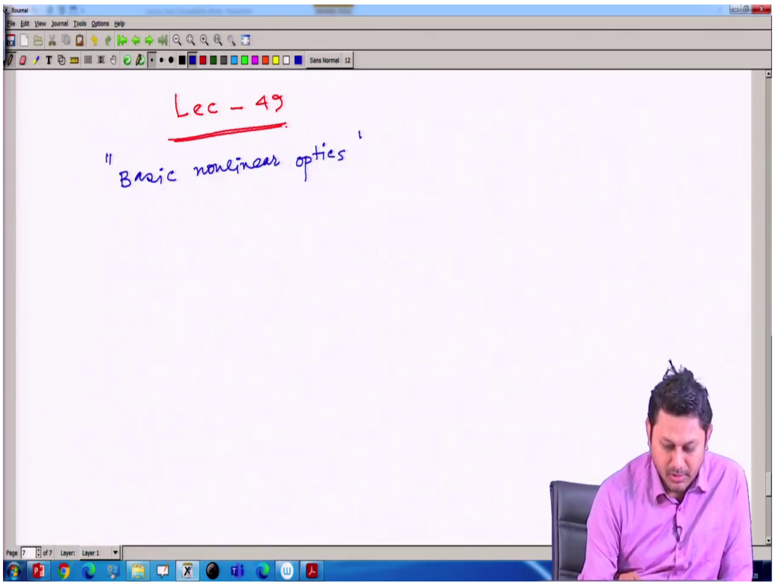
Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 05
Nonlinear Fiber Optics
Lecture - 49
Basic Nonlinear Optics

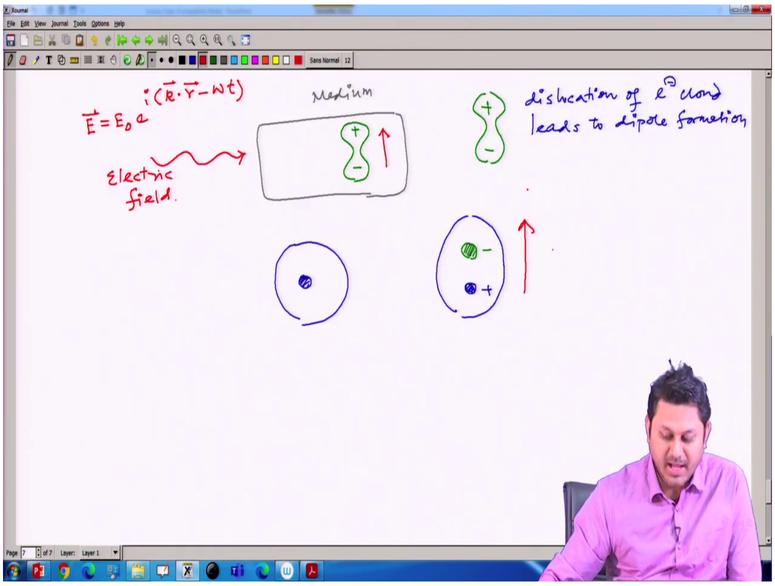
Hello student to the course of Physics of Linear and Non-Linear Optical Waveguide. So, today we will going to start a new topic in this course which is the final topic rather which is Non-linear Fiber Optics and in this non-linear fiber optics we try to understand the non-linear aspects of a waveguide it is in general it is not related to only fiber it can be applied to any general wave guide. And today we start with the basic optical nonlinearity ok.

(Refer Slide Time: 00:47)



So, let us start with the basic few basic concept of non-linearity. So, let me write it basic non-linear optics.

