Physics of Linear and Non-Linear Optical Waveguides
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Module - 04
Fiber optics components
Lecture - 48
Bandwidth of Reflectivity

Welcome student to the course of Physics of Linear and Non-Linear Optical Waveguide. Today, we have lecture number 48, where today we will going to calculate the Bandwidth of the Reflectivity of a given FBG ok.

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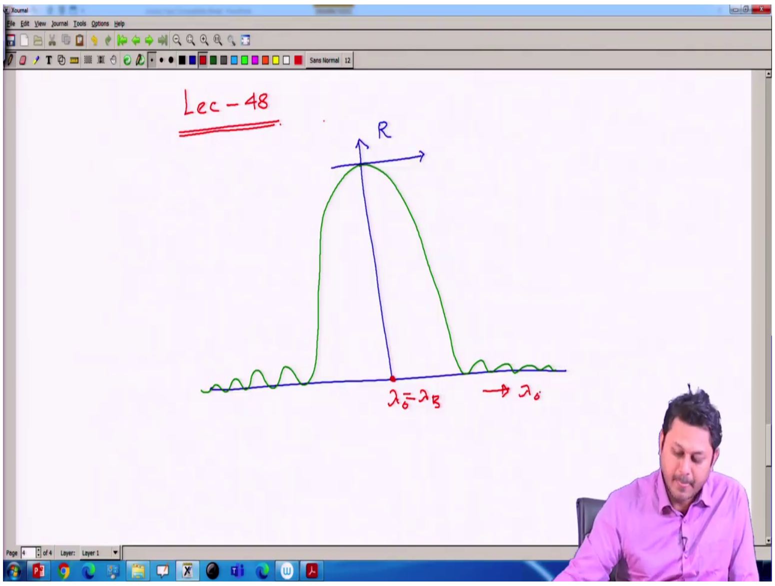

So, if you remember in the last class we had a reflectivity curve. So, let me draw it once again. So, the reflectivity curve was something like this. Along this direction I plot, this is reflectivity R, and this is the peak reflectivity. And, this point I have lambda 0 equal to lambda B, this is the point I am talking about. And along this direction I basically I vary lambda 0.

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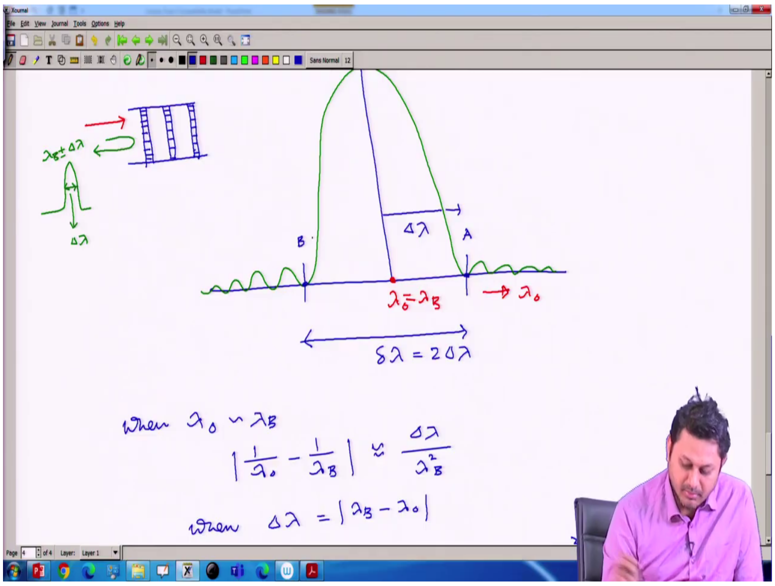

If, I change lambda 0, the expression of the R will going to change, that we have shown in the last lecture. And, if I plot accordingly then this should be the typical curve of that reflectivity curve. Now, if I look carefully if I move towards left or right of lambda B. Then, I still have a range of wavelength where certain amount of energy is reflected web back from the grating.

So, let me draw the grating here, side by side. So, this is the typical structure of the grating. So; that means, when I launch a light here with a range of lambda, then due to the Bragg grating something is reflected back and this reflected back thing has a bandwidth, because the reflection curve tells me that. So, this bandwidth is lambda B plus minus of delta lambda.

So, I have not a single wavelength, but a range of wavelength here I am talking about this is delta lambda, which we will going to reflect back. Lambda B is a big value and then I have a range of wavelength a width of wavelength. And, this width we will going to calculate today. So, let us say this is my small delta lambda. And, from here to here to this point actually this is my delta lambda. So, this; that means, this is equal to 2 of delta lambda big delta lambda that is all.

