

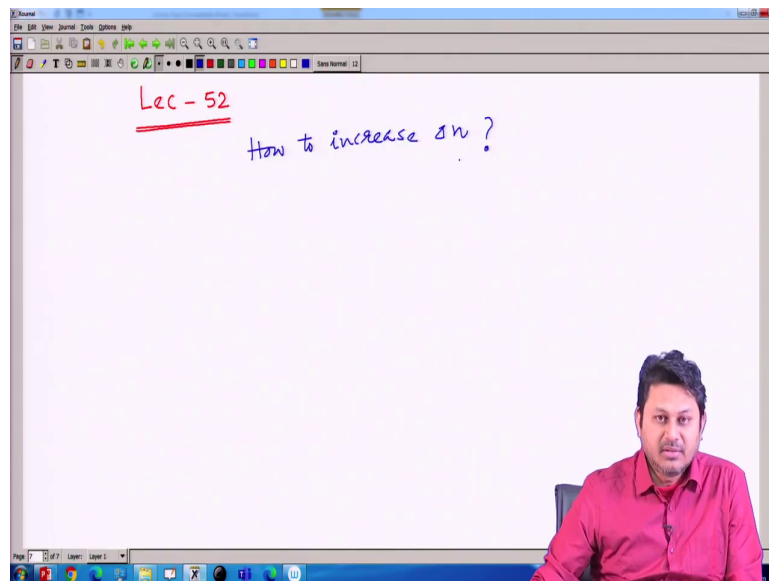
Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 52
Self Phase Modulation

Hello student, to the course of Physics of Linear and Non-Linear Optical Waveguide. Today we have lecture number 52 and we will going to start a new topic today which is called the Self Phase Modulation which is related to the fact that how one can increase the refractive index that is generated by the non-linearity.

So, last we put some kind of calculation to show that in normal fibres, the refractive index change is very very small if we use the standard parameters. But there is a way to increase that. So, today we will going to discuss that first and then try to understand what happened when the light is propagating inside a medium which is having certain non non-linearity.

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In the last class we find that the value of Δn is very very small. So, the next question is how to increase, how to increase Δn . Normally, I like to mention here that normally we do not want the non-linear effect in the in any systems because it is creating certain problems. But, here I am saying that how to increase Δn . So, that is there is some reason behind that, in many many applications nowadays the non-linearity of the system becomes very very important.

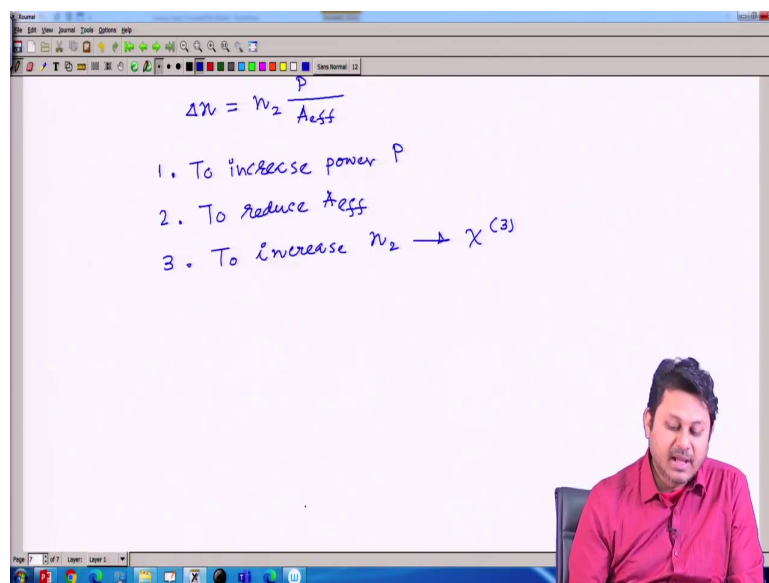
So, one of the major application is the generation of optical soliton, where you can launch an optical pulse which will be going through a very very long distance without any kind of distortion in it.

So, it is called the optical soliton. In order to generate the optical soliton, non-linearity is very very important. Also if you use non-linearity to generate new frequencies, then there is a

possibility that you can generate wide range of new frequencies. Generally it is called the super continuum generation.

