Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 53
Self Phase Modulation (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we will have lecture number 53 and in this lecture we will continue the topic Self Phase Modulation, the calculation we have started in the last class. So, we will going to continue the that calculation ok.

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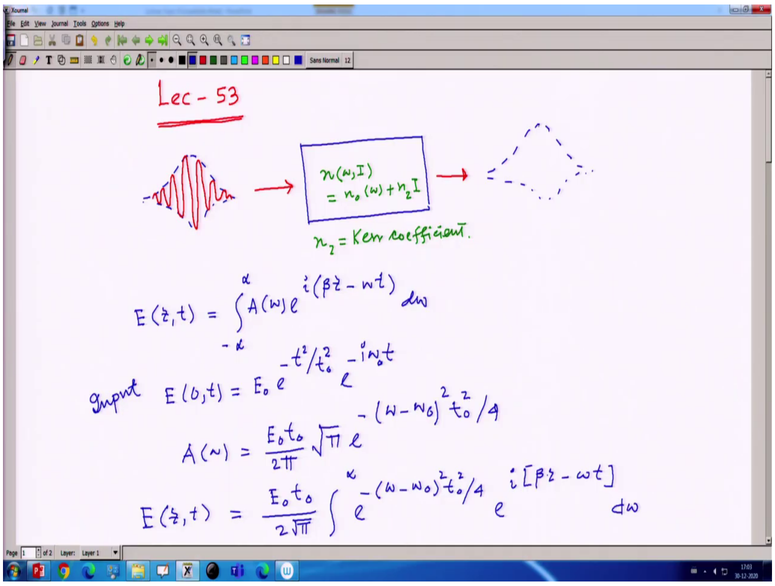

So, let me remind you that what we have done so, far. So, suppose this is a waveguide may be fiber maybe other kind of waveguide and in the input I had a pulse like this shape.

This is the envelope I am drawing and now this is propagating through this medium which is non-linear in nature and non-linear means, the refractive index is now represented in this form. This is called the Kerr coefficient n 2. So, the goal was to find out what happened when the pulse is propagating through this kind of waveguides where we have certain non-linearity.

And when it is coming out from these waveguides, then what should be the change? And already we mentioned that there will be a change in the phase and this phase is changed by the intensity of the pulse itself that is why it is called the self phase modulation, but we will do the detail calculation.

We have already started the calculation. So, let me remind once again. So, the electric field E which is a function of z and t can be represented in this form, we mentioned that several time this is step 1 and then we mention that E 0, t which is the input this is basically the input. So, I defined the input and this input is having a Gaussian shape. So, it should be something like this.

A pulse with Gaussian envelope with initial frequency say omega 0. So, that is the input and then after putting the input I can figure out what is A omega. So, A omega I already derived. So, A omega that will be a Gaussian because it is related to the Fourier transform of that.

So, A omega was something like E 0 t 0 divided by 2 pi and then root over of pi and then e to the power of minus omega minus omega 0 square of that should write this square in a proper way. So, square of that and t 0 square divided by 4. So, that is that was the form of A omega.

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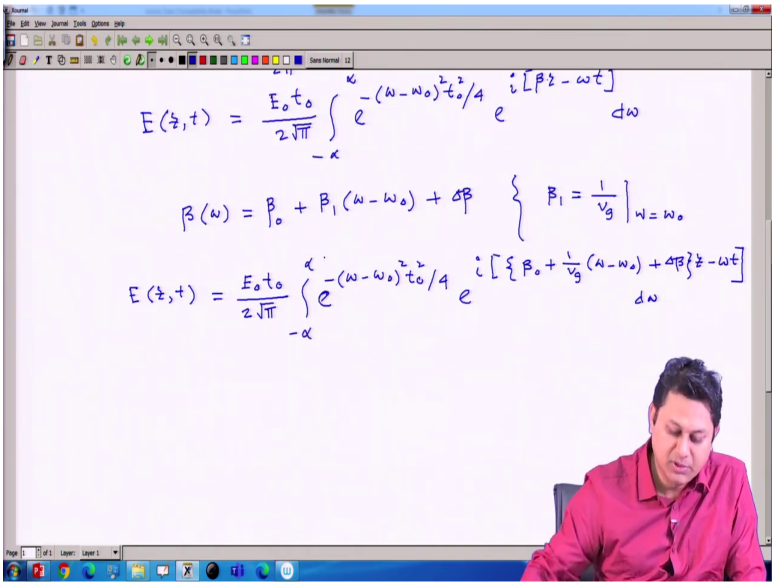