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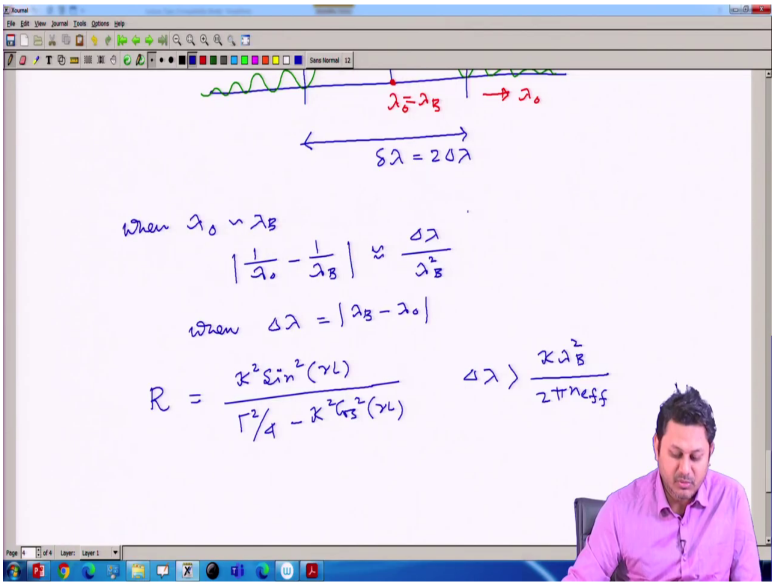

Well, I know, so, when lambda 0 is very close to lambda B nearly equal to lambda B, then this expression 1 divided by lambda 0 minus 1 divided by lambda B can be simply written as delta lambda divided by lambda B square. Where delta lambda, when delta lambda is equal to the difference between the Bragg wavelength and the launching wavelength ok. So, in terms of delta lambda, if I write my R, I know this is when this condition is greater than.

So, it should be sin of sin square of gamma L whole divided by, gamma squared divided by 4 minus kappa square cos square gamma L. This condition is for when delta lambda when greater than kappa of kappa lambda B square whole divided by 2 pi n effective. This is the condition I derive earlier and then I modified this portion and this portion if I write so, in terms of delta lambda.

So, that is the case so; that means, when the difference between lambda and delta lambda 0, a and lambda B is relatively larger, than the expression sin square is coming to the picture. Previously it was the sin hyperbolic form; that means, this way we thing now come into the picture because of this sin square variation, sine and cosine variation. So, for this case now we will going to calculate.

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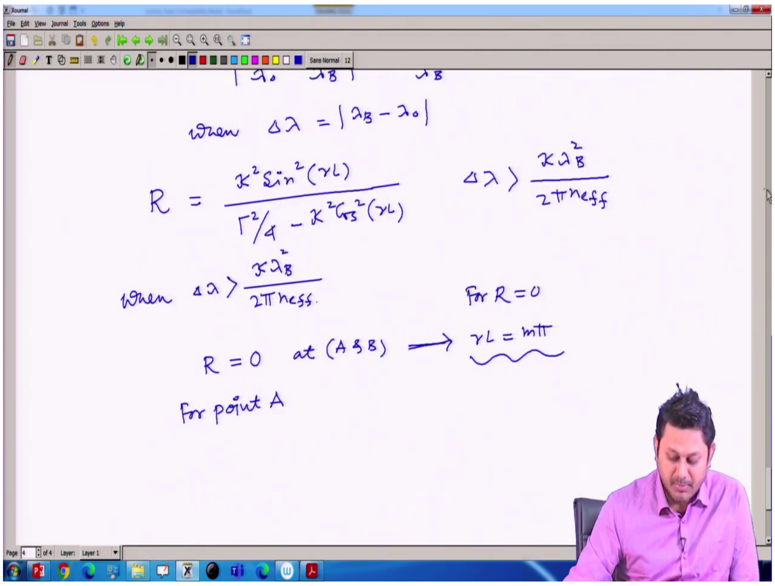

So, we have so, when this condition is valid is greater than kappa lambda B square divided by 2 pi n effective. This condition is valid, then what happened we can have certain value of R and R equal to 0 1 can have here in this point, this point and this point. So, suppose this is A point A and this point is B. So, R equal to 0 at point A and B and for 0.1, for point A, I have so, this condition is when gamma L in general gamma L is equal to m pi then only this happens.

For R equal to 0, whenever we have R equal to 0, gamma L has to be m pi from this expression if I put gamma L equal to m pi. So, the sin square term is always 0. So, we have these 0 points here, then another 0 point here and so on. So, if I want to find out what is what is the value of a gamma L? When I am talking about only this point A, one can readily find out that, for point A m is equal to 1.