So, for that you need a very very high non-linearity in the system. So, these are the few applications where indeed the non-linearity is required. So, that is why we look for how to increase the value of delta n.

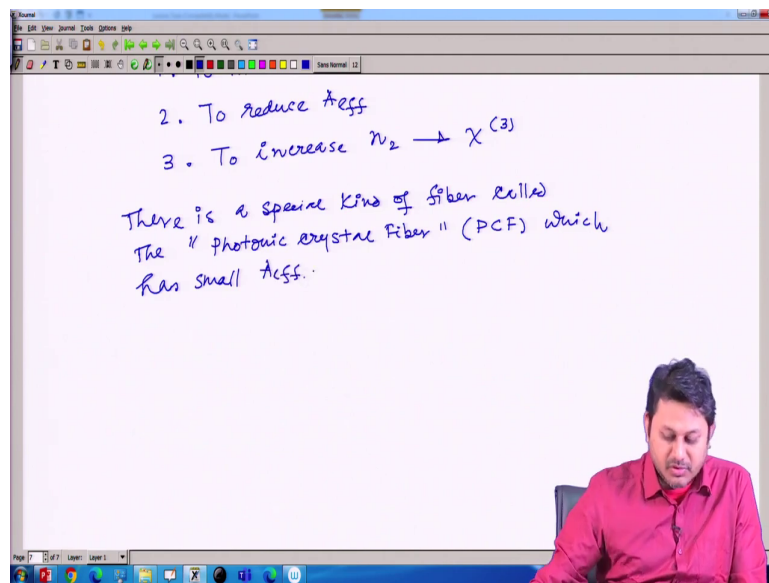
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Well, delta n which is a change of refractive index and that basically creates certain non-linearity, non-linear effects rather. So, I can write delta n as $n_2 P$ divided by A_{eff} and lastly we find that when we put these values of P and A_{eff} for a standard fibre, then this value is very very small.

So how to increase that? So, 1 to increase power to increase power P that is 1, 2nd to decrease to reduce $A_{\text{effective}}$. So, I need to increase power or need to reduce $A_{\text{effective}}$. So, if I do that then only Δn can be increased. Another trivial way to increase that to increase need to increase n^2 ; so that means, I need to increase this value 3rd order of susceptibility.

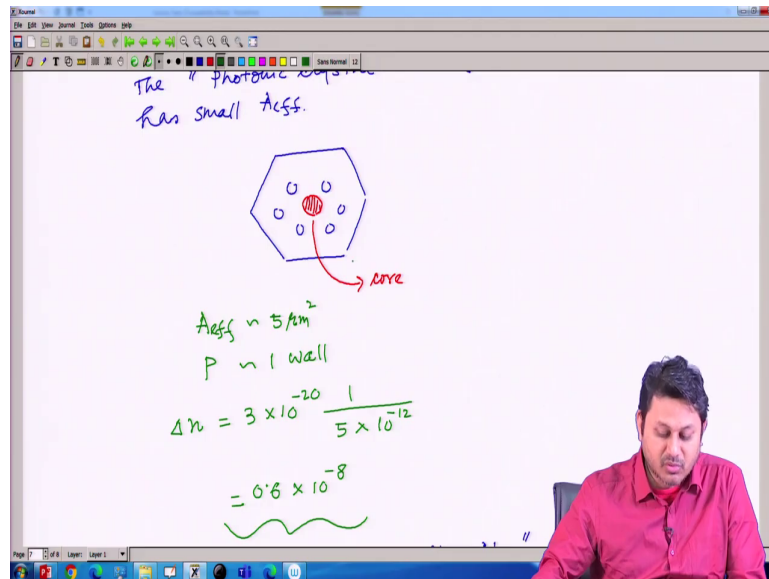
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So, there are many materials for which the non-linearity is high compared to the silica. So, I can use also that that kind of material for which χ^3 is high and if χ^3 is very high if I increase the power and reduce the effective area then I can then I can have a value for which Δn may be significant.

So, here I can say there are few kinds of I mentioned last day. So, there is a specific, there is a rather special kind of fiber called the Photonic Crystal Fiber or in short PCF which exhibit which basically has small A effective area.