And from that we can find out what should be the E z. E z, t should be something like E 0 t 0 divided by 2 root over of pi, then integration of minus infinity to infinity e to the power of what whatever the value of a we have minus omega minus omega 0 square, then t 0 square divided by 4 and the propagation part e to the power i beta z minus omega t and then d omega.

That was the form of E z, t. So, after that we find what is the E z by putting the value of beta. So, beta if you remember which is a function of omega is expanded in this way. So, first term is beta 0 and the second term is beta 1, omega minus omega 0 that is the expansion and then I have delta beta and this delta beta is the modification of the propagation constant due to the non-linearity.

So, if there is a non-linearity. So, it should experience a different propagation constant and this different propagation constant can be taken care by this delta beta term. Well after that we can find E z by the way beta 1 here is 1 divided by v g as usual at the operating wavelength. So, E z finally, is something like E 0 t 0 divided by 2 root over of pi then integration of. So, this term is here and then I should write this term.

So, e to the e to the power of minus omega minus omega 0 square, then t 0 square divided by 4 and then e to the power of in place of beta I need to write this expansion whatever the expansion I have here. So, it is simply it is simply beta 0 plus say 1 by v g and then omega minus omega.

And then plus delta beta over z that is one term and then after that this omega t. So, minus of omega t with the close bracket and integration should be over d omega integration minus infinity to infinity. So, that should be the value of my E z.

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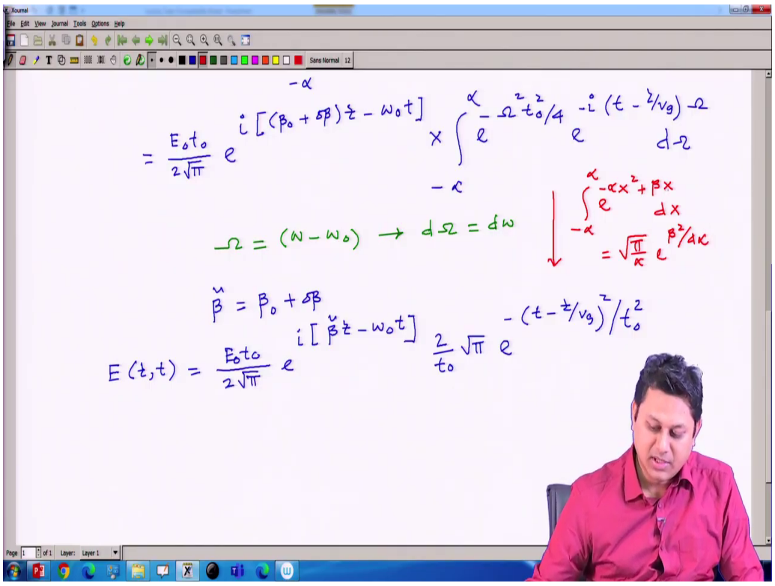

Well I can simplify a bit. So, let me do that. So, this term I should write. So, calculation again I am saying that this is the same calculation we are doing when we are dealing with the dispersion problem the pulse propagation in a dispersive medium. So, here we are using the same thing only difference is instead of having dispersive medium now we have a medium having non-linearity.

So, I launch a pulse and I want to know what happened to this portion. So, what happened to this pulse which is coming out from the waveguide. Well I can write here say e to the power of i, then this term which is not depending on omega say beta 0 plus delta beta z I can take this term outside and then I put an additional term omega 0 t this term I just add from outside. So, that I can have a have the term omega minus omega 0 throughout inside this integral.