(Refer Slide Time: 01:23)



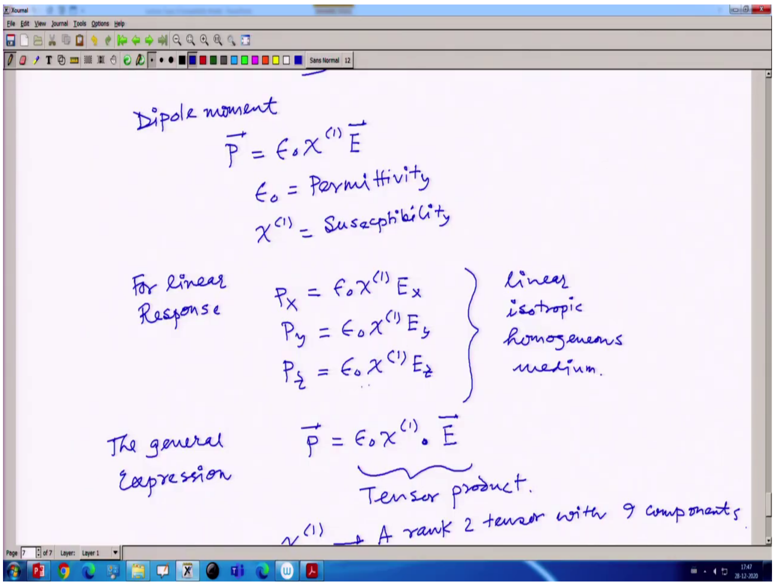
So, let us start with this figure, where I have a substrate like this or a material or a medium whatever you can consider and draw it. And here I launch an electric field having the form say E vector is equal to E 0, E to the power of i k dot r minus omega t. So, a moving plane wave which is the electric field that I am launching. So, this is my electric field.

So, when I launch this electric field. So, suppose this is our medium. So, when we launch the electric field what happened as a result? So, there should be a dislocation of the electron cloud and they basically form a dipole like this and if the electric field is along this direction. So, it will be like this.

So, what is this quantity? So, the dislocation of electron, dislocation of say electron cloud leads to a dipole formation. So, initially we had a atom here, where the nucleus and is like this. So, electron cloud is just over the nucleus so, that there is no net separation of the charge and there is no electric field. Now, under the electric field what happened? There is a separation of charge.

So, this is a nucleus. So, nucleus can sit here say positive and the overall charge of the electron is somewhere here which is negative like that. So, that is why one dipole one can have this is along the electric field say. So, there is a separation of the charge and that is as a result we have dipole formation.

(Refer Slide Time: 04:48)

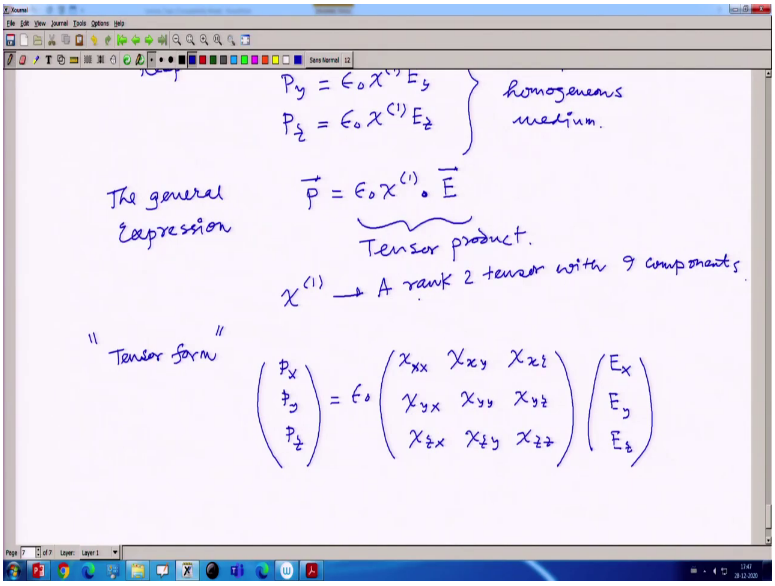


So, we know that expression of the dipole moment. So, dipole moment P is simply epsilon 0 chi 1 E it is related to electric field in this way, where P is a dipole moment. Now, this equation one can write as a linear response. When there is a electric field is applied and as a result I am having a dipole moment and if this relation is linear one can write this.