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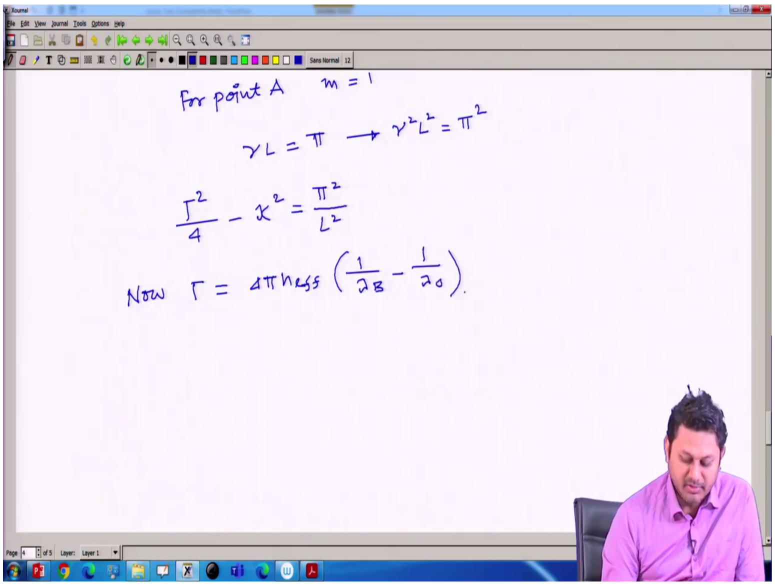

So, I can have gamma L is equal to simply pi or I can write gamma square L square is equal to my pi square. So, I have gamma square L square equal to sorry this is pi square so, pi square. So, I will now, I am going to use the expression of the gamma.

So, this is 4 minus kappa square is equal to pi square divided by L square, because small gamma square is big gamma square divided by 4 my kappa square, that I already defined earlier. After that I can now right so, now, gamma also I derive this is 4 pi n effective into 1 by lambda B minus 1 by lambda 0, that we already derived.

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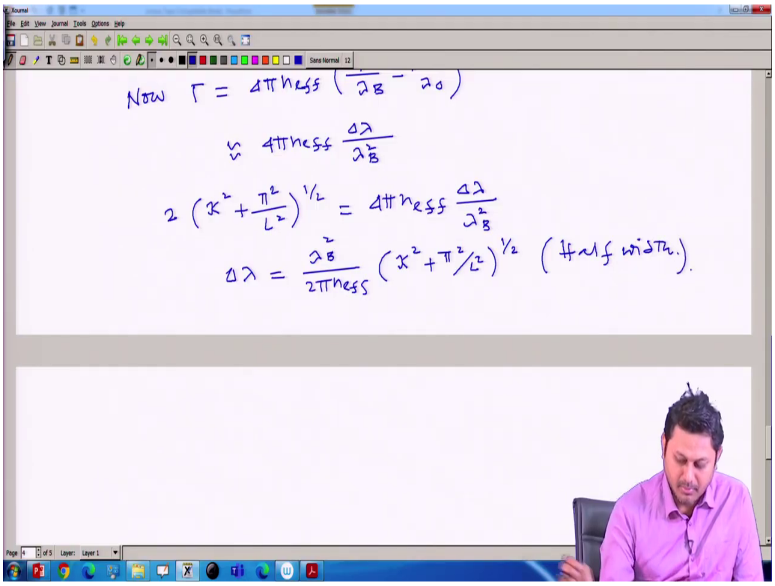

So, this quantity is nearly equal to 4 pi n effective. Then, delta lambda divided by lambda B square, where delta lambda is lambda 0 minus lambda B. And, lambda 0 is close to lambda B, so, that is why I can write it. So, now, I will going to use this stuff here and 2 of this quantity, say kappa square 2 of kappa square plus, gamma square pi square divided by L square whole to the power half, this is gamma divided by 2.

So, 2 if I multiply these sides and it should be this is nothing, but gamma that is equal to how much that is equal to this quantity, which is 4 pi n effective and then delta lambda divided by lambda B square ok. So, delta lambda I can calculate delta lambda is the. So, value that I wanted to calculate. So, that is the expression of my half width.

So, this is simply lambda B square, then divided by 2 pi n effective 1, 2 will cancel out and this term will be 2. So, it should be 2 by n effective then multiplied by kappa square plus gamma square divided by L square. So, we are having an expression of half width like this. So, this is half width. Now, I can calculate the full width by just multiplying it is the value, which 2.

