Physics of Linear and Non-Linear Optical Waveguides
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Module - 05
Nonlinear Fiber Optics
Lecture - 50
Frequency Mixing, Optical Kerr Effect

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguide. So, today we will have lecture number 50 and in this lecture we will going to study Frequency Mixing and Optical Kerr Effect both are non-linear effects. So, we will going to study this in today's class ok.

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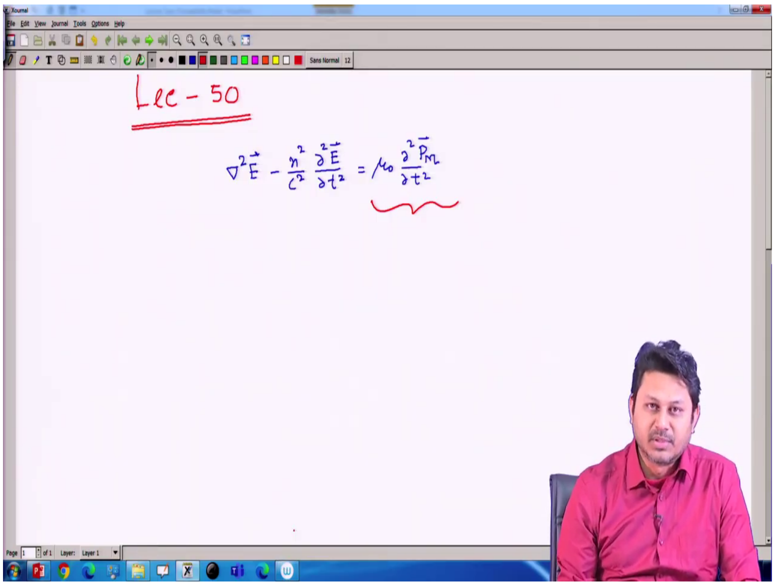

So, in our last class we derived the Maxwell's equation wave equation with this particular form. And this term is a very important term in the right hand side because it associate with something called P non-linear. So, P non-linear is behaving like a source term here in this propagating equation.

So, that we need to consider here for non-linear optics and because of this source term we will going to have few thing called frequency mixing. So, one important thing we will have is called frequency mixing. So, that we will going to discuss today. So, we will study something called frequency mixing.

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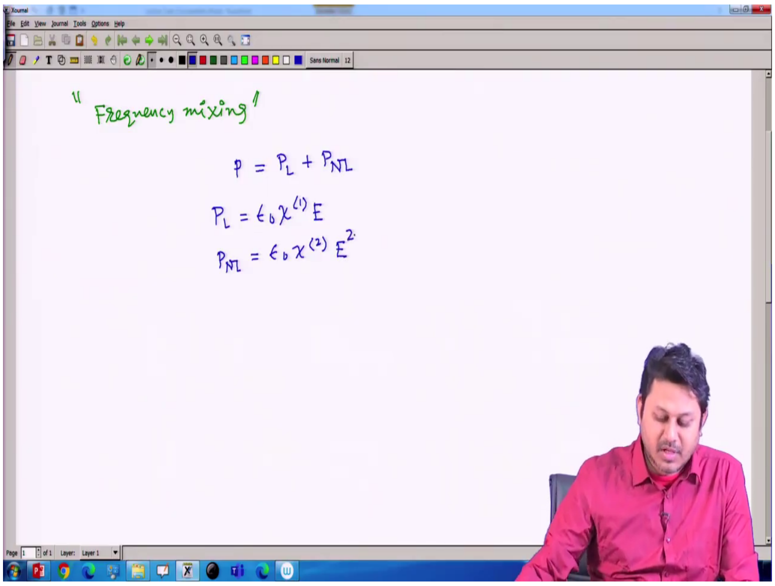

So, last class we mention that total polarization can be written simply into 2 part at linear polarization and non-linear polarization. The linear polarization is having a term like epsilon 0 chi 1 E, whereas, P non-linear is having many terms. If I take only the first term it should be epsilon 0 chi 2 and then E square. This is the first term in P non-linear. Let us start with these first term and try to understand what is the consequence. So, this is only the first term.