Epsilon 0 is the permittivity and chi 1 is the susceptibility. Now, for as I mentioned for linear system this equation one can have. So, for linear response when the system is linear isotropic and homogeneous, one can have an expression like this. So, if I now write component wise, this equation is written in vectorial form if I write the component wise the P x component should be related to E x component, P y component should be related to E y component.

And P z component is related to E z component. The most simple case one can have with this, but x, y and z component of the polarization vector P depends only on the x, y, z component of the electric field and that happens only for linear and isotropic homogeneous medium.

(Refer Slide Time: 08:07)



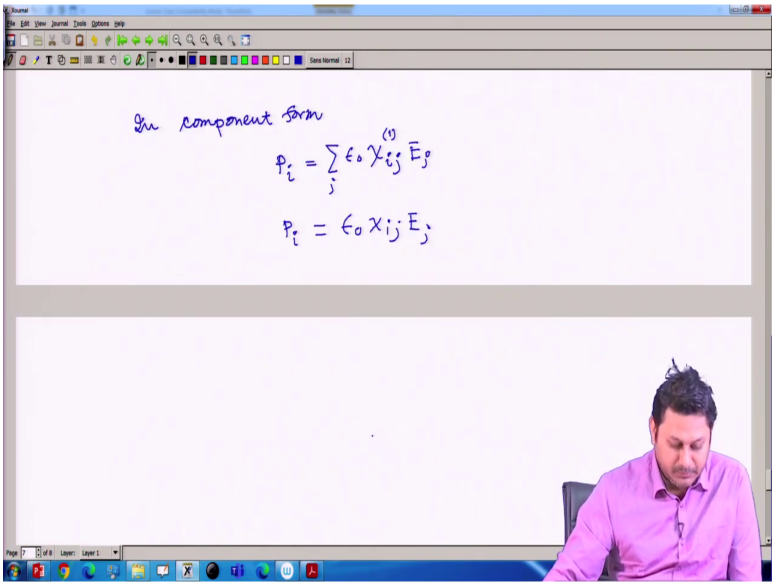
But I can have a general expression one can have a general expression. So, the general expression because it is not always true that you have a linear isotropic and homogeneous medium in your hand always so; that means, there should be some general expression of p n e general relation rather. And this general relation I can write in this form. I put a dot here if this dot has a significance, this is basically tells a tensor product.

This is not a straightforward relation, it is basically a tensor product and that is why this dot signifies this is a tensor. This is not a dot product like dot, but a dot which gives us the idea that ok these things are a tensor product. So, let me write it here this notation is for as a tensor product, where this chi 1 is a rank 2 tensor with 9 components, this is the rank 2 tensor with 9 components.

So, a complicated kind of structure one can have when the system is not linear or isotropic in nature; that means, the P x, P y, P z will not depend on any more the E x, E y, E z component respectively. So, one can write these tensor product in this way. So, the tensor form like this. So, these are the nine components and then I have E x, E y, E z. Well, you can see that this is not the P x is now not depend on the E x anymore, it is also depend on the E y and E z.

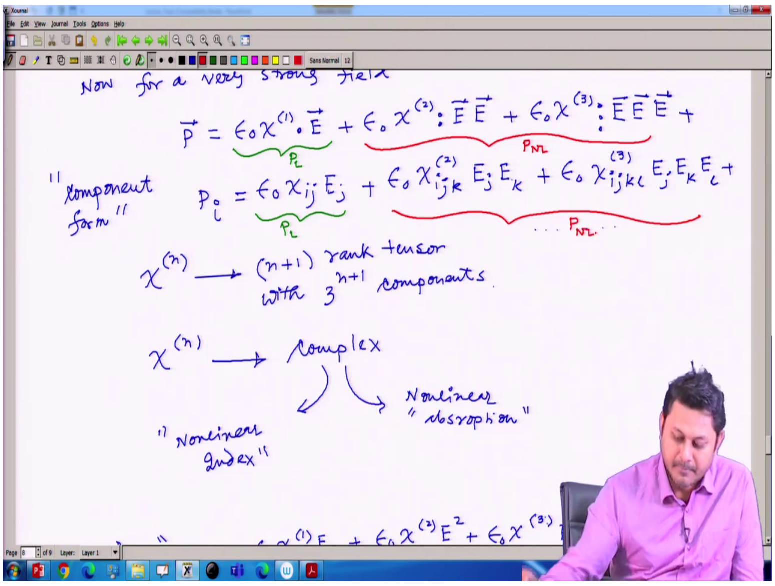
And that is why the system become a little bit complicated because it is not following the linear relationship that we had in the previous case in the simple case that P x is just proportional to E x, P y is just proportional to E y and P z is just proportional to E z; that means, these two vector E and P are parallel to each other, but here this is not the case. So, in component form, I can write this in component form. So, there is a way to write this entire stuff in a more compact form.