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So, the structure of the PCF you can find it in the google. I can show you roughly what is the how the structure is there. So, it is one very very standard structure is this called honeycomb like structure. So, this is the core part of the fiber and in the core part what happened that we had the holes like this. So, it is like.

So, here, here, here, so normally these kind of holes are there. So now, if I increase this hole so, this is the area. So, this there is a effect defect here. So, these holes are there and here in this region in this tiny region the fields are confined. So, this behave as a core and whatever

the hole we have in the outside behave like a cladding because the refractive index here is small.

So, because of the placement of this hole and we are allowed to increase the size of this hole. So, we can produce a system where the core can be very very small so that means, effective area can be reduced very very efficiently with this kind of structure, it is called the photonic crystal fibers.

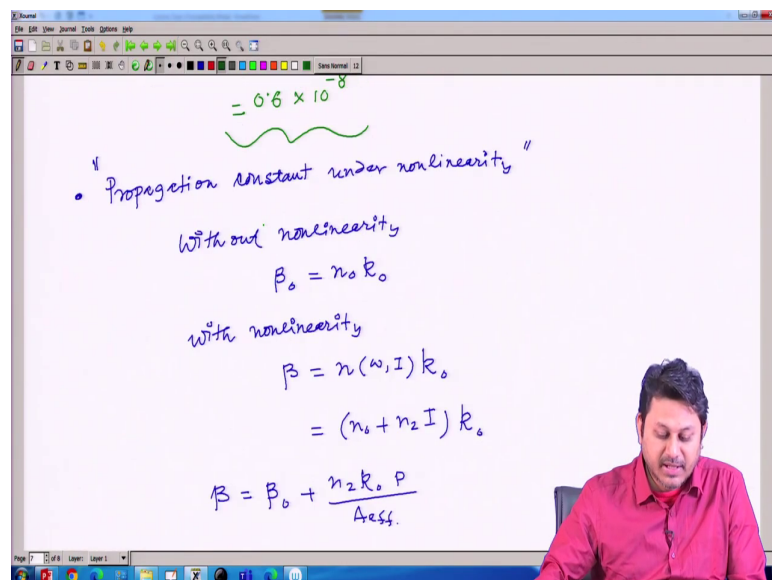
So, in photonic crystal fibers, the $A_{\text{effective}}$ become of the order of say 5 micro meters very very small previously it was 150 micro meter square. Now it reduces to 5 micro meter square and in this kind of fiber we can put a very very high value of power.

So, power say of the order of 1 watt. So, if I just make these two changes, then I can find that my Δn can be increased to few fold. So, it is into which is 3 into 10 to the power minus 20. Then 1 watt divided by this micron. So, 5 into 10 to the power of minus 12 even though the n is small but it is greater than the previous value. So, it should be around say 6 into 0.6, 0.6 into 10 to the power say minus 8 roughly.

So, this is compared to the previous value this is rather a high value and for this value what happened that when the light is propagating it will going to experience a different refractive index by its own presence. So, due to that something will happen here and this is called the self phase modulation.

So, we will going to study that we will going to start the calculation today. So, few we need to understand that how the pulse is going to propagate and how it going to experience the change of refractive index and that is why how the phase will going to modify for this pulse. So, we will going to study that.

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So, let us start with the expression. So, the propagation constant under non-linearity, the propagation constant under non-linearity is modified. So, the propagation constant under non-linearity we will going to calculate today. So, without nonlinearity we have beta 0 is equal to n 0 k 0.