Then this should be multiplied by the term like minus infinity to infinity e to the power of minus I write a new variable omega square which is omega minus small omega minus omega 0, then t 0 square divided by 4 and then e to the power of i and e to the power of minus i rather then t minus z by v g this quantity and then big omega and d omega.

So, here I should emphasize that my omega, here is omega minus omega 0 which gives me simply d of omega is equal to d omega. Well after that let us take beta tilde as beta 0 divided by delta beta. I just write a new notation beta tilde which is sum of these two then my E z should be equal to E 0, t 0 divided by 2 of root over of pi first term then e to the power of i beta tilde z minus omega 0 t and then the integration that I have.

So, this integration again I know what is the recipe because this this kind of integration we have done several time. So, I just directly write the solution. So, it should be 2 divided by t 0 root over of pi and then e to the power of this square. So, it should be e to the power of beta square divided by 4 alpha as per the integration rule. So, I will going to use that. So, it is minus of t minus z by v g then square of that divided by t 0 square.

So, that is the ultimate e z. So, from here to here we use the integral form like this standard integral minus infinity to infinity e to the power of say minus alpha x square plus beta x d x is equal to e to the power root over of pi by alpha. So, there was a term like root over of pi divided by alpha then e to the power of beta square divided by 4 of alpha that is the result of this kind of integral.

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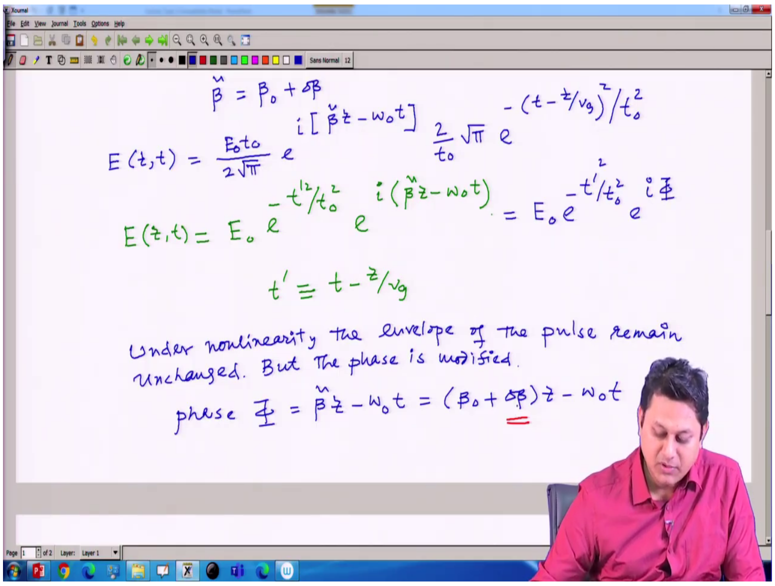

So, I am using this and then I will going to get this result. So, eventually I will going to get the value of E z as few terms will going to cancel out for example, this 2 pi 2 pi cancel out t 0 t 0 cancel out. So, I will going to get a result which is quite straight forward. So, this result says this is nothing, but E 0 e to the power of minus t prime square divided by t 0 square and then e to the power of i beta tilde z minus omega 0 t where t prime is t minus z by v g.

So, we know that t prime is a reference frame that is moving with the group velocity that of the pulse that is why this term is coming. So, what we can infer from these output result and that is from the expression it is clear that under the non-linearity, the shape of the pulse remain unchanged. So, under non-linearity the envelope of the pulse remain unchanged.

So, the envelope remain unchanged. So, initially it was a Gaussian pulse and finally, also we are having a Gaussian pulse. So, what is changing? But the phase is modified the phase is modified. So, phase what is the phase here? Say big phi which is beta tilde z minus omega 0 t and beta tilde if you remember it is beta 0 plus delta beta z minus omega 0 t.