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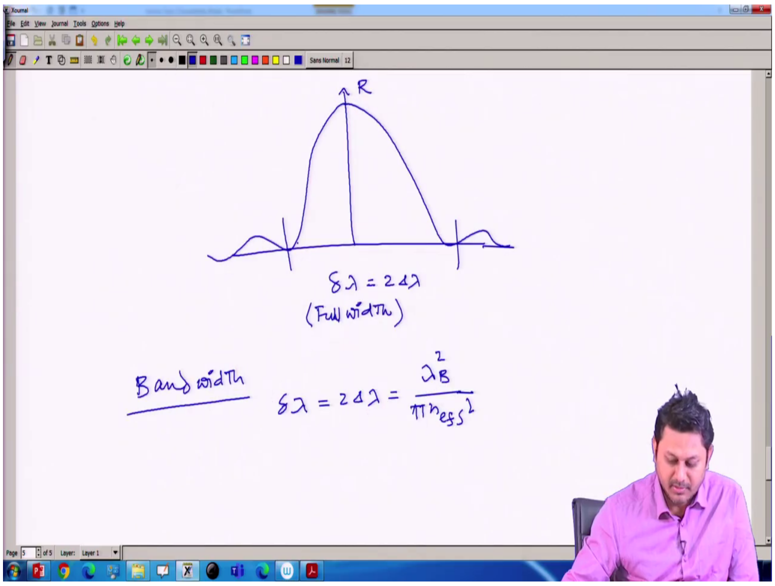

So, this is the reflectivity curve. So, I so, this is the full width according to our notation it is delta lambda, which is 2 of this lambda and this is the full width. So, an along this direction I have reflectivity R.

So, the bandwidth: the bandwidth is simply delta lambda which is 2 delta lambda and this big delta lambda I already calculate. So, it should be simply lambda B square divided by pi n effective I just simplify it little bit divided by L I take this L common from this. So, ok I should write properly here it should be L, n effective multiplied by L.

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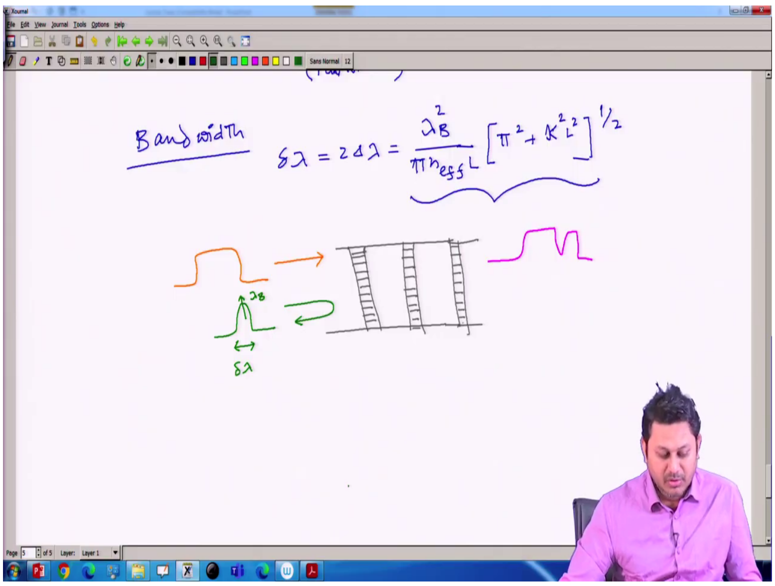

And, the rest of the term is pi square plus kappa square, L square, whole to the power half. So, this is the expression of the bandwidth so; that means, I already mentioned earlier, that if I have a grating structure say like this periodic variation of refractive index. So, the light that is in the incident light have a spectra suppose this is the spectra of incident light. So, this is in ongoing.

So, when it is coming outside, it should have a structure like this. And, the reflected light will be like this. So, this is the width we are calculating here. So, this width is simply delta lambda. So, and here we have a Bragg wavelength. So, the central wavelength is the Bragg wavelength. So, this is lambda B. Surround lambda B there is a delta lambda, which is also reflecting through this Fiber Bragg grating structure, ok. So, now, we will jump to the last part of the Fiber Bragg grating and this last part is the sensor.

