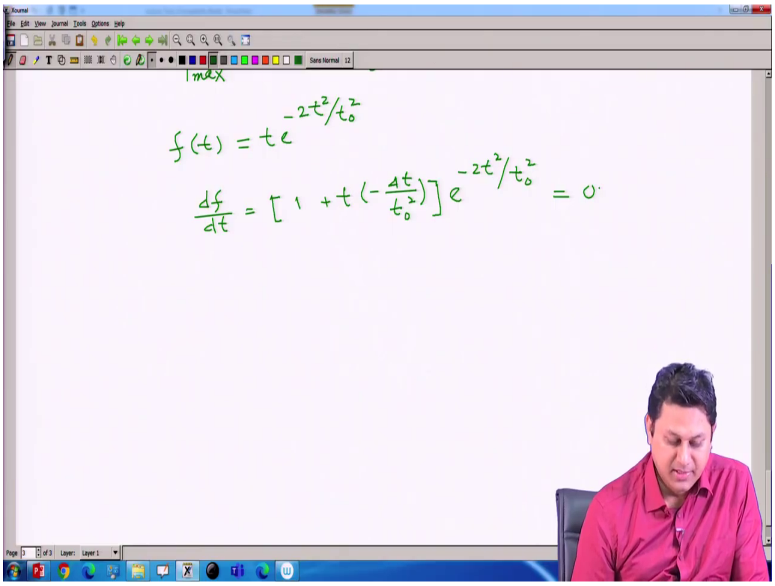

Well, if I write a function f t, which is t e to the power of minus 2 t square divided by t 0 t 0 square then, if I want to find out the maximum this function. So, the simple thing is to make a derivative with respect to t to find out the condition of extrema. So, I will going to make a derivative of this function with t.

So, d f d t will be simply the first derivative of t. So, this is 1 and then plus derivative of this quantity. So, t multiplied by this. So, it should be minus of 4 t divided by t 0 square and then, e to the power of minus 2 t square divided by t 0 square. So, this will be the derivative of this quantity. Now, this is 0 and it is the condition for extrema.

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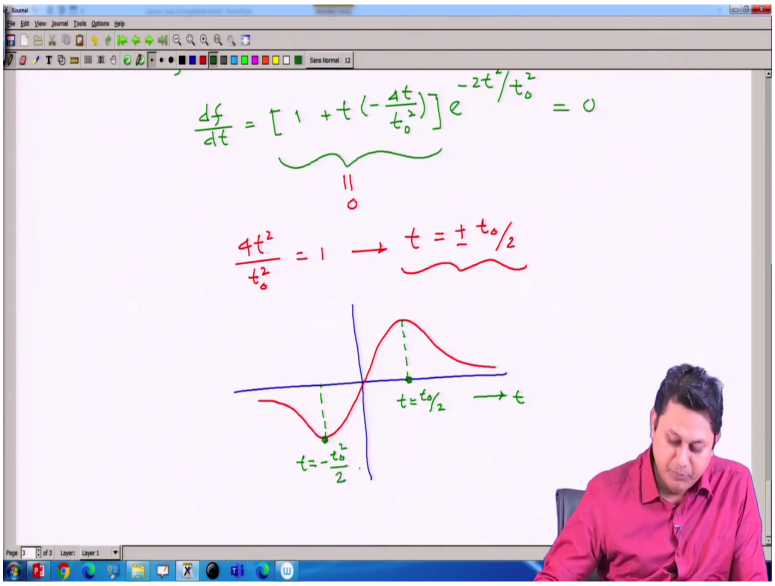

So, this will going to be 0 when. Then, if it is 0 then, this quantity has to be 0 this has to be 0. So, if I put that condition then, I simply have I simply have a condition like 4 t square divided by t 0 square is equal to 1. So, the maxima the function will have a maxima, when the value of t is plus minus of t 0 divided by 2. So, that is the point that is the t for which the function is maxima.

So, now, if I look to the plot of this function, so, it is something like this. So, over these direction I have t; the function is wearing that like this. So, this is the point where we have the maxima. So, this one point is t equal to t 0 divided by 2 and another point should be sitting exactly the opposite direction this is the maxima.