So, we have a phase here and inside the phase I am having this modification and due to that there is a change in phase that is all. So, these I can write here as a form like E 0 e to the power of. So, if I move with the same velocity same group velocity of the pulse. So, it will be the same pulse having a Gaussian envelope with that we have a phase term here which I write i phi and this phase is now going to modify because of the presence of this delta beta.

So, in this figure now I am having a same envelope in the output. So, if it is a Gaussian. So, it will it will remain Gaussian. So, the envelope will remain same the thing that will going to change is phase and we know that the rate of change of phase with respect to time give us the frequency. So, there will be a certain change in instantaneous frequency because of this. So, next we going to try we try to find out that what kind of change one can expect due to this.

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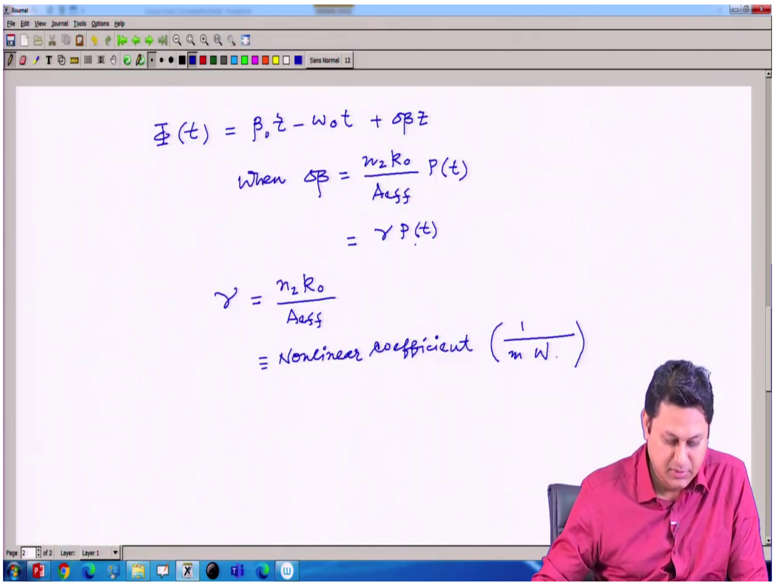

So, let me write down the phi. So, phi which is a function of t is now simply beta 0 z minus omega 0 t plus this additional term I can write this in this way delta beta and then z. When delta beta from the previous class I can write is at n 2 k 0 divided by A effective and then the power which is a function of t. So, this again we write as gamma P t and gamma is simply into k 0 divided by A effective we call this as non-linear coefficient.

So, this is non-linear coefficient it will going to have an unit. So, this unit of distance is meter watt. So, I can have my phi as a function of t and it is coming through this power.

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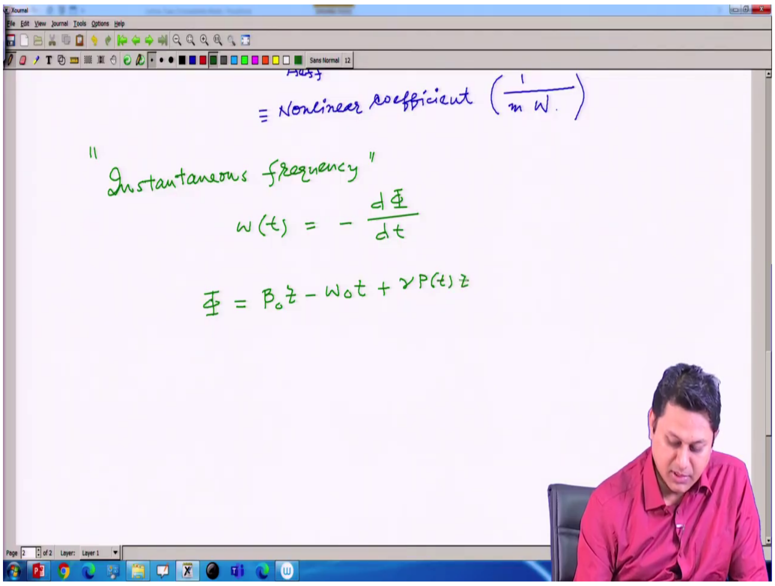

So, now if I calculate the instantaneous frequency which is interesting, it should be omega t equal to the rate of change of the phase with respect to time. So, phi we know already I have written here this is my phi. So, let me write it once again.