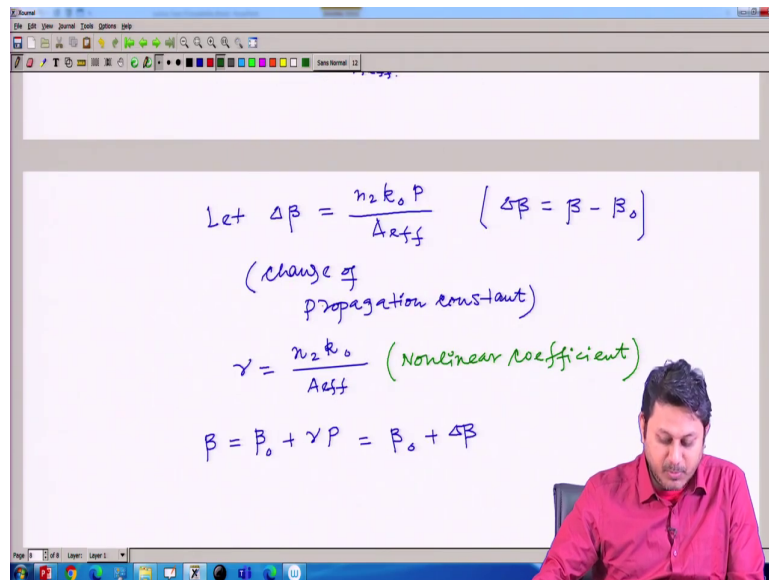
There is no non-linearity so we have my propagation constant in this form only. With non-linearity this propagation constant we will going to modify.

Because n will now going to replaced by n 0 will now going to replaced by n which is a function of omega frequency and intensity I multiplied by k 0.

So, it is simply n 0 plus into I this is the functional form, explicit form of n multiplied by k 0. So, beta is simply beta 0 plus because beta 0 is n 0 multiplied by k 0. So, I can write it plus n

$2k_0$, $n^2 k_0$ and intensity I can be written as P divided by $A_{\text{effective}}$. This is the way one can write.

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$$\text{Let } \Delta\beta = \frac{n_2 k_0 P}{A_{\text{eff}}} \quad (\Delta\beta = \beta - \beta_0)$$

(change of propagation constant)

$$\gamma = \frac{n_2 k_0}{A_{\text{eff}}} \quad (\text{Nonlinear coefficient})$$

$$\beta = \beta_0 + \gamma P = \beta_0 + \Delta\beta$$

Now, let $\Delta\beta$ be $n^2 k_0 P$ divided by $A_{\text{effective}}$, $\Delta\beta$ is nothing but the change of refractive index β minus β_0 , β_0 change of propagation constant. So, $\Delta\beta$ is a change of change of propagation constant. So, I can write another parameter say γ which I defined as $n^2 k_0$ divided by $A_{\text{effective}}$ keeping P aside. So, that my β will be simply β_0 plus γ and then P , I can write it in this way also which is equal to β_0 plus $\Delta\beta$.

So, $\Delta\beta$ is a change and this change I can write in terms of a parameter γ which is $n^2 k_0$ divided by $A_{\text{effective}}$. This γ parameter is very very important and we call this as non-linear coefficient. So, I defined a non-linear coefficient as γ which is related to

the Kerr non-linearity n^2 as well as the effective area of the system. That means, it related to certain geometry of the system as well.

So, if $A_{\text{effective}}$ is small, then for the same n^2 , I can increase the value of γ . So, for normal silica I can have a value of γ and this silica if I make it as a photonic crystal fiber. I can use a fiber which is the photonic crystal fiber for which the effective area I just draw that the core area or the effective area is very small, then automatically the γ will going to increase even though the n^2 is not effected n^2 is same here.

Well so $\Delta\beta$ I already mentioned this is a change of propagation constant or the modification of the propagation constant with the original propagation β_2 . So, I just add this to find out the original propagation constant under non-linearity. So, that is the that is the case.