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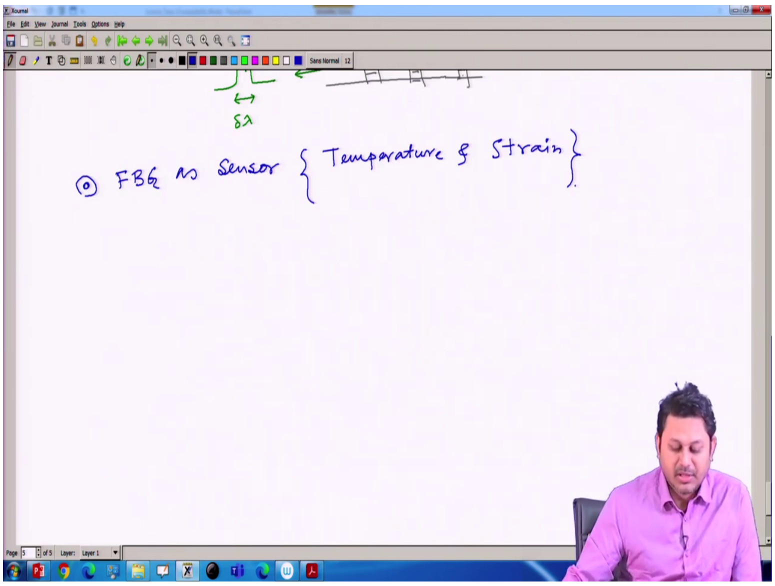

So, FBG as sensor: So, there are two kind of sensing one can have using the Fiber Bragg grating one is temperature and strain. So, two kind of sensor one can mate in principle with FBG one is the temperature and another is the strain.

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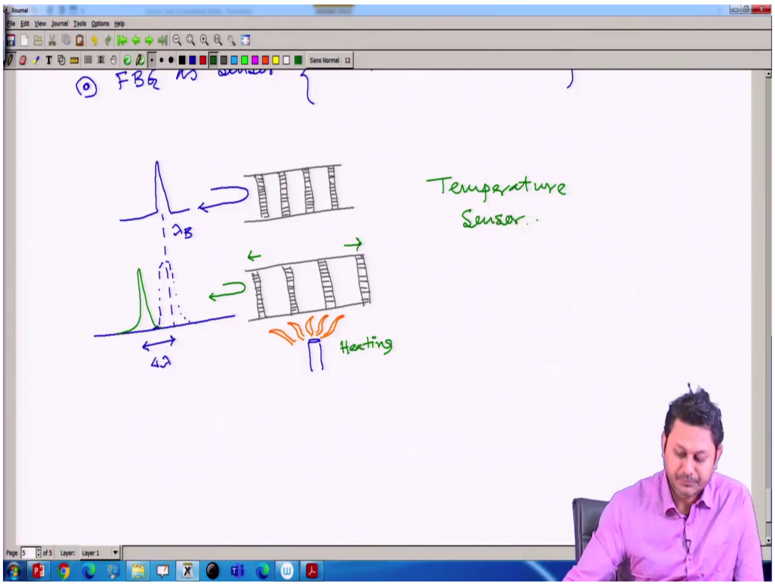

So, what is temperature sensor? So, quickly let us draw the Fiber Bragg grating structure. Suppose this is a Bragg grating we have I am just drawing the concept schematically. So, that, you can understand what is going on here inside. So, this is the Bragg grating. And, you have a similar Bragg grating here, but now what happened at this Bragg grating is now heated up.

So, once it is heated up. So, what happened? There is a change of it is period, that is the essential thing the grating period big lambda will going to change, because of this heating. So, schematically I if I draw it should be something like this. So, I have some say heating substance here and some heating is provided here like this, burn or a something.

So, what happened if I now so, I should write it here it is heating. Due to the heating what happened there will be elongation of Bragg grating. Previously, if I draw the previous curve, so, suppose one wavelength is reflecting back like this. So, this is the wavelength that is reflecting back and this is our Bragg wavelength.

So, this wavelength is lambda B. Now, due to the heating what happened that a new wavelength will going to reflect back, because the grating period is now changed. And, I can have a shifting here. Initially the wavelength that was reflected from the structure before heating was something like this, dotted one and now there is a shift there is a shift of wavelength, due to the heating.

So, if somebody measured this shifting then they can find out what should be the value of the temperature. Because, they can calibrate with temperature they can calibrate how much shifting is there and from that they can use this Fiber Bragg grating as a very sensitive temperature sensing. So, this is a schematic diagram how the temperature sensing works? So, this is temperature sensing or temperature sensor.