(Refer Slide Time: 12:26)



So, in component form: So, in I can write it simply the i th component of the P should be simply the sum of epsilon 0 over chi say ij and E j, where we should right this is a chi 1 sum over j. Now, normally we never use this sum, we use the Einstein notation and this Einstein notation is simply when you have a repetitive index like j here. So, I will just remove the sum. So, make it more it makes the thing more convenient. So, I simply write P i is ij E j using the Einstein notation, ok.

(Refer Slide Time: 13:43)



So, now if I launch a very very strong field; so, now, for a very strong field; very strong field what happened? The relationship between E and P is not more linear no more linear. So, there will be a general form. So, for a very strong field what happened? The relationship between P and E will going to change. So, we have P as, this is the first order term which we already wrote.

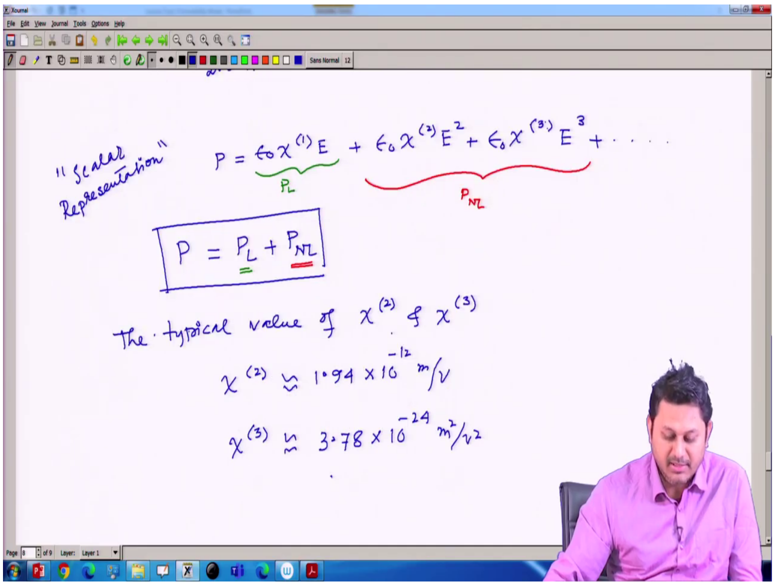
But if the E is strong then I need to add more terms like this. These two dots are representing a tensor product like this and so, on. It is much much more convenient to write in a component form. So, let me write down the component form here much more convenient component form. So, the component form this is basically the tensor form we are using. So, the component form is much more convenient and one can have the i-th component of P the left hand side is epsilon 0 ij E j that is the first term.

What is the second term? Epsilon 0 chi 2 ijk then E j, E k, then epsilon 0 chi 3 ijkl then E j E k E l and so, on. So, this is the component form and much more easier than the previous one. Here mind it chi n because I am now increasing this susceptibility the weightage of the susceptibility rather. So, this is a n plus 1 rank tensor, this in with 3 to the power n plus 1 components. For example, if I have chi 2 then n equal to 2.