So, my phi is big phi is beta 0 z minus omega 0 t plus delta beta z. Delta beta z means gamma delta beta is gamma P t and z. So, this is my new phi which is changing with respect to time.

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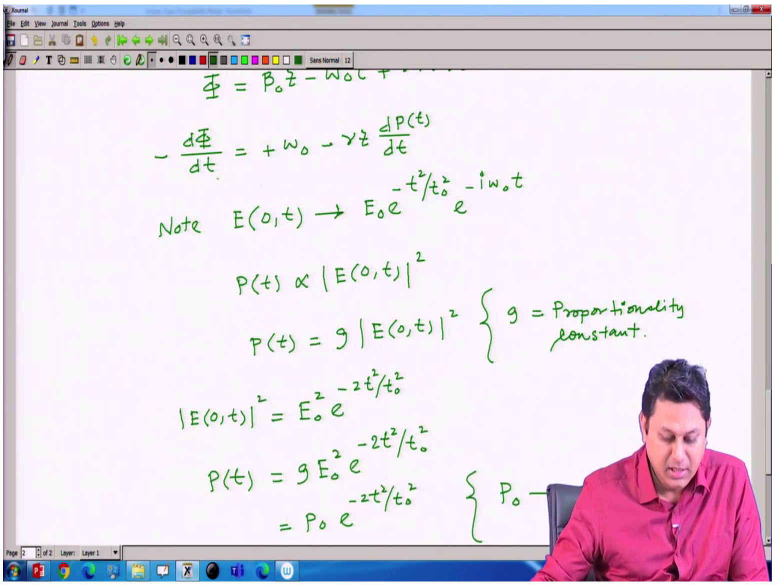

Now, if I if I want to have this quantity. So, this quantity becomes del phi del t becomes minus of omega 0 this result we have already a negative sign.

So, if I put a negative sign here. So, it should be plus of these things and then minus of gamma z and del P del t where P is a function this is a power. So, now, we know what is the envelope and that is why P the functional form of the P one can extract out of that. So, note my E which is the input is given. So, this is a Gaussian. So, this this was a Gaussian e to the e to the power of say minus of t square t 0 square then e to the power of minus of i omega 0 t.

So, that was the full structure of the e at input. If that is the case then what happened? P I can find out. So, P which is a power is proportional to mod of E 0 t square. So, power should be proportional to this thing. So, if I just remove the proportionality constant, then P t can be related to mod of e with certain constant like g maybe g is a constant. So, g proportionality constant. So, this is a constant, this is a proportionality constant.

So, P I can write as if as e in this way. So, now, e is known. So, mod of E 0 t mod square is equal to E 0 square e to the power of minus 2 2 t square divided by t 0 square. So, that is the form of mod of v square. So, my P will be simply P I can write which is a function of t as this proportionality constant whatever may be then E 0 square and e to the power of minus of 2 t square divided by t 0 square.

So, this I can write simply as something like P 0 e to the power of minus 2 t square divided by t 0 square.