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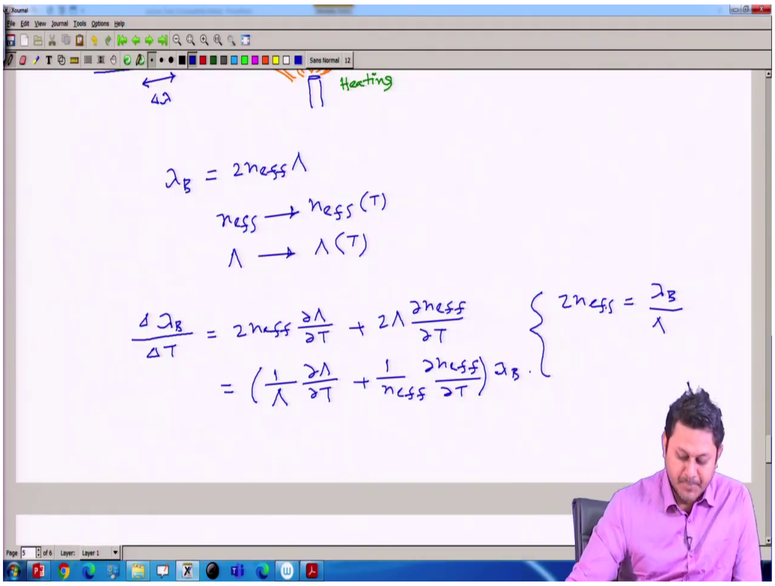

Well, we can find out a mathematically what is the formulation of this, what is the principle of this sensing? So, lambda B which is 2 n effective and then big lambda, that is the expression. Now, because of the temperature what happened n effective we need to take account that n effective as a function of temperature.

And, grating period is also function of temperature. Both are function of temperature. Now, when these two are function of temperature, if you change the temperature definitely lambda B will going to shift, so, that will going to calculate. So, lambda B the change with respect to temperature will be simply 2 n effective the rate of change of the period with respect to temperature plus, 2 of gamma the rate of change of n effective with temperature.

2 n effective I can use from here the value lambda B divided by so, I can from here I can have 2 n effective is simply lambda B divided by big lambda. Using that and also 2 lambda is equal

to lambda B divided by 2 n effective. Using lambda B divided by n effective, using that I can write simply this quantity as 1 divided by lambda, delta lambda delta T plus 1 divided by n effective, n effective, by delta T and then I have lambda beta finally.

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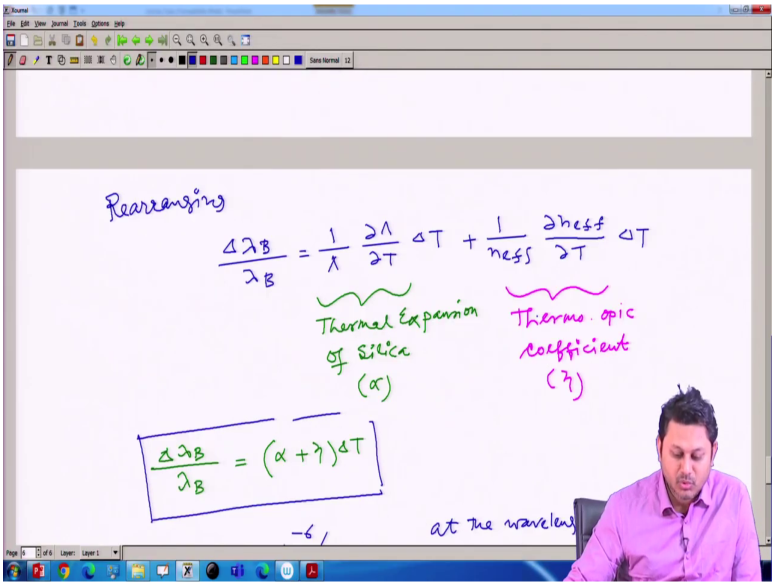

So, this quantity the first one, so, rearranging if I now rearrange. So, rearranging I can have delta lambda beta, divided by lambda beta is simply equal to 1 by lambda, delta lambda delta T, this T plus 1 by n effective delta T, delta T. So, this quantity is some sort of known thing, that what is the change of period with respect to temperature? So, this is nothing, but the thermal expansion of silica is simply thermal expansion of the silica.

And, this quantity is how the n effective is changing with respect to; that means, the refractive index is changing with respect to temperature. It is thermo optic coefficient, what is the change or rate of change of refractive index with respect to temperature divided by 1 by refractive index. So, these quantities normally defined as eta and this quantity is normally defined at thermal expansion as defined at alpha by definition.

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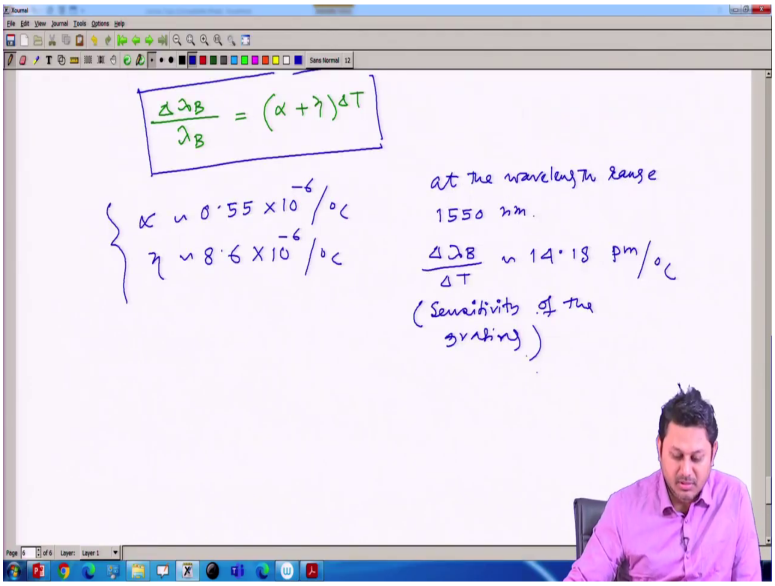