So, it should be have a 3 rank it is chi 2 is a three rank tensor and it will now going to have 3 multiplied by 3. So, 3 to the power 3. So, it should be 27 components for chi 2, for chi 1 we have 9 component and so, on. It will going to increase if the order of chi is increased. Also one thing you should note that this chi n this can be a complex term, complex in nature where we have two part one is real and the imaginary. So, the real part basically tells us the non-linear index and the imaginary part basically gives us the non-linear absorption.

This is pretty much same with the linear case because in linear case also we have the refractive index and absorption inside this chi. Using the Lorenz model you can have these terms. Apart from these representation one is the tensor representation another is the component form we can also write this equation as a more simpler way which is this which is which I call say the scalar representation.

(Refer Slide Time: 19:12)



The scalar representation of the equation whatever the relation I have here. And in this scalar representation I can simply write it as this. P is epsilon 0 chi 1 E plus epsilon 0 chi 2 E square plus epsilon 0 chi 3 E cube and so on. So, we are using this form as a simplicity and it is in few cases in understanding the higher harmonic generation and all these things, this equation is found to be a little bit simpler.

And gives us certain result certain meaningful results. So, to make life simple we use this scalar representation ok. So, next thing we will do to find out what is the. So, here from this expression you can one can find that this portion is the higher order effects, this portion here is the higher order effect, this portion here is the higher order effect. And here in the scalar representation these are the higher order effect.

So, I can divide my P into two part. So, P I can write it as P linear plus P non-linear. So, this is the two part I can divide the P. So, in three representation; in these three representation here I have the P linear part. So, this part is P linear and this red part is my P non-linear in this representation. In the similar way for component form, I have this first term which tells us this is P linear and this term is P non-linear.

Here also this term is P linear term and this term is P non-linear. So, I can have a P linear here and P non-linear here. So, let me change it like this. So, when the electric field is high then only the effect of P non-linear became dominant. Normally, this value of chi 2, chi 3 are very very small so; that means, normally P non-linear term is not that much of significance and that is why it is sufficient to take only the linear term of this relation.

That means, P is a linear relation of E here is sufficient and it basically explained almost all the phenomena, if the electric field that is launched into the system is having a lower amount of energy, but if the power or the energy is very very high, then you need to take account all these higher order effects to find out what is going on inside the medium then the non-linear effect will be dominating.

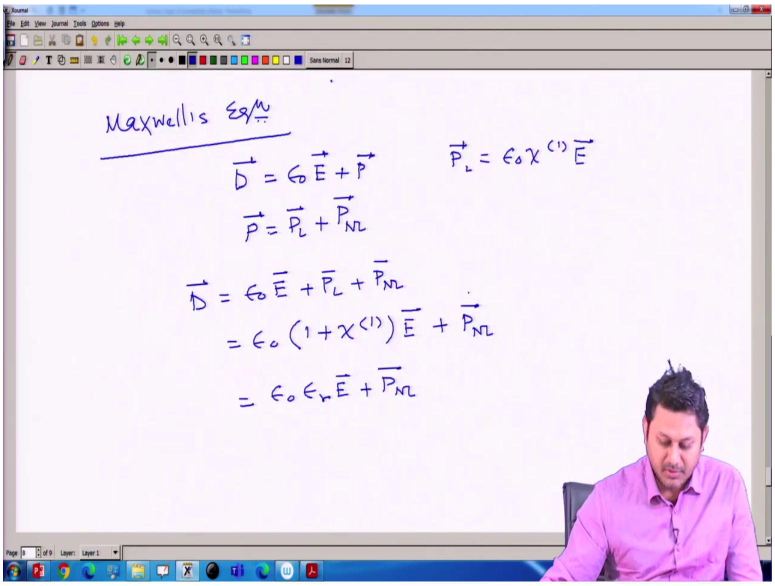
In order to understand what is going on I mean what is the weightage of this P chi 1 and chi 2, I can list this value. So, the typical value the typical just to give the order. So, chi 2 is roughly the order of say for normal silica fiber or normal silica, it is of the order of 10 to the power of say minus 12 in unit meter per volt, chi 3 on the other hand is 3.78 into 10 to the power of minus of 24-meter square per volt square.