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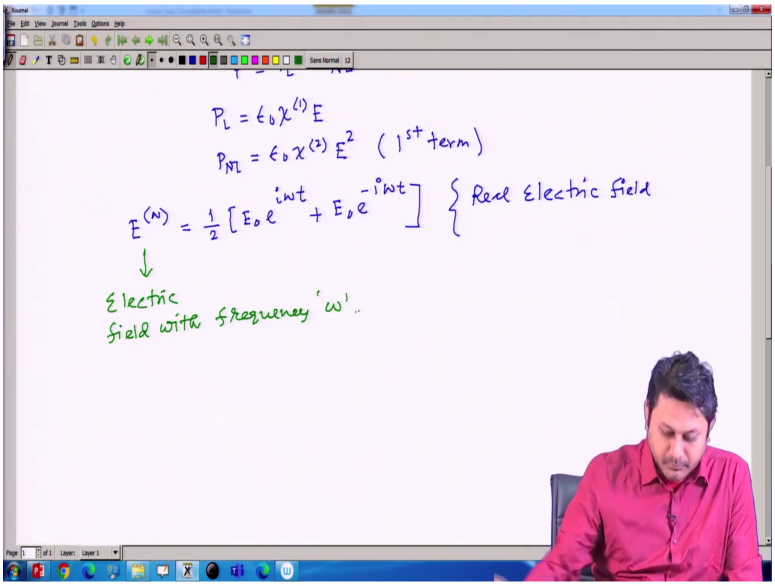

So, first term, there are also higher order terms E cube, E to the power 4, etcetera. I am not going to take this term this higher order term right now. We only take this first term and then we launch an electric field E having a frequency component say omega and I can write this real electric field as a combination of 2 terms like E 0 e to the power of say i omega t plus E 0 e to the power of minus i omega t. So, this is a combination of 2 terms such that E omega is a real quantity.

Because electric field should be a real quantity. So, this is a real electric field. So, and this is electric field with frequency omega electric field with frequency omega. So, I defined an electric field E in this way and now after that we will going to put this electric field here to find out what is P non-linear.

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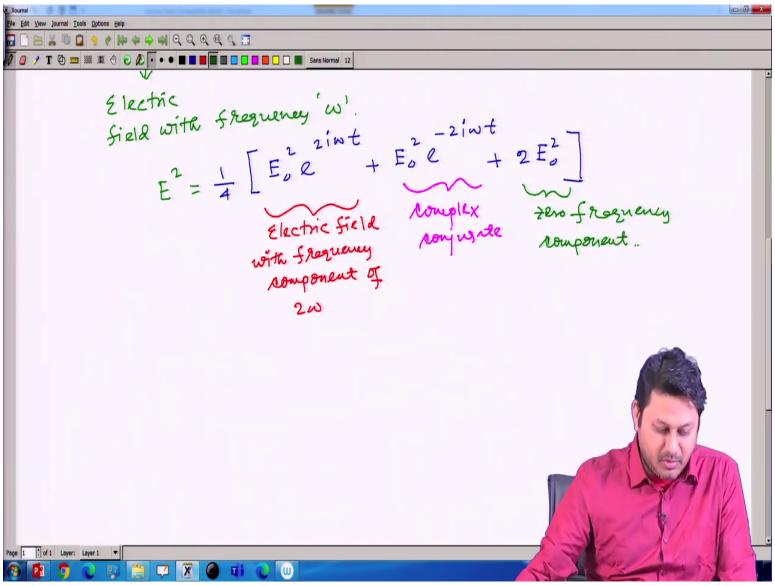

So, E square let us find out what is E square here. So, E square is equal to 1 by 4 then there are few terms. One term should be E 0 square e to the power of 2i omega t that is one term. Another term will be E 0 square e to the power of minus 2i omega t. Another term will be 2E 0 and square of that if E 0 is a real quantity.

So, it should be simply square. So, I have 3 terms. Two terms is having some phase and another term is without phase. So, these three terms is having certain physical consequences. So, what is the physical meaning of that? So, this is the term which is a electric field with the frequency component of 2 omega.

So, the electric field with frequency component of 2 omega, this term is essentially the complex conjugate of that of the first term. So, this is nothing but the complex conjugate of the first term. Whereas this last term this is the complex conjugate of the first term and the last term is having 0 frequency component is having 0 frequency component.

So, in so, P if I want to find out what is P non-linear because E square I figure out and if I put this E square here in this equation of P non-linear.