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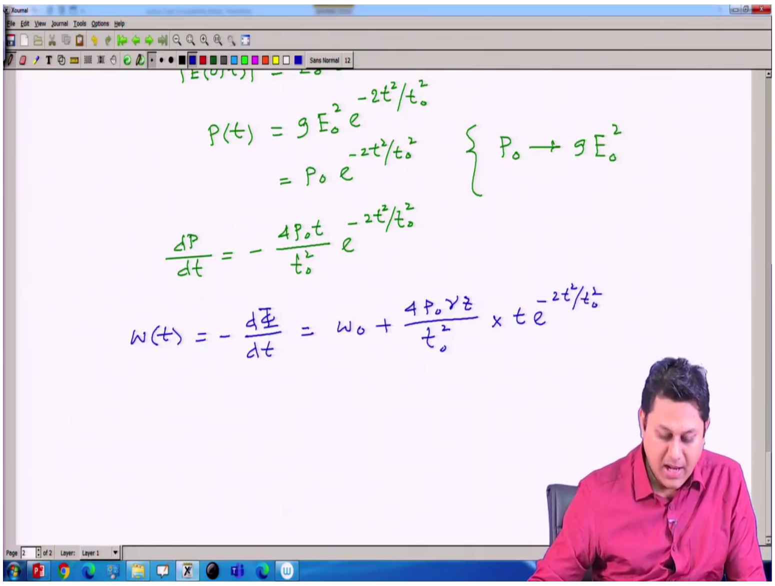

Where my P 0 is simply this associate this constant associate to E 0 and g I can write a new constant say P 0 ok. So, in the equation I need to make a derivative of this quantity. So, if I make a derivative del P del t then this derivative comes out to be minus of 2 t. So, it should be simply minus of 4 P 0 then 1 t will be here and then t 0 square will be in the denominator and I have simply e to the power of minus 2 t square divided by t 0 square.

So, I have this. So, I now derive the value of dP dt then I can now then I can put it here in this equation and find out what is the instantaneous frequency. So, my instantaneous frequency omega which is a function of t that is equal to minus of d phi d t and this value is simply omega 0 plus 4 P 0 then gamma, then z divided by t 0 square and then I have a function which is depending on t and this function is this one.

Well, I figure out what is the instantaneous frequency and you can see that instantaneous frequency is now changing.

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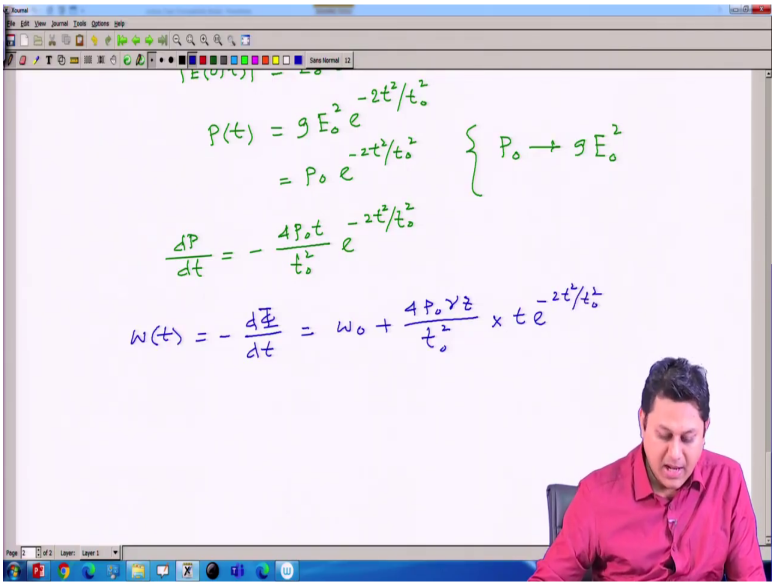

So; that means, if I have a power with this form since the pulse is Gaussian. So, I can have this over the t. So, this is the P t, I draw simply P t and this value is the highest peak power. So, P 0. If the if the power is distributed over time in this this way then I can also draw the derivative of this quantity and if I draw the derivative of this quantity which is del P divided by del t that I am drawing now the drawing should be something like this and along this direction this is t.

So, there is a distribution over time and this distribution because of this distribution what happened that we should have a distribution of the frequency inside the envelope. So, today we don't have that much of time to discuss this in detail. So, in the next class we will going to discuss how the frequency distribution will be there because of this derivative. So, with that note I will like to conclude my class today.

Thank you very much for your attention and see you in the next class.