So, you can readily understand that if I have a system where these are the typical value of chi 2 and chi 3 in order to excite the chi 3, you have to put lot many of energy. So, I can read I can from this value of chi 2 and chi 3 I can readily remove the effect of chi 3 compared to the chi 2.

So, in order to excite the higher order effect say chi 2 or chi 3 we need to launch in light having high energy very high power or high energy that we will going to discuss in the next few classes. But you should note that the value of in general the value of chi 2 and chi 3 are very very less that is why inside the fiber normally we do not bother about this higher order effect.

But if somehow the launch electric field is very high then because of the presence of this higher order effect, we will going to get something inside the medium and this something is nothing, but the non-linear interaction between the light and the medium ok. So, the effect of let us try to understand what is the effect of this higher order polarization. So, I will going to rewrite the Maxwell's equation so, Maxwell's equation now going to have a new kind of form.

(Refer Slide Time: 26:24)

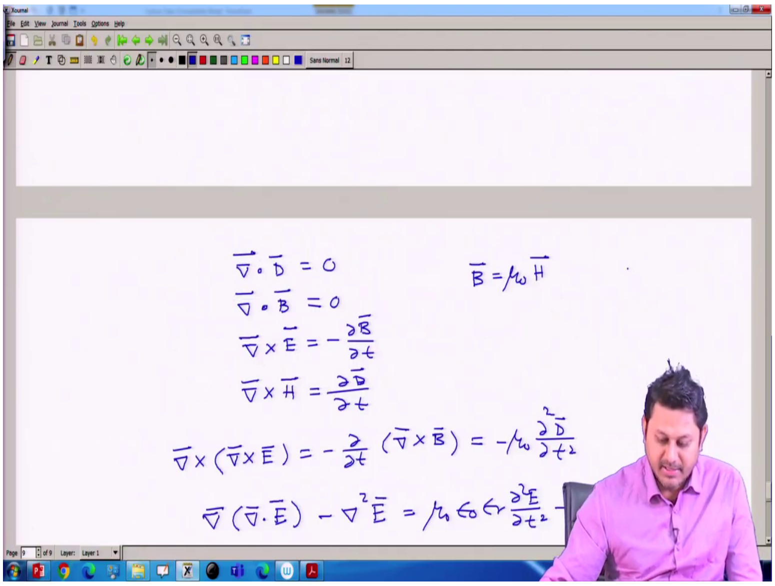


So, let me go back to the Maxwell's equation. So, we know the Maxwell's equation in the medium. So, I can have a D which is epsilon 0 E plus polarization P inside a dialectic medium say I can have the value of D like this, where polarization P can have a linear part and non-linear part.

So, P L I can write it as epsilon 0 chi 1 E it is the linear part of the polarization it can have also the non-linear part. So, let me write it here clearly. So, P is now having two part one is the linear part and another is the non-linear one. So, linear part and non-linear part we have for P.

D now I can write it as epsilon 0 E plus P linear plus P non-linear. Epsilon 0 E plus p linear is again epsilon 0 chi 1 E. So, I can write it as simply epsilon 0 1 plus chi 1 E and P non-linear. This quantity is epsilon 0 epsilon r because this is epsilon r E plus P non-linear.