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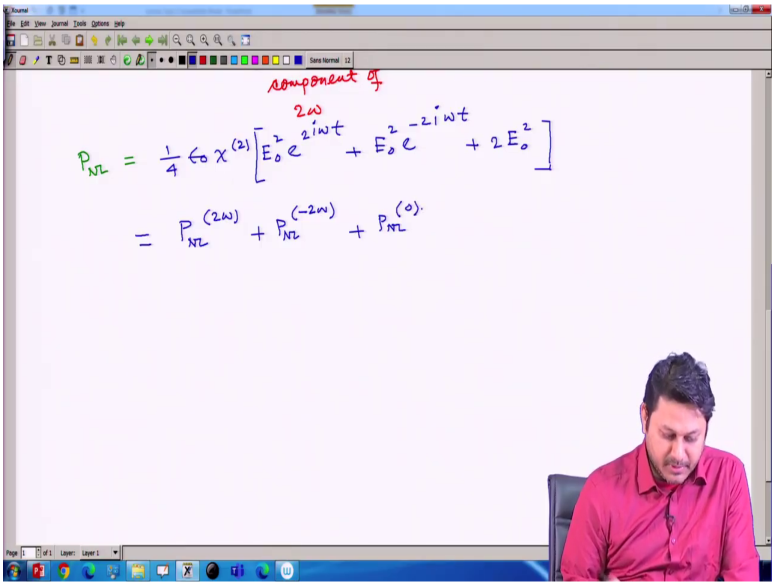

Then the P non-linear simply becomes. So, let me write P non-linear is how much. It is 1 by 4 then epsilon 0 then chi 2 and E square if I take E square common it should be simply E square. Let us write whatever the equation I have in the earlier. So, e to the power 2i omega t plus.

E square e to the power minus 2i omega t plus twice of E 0 square, this is the expression we already got in the previous line. And now I can write this P non-linear to 3 components like P non-linear with 2 omega frequency component plus P non-linear with minus of 2 omega frequency component and then another term which is also associated with P non-linear and that is with 0 frequency component. So, now, this term if I look back to my Maxwell's equation.

Here you can see the term P non-linear is sitting in the source term as a source term. So that means, now the P non-linear will going to vary in these frequencies. So, if P non-linear varies as a 2 omega frequency from this equation I can say that it is a source term. So, it basically going to generate a frequency of 2 omega electric field of frequency of 2 omega. So, I launch an electric field E which is having a frequency initial frequency omega, but because of this P non-linear term which is varying is a frequency 2 omega.

It will now going to vary with a going to oscillate with the frequency 2 omega and that is why it will going to generate a new electric field which is of frequency 2 omega.

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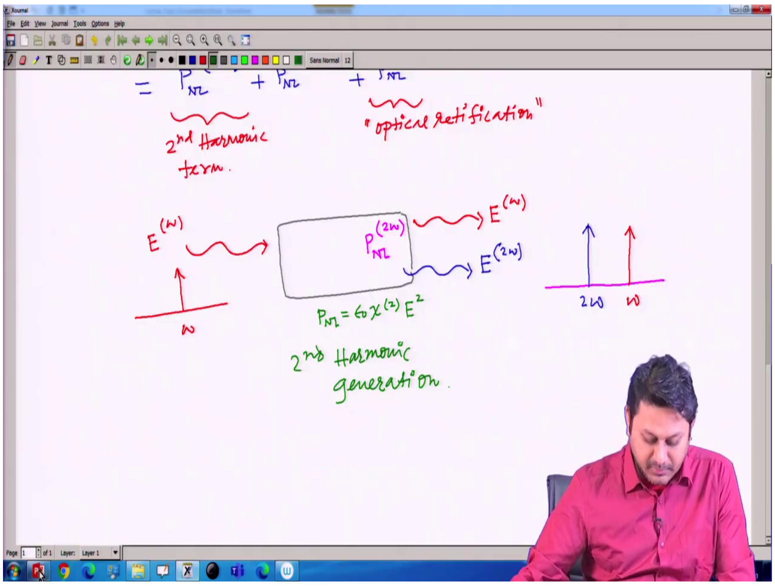

So, this term is the second harmonic generation term because 2nd harmonic term and this term is called optical rectification. So, if I try to understand quickly what is going on here, so, I have a system say a non-linear system and an electric field is launched here with frequency omega and inside the medium there is a non-linear interaction and I have a P non-linear term as E 0 epsilon 0 chi 2 and E square. Because of the E square.

I can generate inside I can generate a term P non-linear which is varying as a frequency 2 omega and this Maxwell's equation I can see that it is now sitting in the source as a source term. So, what happened that it will now going to generate. So, the dipoles are now going to vibrate at a frequency of 2 omega inside the medium and as a result we will going to have a wave which is having a frequency 2 omega. This is called the 2nd harmonic generation.