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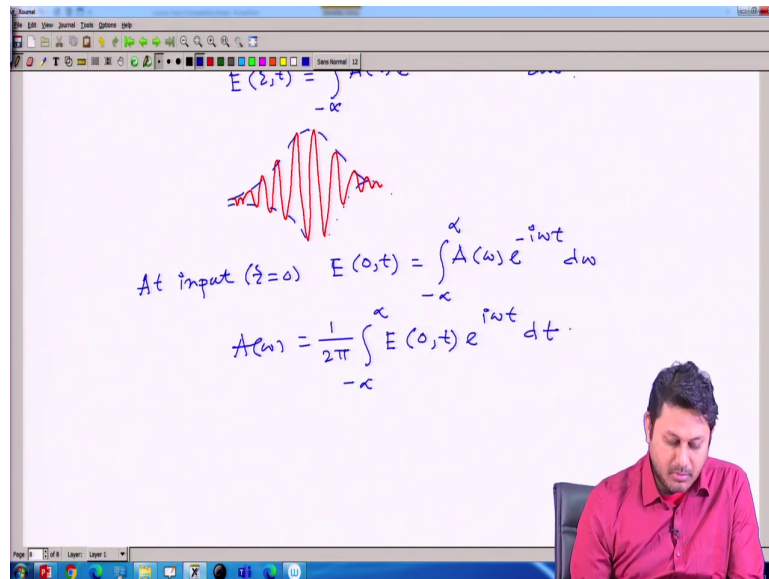
The screenshot shows a digital whiteboard interface. At the top, the title "Pulse propagation under 'Kerr effect'" is written in green. Below it, the text "General form of a propagation pulse" is written in blue. The mathematical expression for the electric field $E(z,t) = \int_{-\infty}^{\infty} A(\omega) e^{i(\beta z - \omega t)} d\omega$ is written in blue. In the bottom right corner, a man with dark hair and a beard, wearing a red shirt, is visible from the chest up, looking towards the whiteboard. The whiteboard has a standard toolbar at the top and a taskbar at the bottom.

Well now we will going to, so after having all these, now we will going to in a position to understand what happened when the pulse propagates ah under Kerr effect.

So, I should write as pulse propagation under Kerr effect. So, Kerr effect will be there and if the Kerr effect is there then what should be the fate of the pulse that we will going to understand. So, pulse propagation under Kerr effect so; that means, certain non-linearities associated with the system and we now launch a pulse here and then how the pulse will propagate and what should be the effects that we understand.

So, let us start with the general form, general form of a propagating pulse. I can write it as E as a function of z t and one can define this as $A \omega e^{i \beta z - \omega t + d \omega}$. So, this is not new, these expression we have been using for long when we are discussing about the dispersion, then we introduce this. So, a optical pulse can be represented in this form where the pulse shape one can have in. So, this is basically the pulse shape.

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So, this is an optical pulse that is propagating. So, this is the optical pulse, the shape of the optical pulse with this mathematical form. Well at input, so at input; that means, at z equal to 0 what happened that $E(0,t)$ will be simply minus infinity to infinity $A(\omega) e^{-i\omega t}$. It is the same treatment that we did during the calculation of dispersion we will follow that. Well $A(\omega)$ will be related to this fourier transformation and it should be this.

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Input pulse shape

$$E(0, t) = E_0 e^{-t^2/t_0^2} e^{-i\omega_0 t}$$

(A gaussian envelope with frequency ω_0)

$$A(\omega) = \frac{E_0}{2\pi} \int_{-\infty}^{\infty} e^{-t^2/t_0^2} e^{i(\omega - \omega_0)t} dt$$

Note

$$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\beta^2/4\alpha}$$

$$A(\omega) = \frac{E_0 t_0}{2\pi} \sqrt{\pi} e^{-(\omega - \omega_0)^2 t_0^2 / 4}$$

Now, the input pulse I need to put certain shape. So, input pulse shape I can put I can the input pulse shape I can assign something like $E_0 e^{-t^2/t_0^2}$ to the power of minus t square divided by t_0 square. So, this is essentially a gaussian pulse with a frequency component like this. So, this is the input pulse.

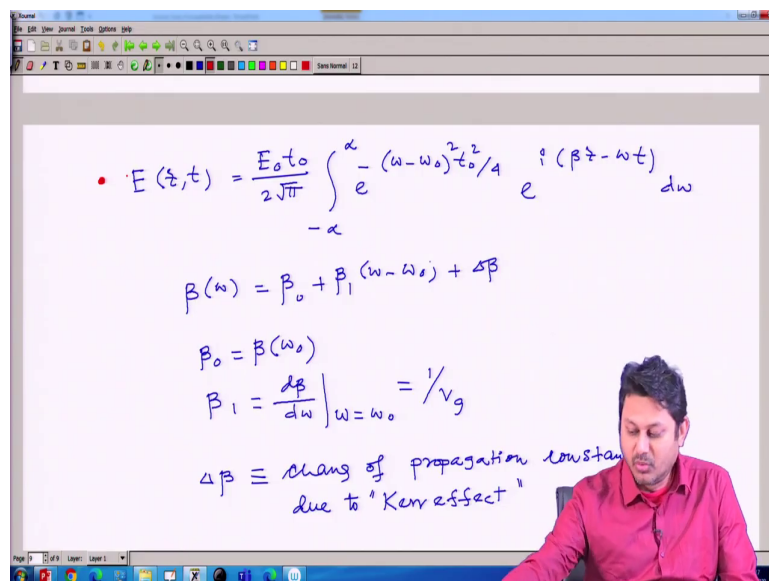
So, it is a gaussian envelope with frequency ω_0 . So, we have a gaussian profile with frequency ω_0 that I assign as a input pulse shape. So, $A(\omega)$ if I want to calculate what will be the $A(\omega)$. So, it should be simply E_0 divided by 2π then integration of $e^{-t^2/t_0^2} e^{i(\omega - \omega_0)t} dt$.