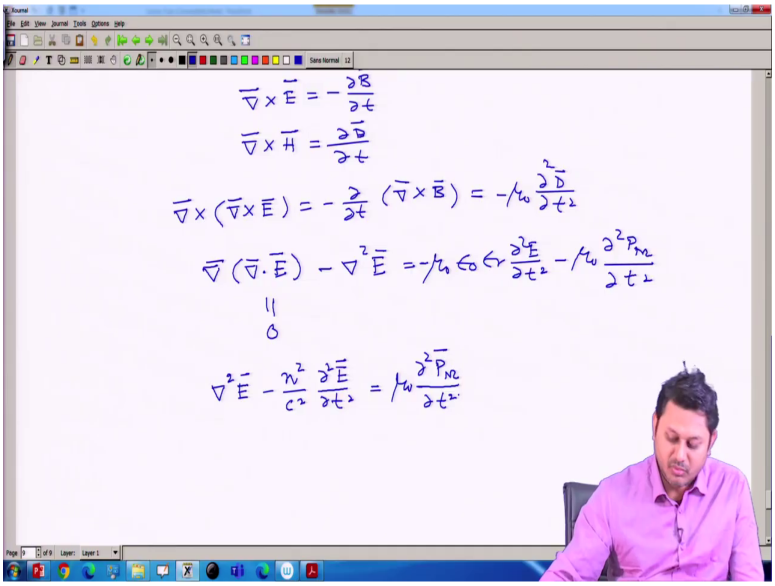
(Refer Slide Time: 28:34)



Well, in absence of any source the Maxwell's equation I can write once again in terms of D and all this. So, I have this equation first equation, this is the second equation curl of E so, equal to del B del t. And then curl of H is del D del t. So, this is D ok. So, these are the four Maxwell's equations we have also additionally I should write B is equal to mu 0 H ok.

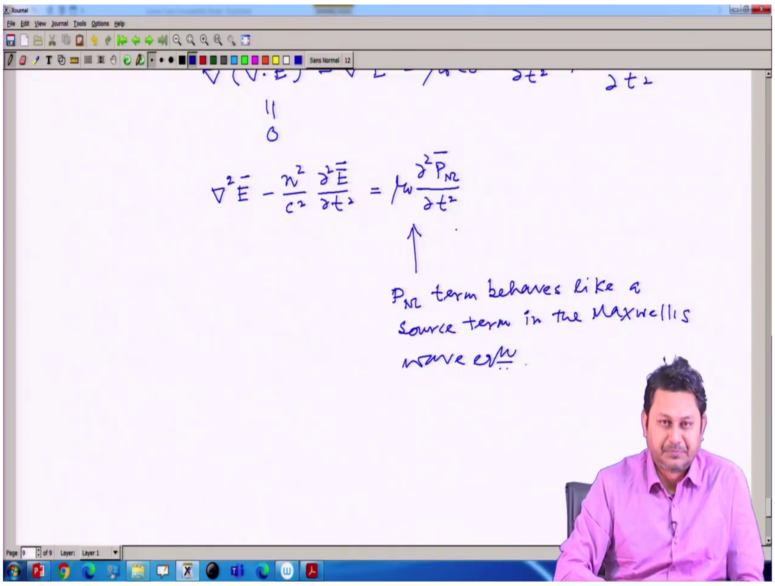
Now, if I take the cross product of the second equation then I can have both side I can have a curl of this equation, third equation I can have this one. Curl of E is equal to minus of del del t and curl of B which is minus of mu 0, curl of B I can write from this equation has del 2 D del t square. This quantity I can have mu 0 epsilon 0 epsilon r del 2 E del t square minus mu 0 del 2 P NL del t square because I just write d in terms of this. So, I just write this here ok. Here I should have a negative sign.

(Refer Slide Time: 31:08)



So, this quantity is 0 because there is no source. So, I can have an equation something like this, this is the wave equation, but apart from that I have something in the right hand side that is important. So, I am having a source term in here which is associated to the contribution of the non-linear polarization that is important.

(Refer Slide Time: 31:45)



So, I should write it here the P non-linear term behaves like a source term in the wave equation in the Maxwell's wave equation. So, I derived the Maxwell's wave equations in a straightforward manner, but normally this right hand inside is 0, but here I find that instead of 0 I am having the contribution of the non-linear polarization.

So, in the next class we will try to find out what is the consequence of these source term which is coming through the non-linear polarization effect and because of this non-linear polarization effect we will have the higher harmonics. And also the refractive index we will go into modify which is called the car effect. So, that thing we will going to discuss in the next class. So, till then goodbye and see you in the next class.