So, this point is t equal to minus of t t 0 square divided by 2. So, these two points I will have.

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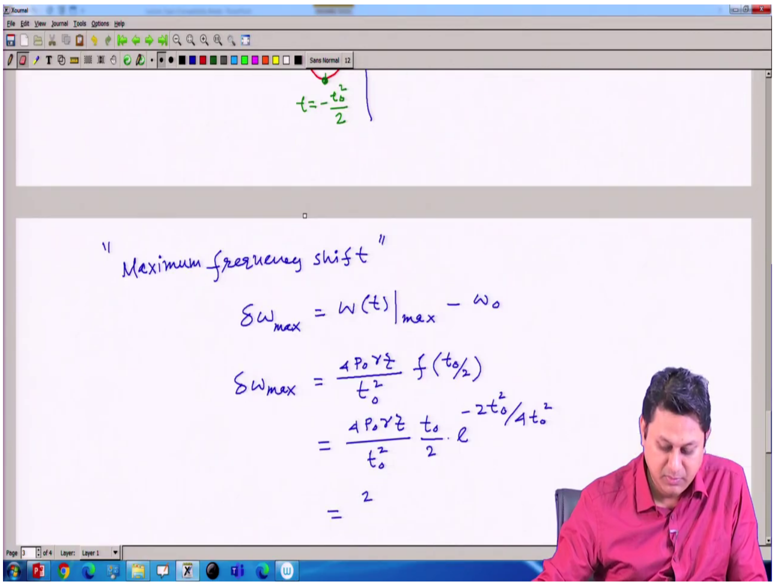

Well, now, if I calculate. So, I know what is the points where I have maxima and if I now calculate what is the maximum value of the frequency it should be. So, maximum frequency shift if I calculate.

So, maximum frequency shift if I want to calculate, it should be delta of omega say, max this is the maximum shift which is nothing but, omega t that is maxima minus the frequency already we have that is the frequency omega 0 that we already have during the input. So, delta omega max is simply 4 P 0 gamma z divided by t 0 square and then, the function where we have a maxima so that means, at this point.

So, this value if I put, I will go to get simply hm. So, here ok here ok. So, let me do that. So, it is 4 P 0 gamma z divided by t 0 square and then the function. So, function, if I put it should be t 0 divided by 2 in e to the power of minus of 2 t 0 square divided by 4 of t 0 square. So, it should be simply 2. So, t 0 t 0 will cancel out one ok this is not t 0 square this is t 0, because if I look to the function here is t multiplied by this one t multiplied by this one. So, in place of t, I just replace t 0 by 2 that is all.

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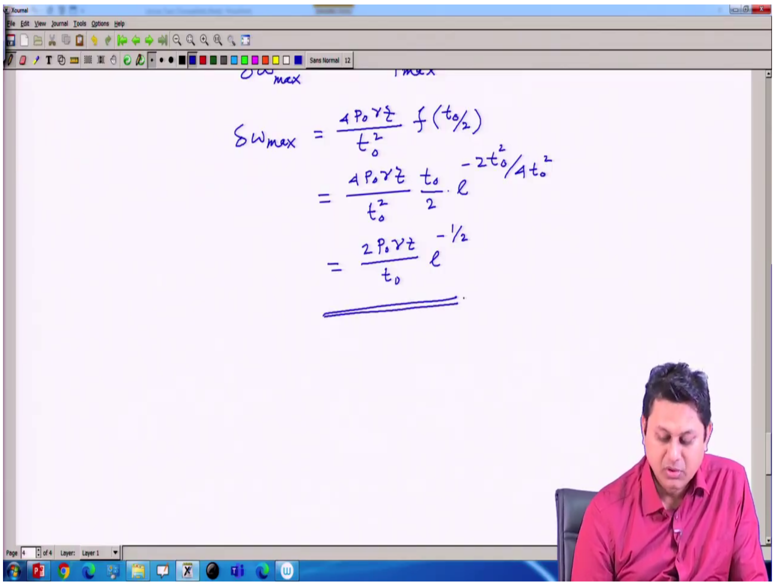

So, it should be 2 P 0 gamma z divided by t 0 and exponential I should have minus of half. So, this is the value of the maximum shift of the frequency this is the maximum shift of the frequency.