So, the equation simply becomes delta lambda B divided by lambda B simply becomes alpha plus eta delta T. So, this is the expression we have finally, in terms of the parameter alpha and eta. Now, typically alpha is of the order of 0.55 into 10 to the power of minus 6, per degree centigrade.

And, eta is order of 8.6 into 10 to the power of minus 6 centigrade; these are the typical values of alpha and eta. So, at the wavelength range say at the wavelength range around wherein say around 1550 nanometer, one can have the typical value of lambda B divided by this is of the order of 14 point say is picometer per degree centigrade.

That means, if I increase the temperature to 1 degree, then there will be shift of delta lambda up to 14 around 14 picometer for this system. So, this basically this quantity is basically the sensitivity of the grating. The next thing that I like to cover today itself and that is another sensitivity which is the stress, another sensor one can use using the Fiber Bragg grating, which is a stress sensor.

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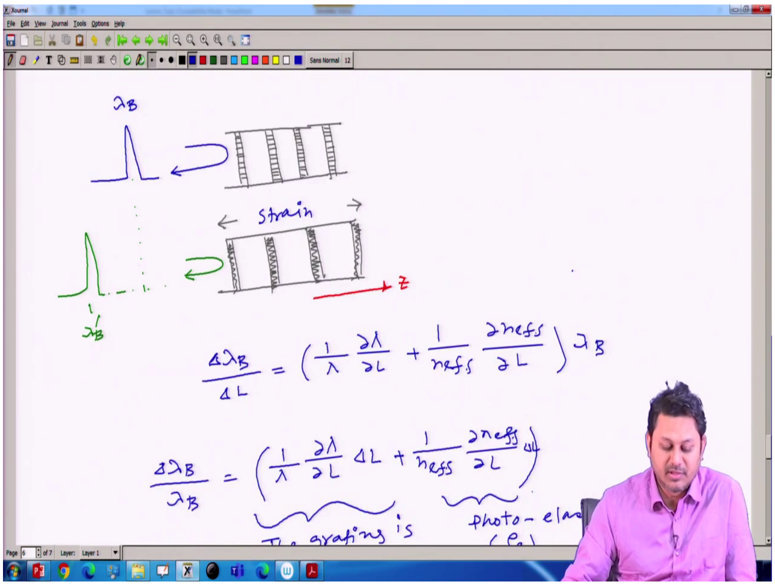

So, let me quickly draw the concept. So, this is the grating structure and exactly the previous way, if I now stretch it. Previously we stretch it by using temperature; there is a elongation due to temperature. Now, this elongation one can have by just stretch this fiber.

So, again one can have a reflected wavelength the same old figure I like to draw this is lambda B. And, now this grating is stretched. So, when it is stretched what happened? So, this is the grating structure. So, it is now stretched, from this side and this side there is a stretching. And, now what happened that the wavelength that will going to reflect is someway different.

So, suppose this is lambda B, previous lambda B, now it will reflect a wavelength somewhere here say. So, this is say lambda B prime. So, there is a change of lambda because of this stretching and this is this principle the same old figure, but the concept is slightly different here, I am having a strain here and previously it was a temperature.

So, the previous way I can write the expression this delta lambda B divided by delta L is equal to L is the amount of stretch, one can have 1 divided by lambda, delta lambda delta L plus 1 divided by n effective, delta n effective divided by delta L. And, then lambda B again one can rearrange and have the form like delta lambda B divided by lambda B is equal to 1 by this plus this.

Now, this quantity is interesting let me quickly write it, what is this quantity? So, you can see there is a change of lambda with respect to L and there is a ratio a lambda is a grating period already written in the, already written in the fiber course. If, I expand this amount of expansion one should have of this grating period with respect to the initial period should be same as the increment of the entire fiber.