So, this calculation is not new again I am saying because during the calculation of dispersion we followed the same calculation that first I introduce an input pulse shape and from this input pulse shape I want to find what happened in the pulse when it is propagating at a distance z . So, $A(\omega)$ is now.

Now, we know the famous integral identity that minus infinity to infinity. So, note e to the power of minus αX^2 plus βX it can be simply written root over of π by α e to the power of β^2 divided by 4α .

This is the integration identity and if I use this integration identity, I can figure out my $A(\omega)$. So, my $A(\omega)$ comes out to be. So, the $A(\omega)$ will be simply $E_0 t_0$ divided by 2π and then I will have root over of π because α mind it α here is $1/t_0^2$. So, that is why this t_0 will be here, e to the power of minus ω^2 minus ω_0^2 square t_0^2 square divided by 4 . So, it will be a gaussian as well but in frequency domain.

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$$E(z,t) = \frac{E_0 t_0}{2\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{(\omega - \omega_0)^2 t_0^2}{4}} e^{i(\beta z - \omega t)} d\omega$$

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \Delta\beta$$

$$\beta_0 = \beta(\omega_0)$$

$$\beta_1 = \left. \frac{d\beta}{d\omega} \right|_{\omega = \omega_0} = 1/v_g$$

$\Delta\beta \equiv$ change of propagation constant due to "Kerr effect"

So, $A(\omega)$ I know. So, once we know $A(\omega)$ then I know what is the value of my E . So, the general form of the propagating pulse. So, my $E(z,t)$ will be simply $E_0 t_0$ divided by 2

root π . Because I just replace the value of $A(\omega)$ here in this equations I $A(\omega)$ I just deduce and just I put here in this equation, in this equation in this equation.

Then integration minus infinity to infinity e to the power of minus $\omega - \omega_0$ then square t_0 square divided by 4 and then e to the power of i then $\beta z - \omega t$ and then $d\omega$. Now here, β which is the propagation constant generally function of ω is written as $\beta_0 + \beta_1(\omega - \omega_0) + \Delta\beta$. This plus $\Delta\beta$ is the fact that it is the modification due to non-linearity.

So, β_0 here is β as ω equal to 0, β_1 here is $d\beta/d\omega$ at ω equal to ω_0 which is nothing but $1/v_g$ by group velocity. So, this calculation we done early actually and here in addition we have $\Delta\beta$ that is the change of propagation constant, the change of propagation constant due to non-linearity or due to Kerr effect.

So, in the system we have the refractive index that is now modifying with the intensity and that is why I need to introduce this change. So, here I have the electric field and then I have the β values. So, today I will not going to complete this calculation because it will be a lengthy calculation.

So, next day I will start from this and then I put the value of this β here and then execute the integration and try to find out what should be the phase of this propagating pulse. Because there will be certain phase because of this the because of the presence of this $\Delta\beta$ that we will going to understand in detail.

So, till then I like to request you that please go through the calculation and try to understand what is happening here. So, the calculation is exactly the same when the pulse is propagating in a dispersive medium.

I am using the same concept that I put an initial pulse, which has a envelope and it is propagating and when it is propagating and experience a Kerr effect in the system. Then how its phase will going to modify that we calculate with starting from the basic principles. So,

with this note I like to conclude this class. Thank you very much for your attention and see you in the next class.