The process is not that simple there is something called phase matching. So, in order to generate the 2nd harmonic one should have the phase matching. So, original wave is also there a part of original wave, this is E omega. So, in the input I have one frequency which is omega and in the output I can have 2 frequencies.

This is in the output and the output I am now having 2 frequencies. One is say omega and another frequency I can have like 2 omega. So, initially I launch one frequency and as a result as a non-linear interaction here into the medium I have 2 frequencies here. One is omega and another is the double of that thing that is why it is called the 2nd harmonic process. It is called the 2nd harmonic generation. So, this process is called the 2nd harmonic generation.

So, it is having a 2nd harmonic generation. Now, since we are dealing with silica, so, what happen in the silica we need to investigate.

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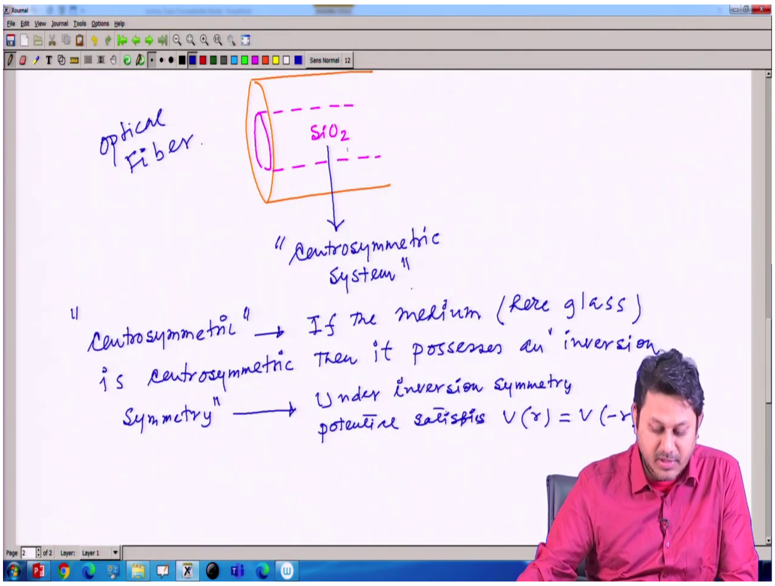

So, for optical fiber now let us go back to our old very old structure of optical fiber that this is a cladding and inside the cladding we have a portion which is core. And now this fiber is made of silica. So, it is made of silica. So, it is essentially SiO 2. So, this is optical fiber. So, optical fiber is made of silica and this silica has a special property. This silica it is a special property, it is a centrosymmetric molecule, it is silica centrosymmetric molecule.

Centrosymmetric, it is a centrosymmetric system. So, what is the meaning of centrosymmetric? So, centrosymmetric is if the medium, if the medium I mean in the medium is made of certain material, so, if that material; if the medium here through which the light is propagating here it is glass.

Because, the light is propagating inside the fiber which is simply made of glass or silica. So, if the medium is centrosymmetric then it possess an inversion symmetry then possess an inversion symmetry, something called inversion symmetry. So, what is the consequence of this inversion symmetry?

So, under inversion symmetry the potential satisfies V r is equal to; see if I just reverse the space from r to minus r the potential is not going to change. So, how these things one can understand in the context of these centro these silicon dioxide system? So, that we will going to.

So, in the silicon dioxide system that means, in silica we have a molecule which is which has a property called centrosymmetric property. And because of the centrosymmetric property we have a center of symmetry and that means, if I reverse the space the potential will going to will remain same.

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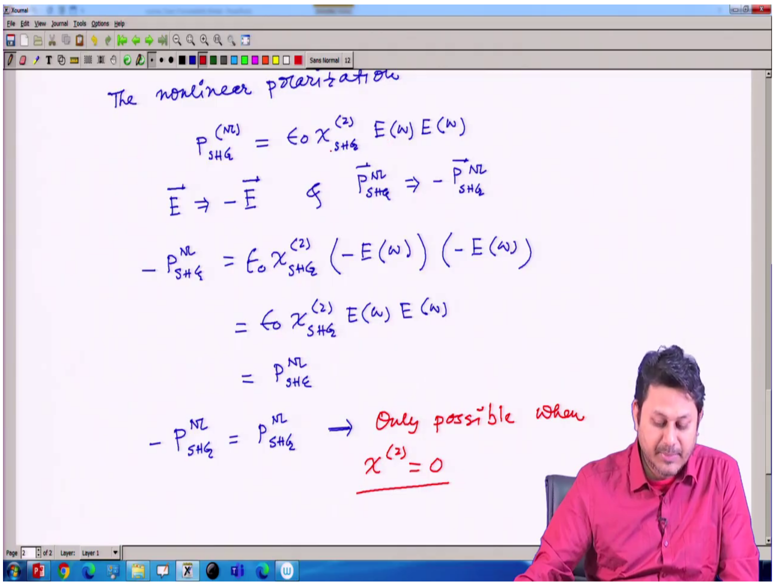