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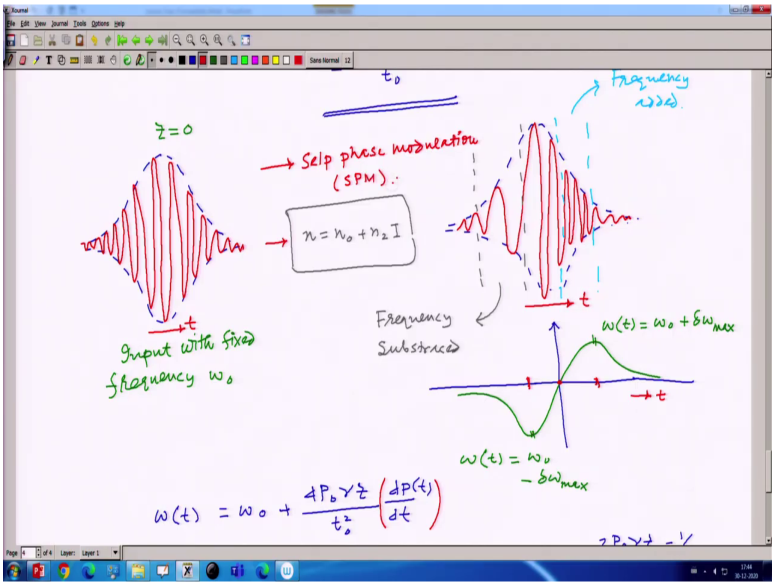

So, let us now, try to understand in a convenient way pictorially that initially we had an envelope like this. So, that was the initial envelope with constant frequency inside. So, these are the frequency distribution I am drawing which is varying constantly there is no chirping. Here, it is called chirping if there is a variation of the frequency.

So, this was the structure this was the pulse. So, the initial frequency [FL], this is input with fixed frequency omega 0. So, I am having a input at fixed frequency omega 0. So, that was the pulse, so, that z equal to 0. So, what happened when it is propagating to this medium I already mentioned?

So, this is the waveguide suppose, where n is modified as n 0 plus into I and the wave is propagating. At the output, we have a distribution and we already figure out that there is a change in the phase and because of that, there is a change in the instantaneous frequency. So, if I plot that there will be no change in the envelops. So, envelope will remain a Gaussian like we have in the input. So, it should be a Gaussian like this, but what change is this frequency distribution.

And from this figure, you can see that ok let me draw this figure here side by side. So, this is the co-ordinate this is over t how it is distributed and the distribution was something like this that was the distribution. And if I draw this side by side, it is distribution here, at this point ok. So, it is erased. So, at this point, I am having the minimum and at this point, I am having the maximum, minimum in the sense and this point in the frequency distribution not going to change.

So, whatever the frequency distribution that will remain same here, but this side there will be a change of frequency, because I am a having negative contribution here. On the other hand, this side I have a positive contribution. See if I plot here, it should be something like that. So, initially and then, there is a rarefication here. In this region, we have a chirping and then, again. So, I have a rarefication here, because of this.

So, whatever the frequency I have. So, in this point exactly in this point, so, this is over time. So, let me draw this. So, this is over time. Here, also this axis is over time it is changing with time this is changing with time. So, at this point we have a frequency maximum frequency. So, this point, we have a minimum frequency. So, frequency here, omega t instantaneous frequency was omega 0 minus of delta omega max; that was the frequency we have at this point exactly this point.

On the other hand, here my frequency would be omega t that is equal to omega 0 plus delta of omega max. So, we have a frequency here, plus and we have a frequency here minus. So, that is why here, inside the pulse in this region we have a less amount of frequency. So, in this region we are having a frequency less amount of frequency or frequency is subtracted. On the other hand, here, in this region the frequency is add up in this region frequency add up.