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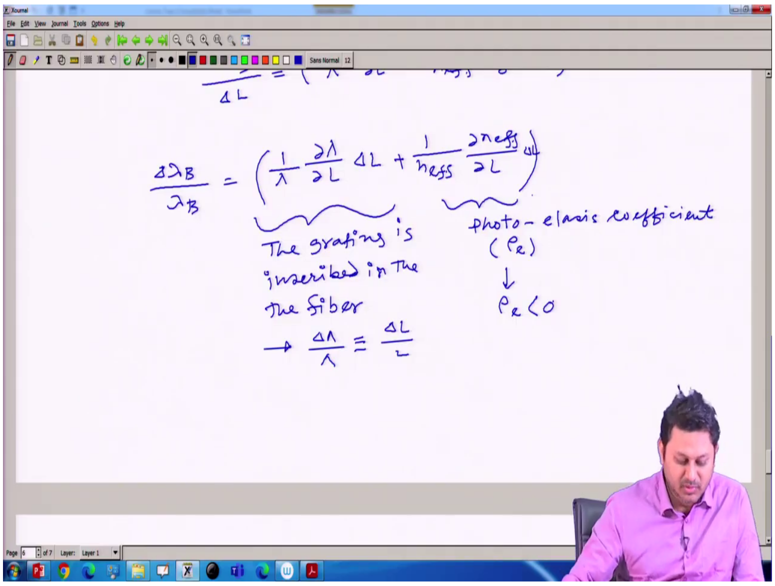

So, the grating is inscribed in the fiber; in the fiber, so, grating is already written in the fiber. So, from that I can write delta lambda divided by lambda is same as delta L divided by L. Hence, this contribution is unity. On the other hand this is the photoelastic coefficient.

This is the, so, I have a delta L missing here, so, I should write here delta L. So, this is simply the photo elastic coefficient that is that change of refractive index with the change of strain. So, this is simply written as rho e and these value is normally rho is negative, why? Because, if I extend the fiber then the normally what happened due to this stretch the refractive index reduces. So, that is why rho is normally less than 0.

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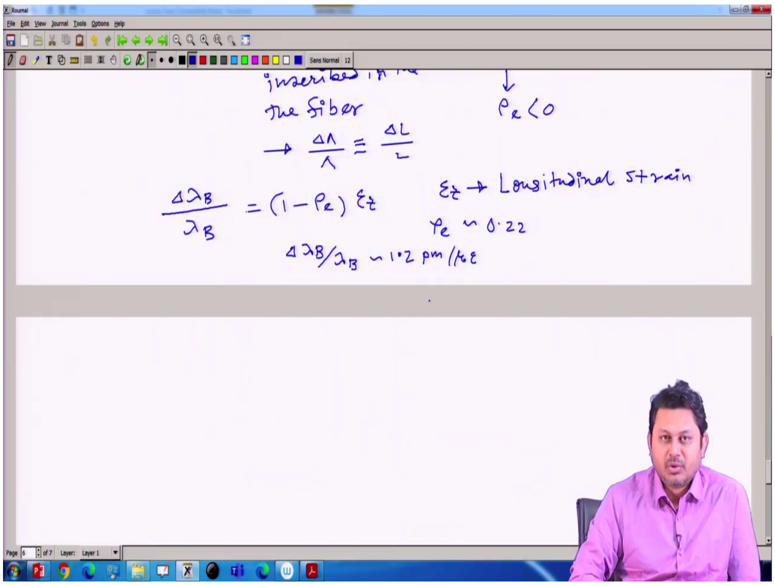

So, since rho is less than 0 and this is unity I can simply have the expression delta lambda B divided by lambda B is simply becomes 1 minus rho e and this is epsilon Z. What is epsilon Z? Epsilon Z is a longitudinal strain, because mind it this is along Z direction, so, this is the this is along Z. So, fiber is placed along Z direction and I am putting my strain along the Z direction. So, that is why this is the longitudinal strain.

And, normally what happened that the value of rho e is of the order of say 0.22 and from that one can have delta lambda B, the sensitivity of with strain is of the order of 1 point say 2 picometer per micro strain. So, this is the sensitivity roughly one can have using the fiber one can measure that amount of sensitivity, that amount of strain by just looking the fact that, how much lambda is lambda B is shifting from the Bragg grating.

So, using these one can use; one can use these as a sensor using this principle this theory. So, with this note I would like to conclude today's class. So, here we will conclude also our module 4, which is based on the fiber component. So, in this module I mainly discuss two component, important component; one is optical coupler and another is fiber based coupler and another is the fiber base Bragg grating.

So, both are very very important in applications and intact theory is discussed with all the required mathematics. So, I hope you have a profound understanding what is going on inside, what is the physics behind that and that is the goal. So, from next class I will start the our final module, which is the Non-Linear Fiber Optics. So, till now goodbye and see you in the next class.