So, here because of that what happened? The non-linear polarization the non-linear polarization which gives the second harmonic; just before we just check that the second harmonic is generating here. So, if I now write these portion as non-linear polarization which generate the second harmonic I can write that this is a non-linear polarization and it gives rise to a second harmonic generation second harmonic generation. So, this quantity is essentially epsilon 0 chi 2 which basically generate the second harmonic.

So, I should write second harmonic generation chi 2 and the 2 fields, E omega squares I write E omega E omega. So, this is the structure we had. Now, the external field is oscillating. So, it changes its sign. So, I can change the sign of e as minus e like this and also the polarization.

So, P second harmonic generation can be replaced by because the polarization and electric field both is changing its signs. So, I can just change these things to plus to minus. So, if that is the case then we can have minus of P non-linear second harmonic generation is equal to epsilon 0 chi 2 second harmonic generation.

And then I reverse the sign of the electric field and I write this, which is essentially epsilon 0 chi 2 this E omega E omega. This things is nothing but P non-linear second harmonic generation.

So, I have an equation which suggests that under centrosymmetric system P non-linear negative P non-linear second harmonic generation is equal to P non-linear second harmonic generation. Minus of P non-linear second harmonic generation is equal to P non-linear second harmonic generation.

So, this is only possible. So, this condition only possible when this quantity is 0, then only I can justify that ok P non-linear second harmonic generation with a negative sign in principle equal to P non-linear is the second harmonic generation.

Because both the terms are 0, both the term; since both since they are 0, so, I can write that E is nonzero quantity epsilon 0 is a constant on term we know which is nonzero. So, only thing that makes this thing 0 is the chi 2 second harmonic generation. So, chi 2 here is 0.

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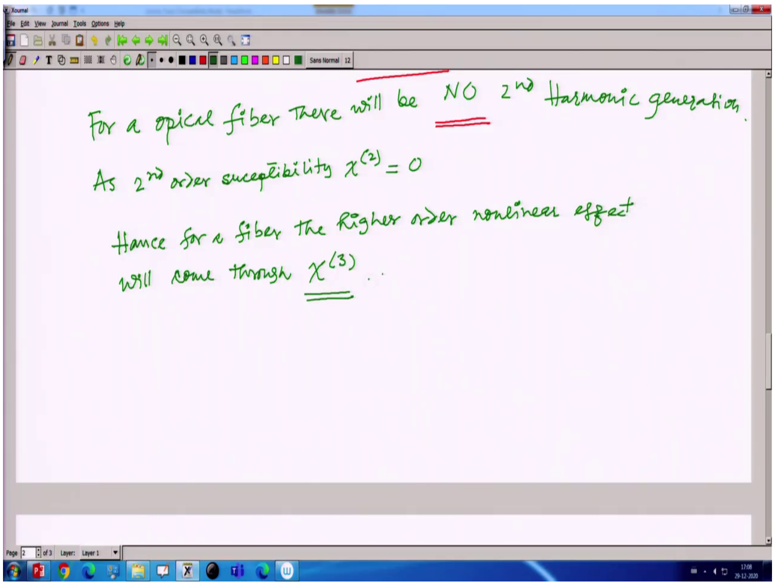

So that means, for a fiber for a optical fiber which is made of silica there will be no second harmonic generation. So, I need to highlight this term no because in silica we do not have any second harmonic generation. So, the 1st, so, the 2nd, so, the 2nd order susceptibility chi 2 is 0 here and it is I should write it as the 2nd harmonic 2nd order susceptibility 0. So, better to write is 0. So, in fiber we find that the 2nd order susceptibility is 0.