So, in this region frequency is subtracted from frequency omega 0 and this region frequency added and the instantaneous frequency, the instantaneous frequency we had already mentioned that the instantaneous frequency in general, it is omega 0 plus 4 P 0 and then, gamma z divided by t 0 square and this portion is in general del P del t.

If I have a value of P explicit form of power P for example, here, it is a Gaussian I have this kind of distribution of energy. The distribution of frequency and the frequency shift. I have delta omega which is simply 4 P 0 gamma z divided by t 0 square and del P del t. Please note that this value of the frequency shift whatever the frequency I have can be increased by increasing the gamma.

If the non-linearity is increased then, you can increase this delta omega max. Also, if z is increased that means, if the pulse is propagating a very long distance then, this frequency subtraction or frequency addition will be more, because this quantity will be increased.

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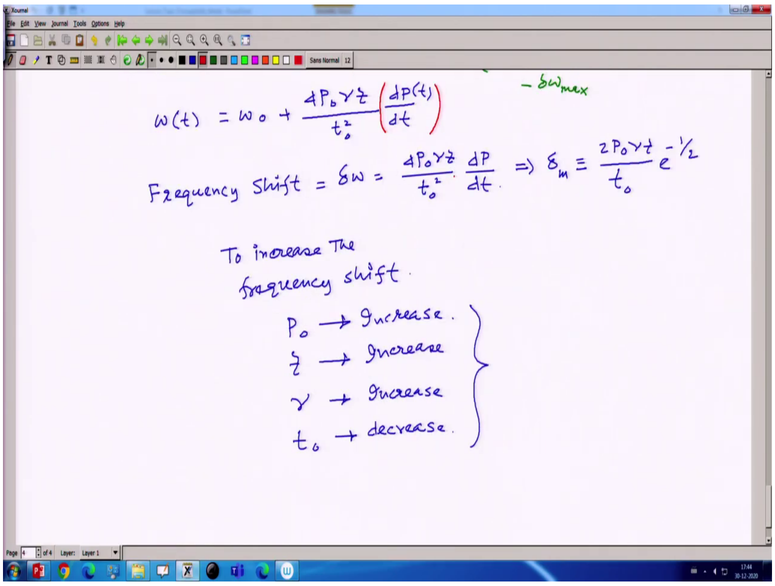

If I increase very high power then, also you can increase that value and also, if you reduce the pulse width initially then also there is a possibility that you can increase this frequency shift.

So, if you want to increase the frequency shift. So, for if you increase the frequency shift. So, f 0 you can increase, z propagation distance you can increase then another parameter gamma non-linear coefficient you can increase, but t 0 need to decrease. So, any of these things can increase this frequency shift and if we have a large frequency shift; that means, I am having a large amount of new frequency that will generate due to this non-linear effect's.

And maximum frequency shift from here, I can write the maximum frequency shift delta m is something like 2 of P 0 for Gaussian pulse it is 2 of P 0 gamma z divided by t 0 e to the power of e to the power of minus of half, and this process through which we are now generating a new frequency or the phase is modulated.

So, that is why this process is called the self-phase modulation. So, because you can see, because of this curve non-linearity what happened that the phase is modulated and this phase modulation is coming through this term?

I should put this in the bracket that, because of these term I am having the phase modulation and P is nothing but, the power of the pulse itself. So, that is why these things is called the self-phase modulation or in short S P M. So, we today's class we roughly understand how the self-phase modulation take places takes place, because of the fact that when the pulse is moving then, it will going to experience the new refractive index which is n 0 plus n 2 I.

This extra part n 2 I which is related to the intensity of the pulse and this intensity basically, modulate the phase of the pulse itself. And as a result, what happened in the output we have the same envelope which is Gaussian initially, it is Gaussian in the output also we have a Gaussian, but with that we have a phase modulation and this phase will be modulated with this fashion that we described today.

So, with this note I like to conclude today's class. So, in the next class we will try to understand what happened when the pulse is propagating. So, what should be the propagation equation for the pulse under self-phase modulation? So, thank you for your attention and see you in the next class.