So, since it is 0, the effect that we will going to consider here is the next one. So, hence for fiber for a fiber the higher order non-linear effect will come through chi 3. So, the first higher order effect that we need to take account here is chi 3. So, let us start what happened of if I introduce the chi 3. So, let us start the effect.

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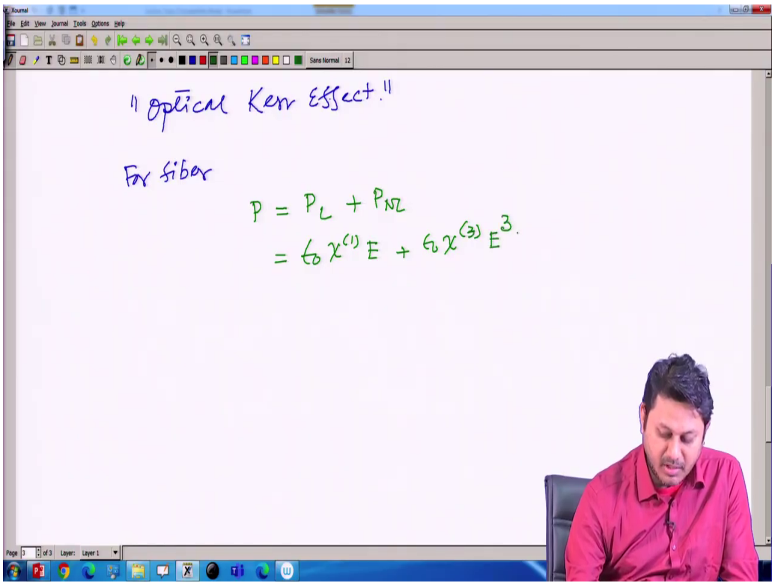

Let us start understand optical Kerr effect. As a consequence that in the fiber we do not have chi 2. So, chi 3 is the is the next higher order effect. So, I can write for fiber I can simply write that P is equal to P linear plus P non-linear. P linear is epsilon 0 chi 1 E plus epsilon 0 chi 2 sorry chi 3 E cube. So, that is the first term we need to introduce here because chi 2 is 0. So, for fiber I can write this P in this particular form.

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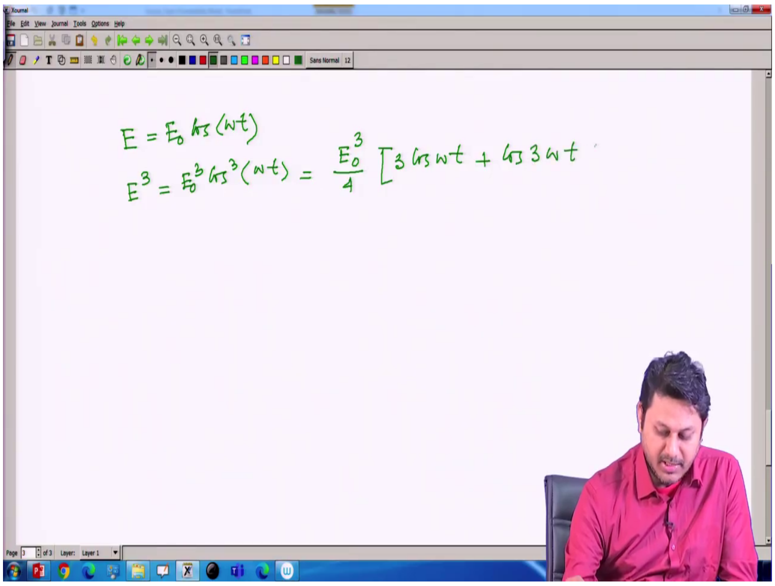

So, this is a scalar form. Now, what we do? We will going to introduce P as E an electric field as E 0 cos of omega t. So, this is a real electric field and I introduce this in this form. So, E cube which is E 0 cube cos s cube omega t. I can write simply using the trigonometric relation as E 0 cube divided by 4 just write this cos cube into this form 3 of cos omega t plus cos of 3 omega t. So, this is the way I can write this cos term cos cube term.

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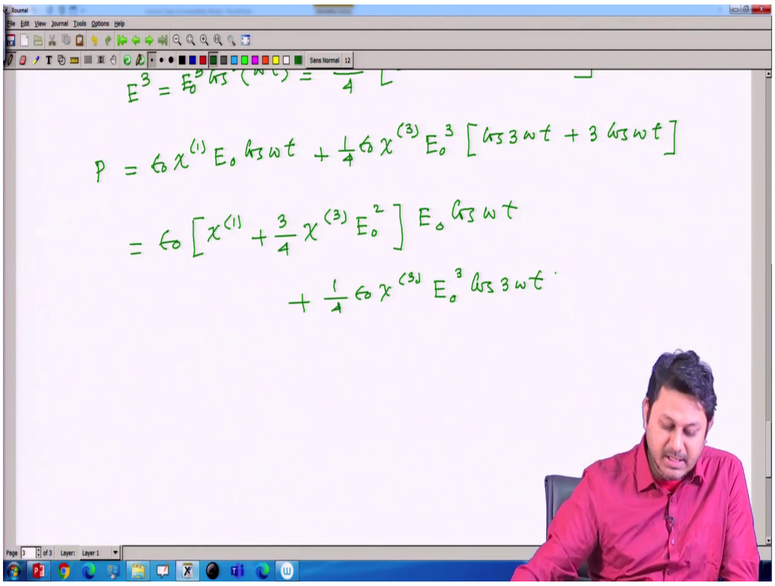

So, now, I will going to write my P which is P linear P plus P non-linear. So, it should be epsilon 1 and E I write simply E 0 cos omega t. So, that is my first term and the second term I can write as 1 by 4 epsilon 0 chi 3. Then E cube E 0 cube multiplied by cos 3 omega t plus 3 of cos omega t.

So, here I can write it as epsilon 0 if I take common then I can have something like chi 1 E 0 cos omega t and 3 E 0 cos omega t I can take it from this side. So, it should be 3 by 4 chi 3 then E 0 square. And I can write it as E 0 cos omega t plus another term 1 by 4 epsilon 0 chi 3 and then E 0 cube cos of 3 omega t. So, this is my total P I am having.

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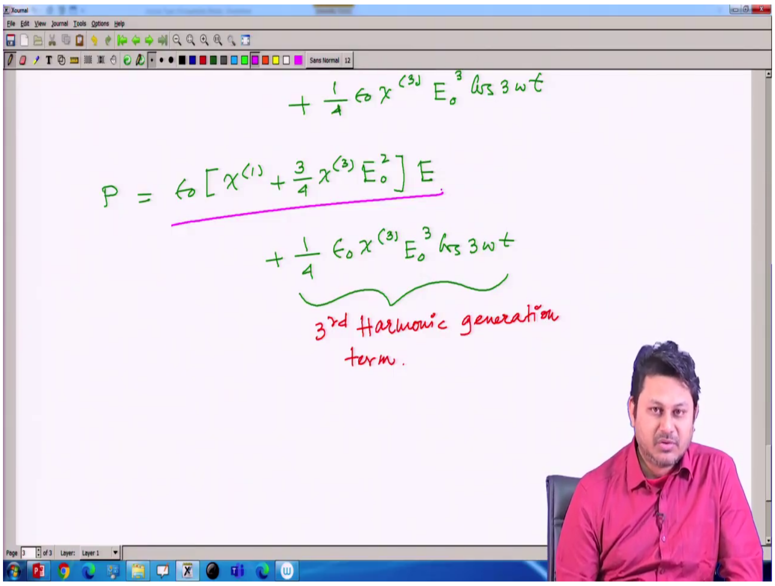

So, I can write this P as this form. Epsilon 0 then this is chi 1 plus 3 by 4 chi 3 E 0 square E that is the first term. And another term I write 1 by 4 epsilon 0 chi 3 then E 0 cube cos of 3 omega t. So, I just separate out the term with E which is launch electric field and another term.

So, this term is responsible for this term is responsible for the 3rd harmonic generation because it is now vibrating as a frequency 3 omega. So, this term is responsible for 3rd harmonic generation. So, I should write that this is 3rd harmonic generation term. This is 3rd harmonic generation term.

So, today I do not have much time to go to the further calculation regarding the Kerr effect. So, I will like to stop my lecture here. So, till now we derived the P which is the polarization into 2 part. One is this one is this portion and another is the 3rd harmonic generation term.

So, normally what happen in order to generate the 3rd harmonic there is some critical phase matching condition that need to be fulfilled, but normally it is not happening. So, we can neglect this 3rd harmonic generation term into this equation. If we neglect then what we will going to get to the first term, that we will going to see in the next class. So, with this note let me conclude. So, thank you for your attention and see you in the next class to understand more about the Kerr effect.

Thank you and see you.