Physics of Linear and Non-Linear Optical Waveguides
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Module - 04
Fiber optics components
Lecture - 47
Reflectivity Calculation of FBG (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. Today we have lecture number 47 and we will going to continue the Reflectivity Calculation that we have started in the last class.

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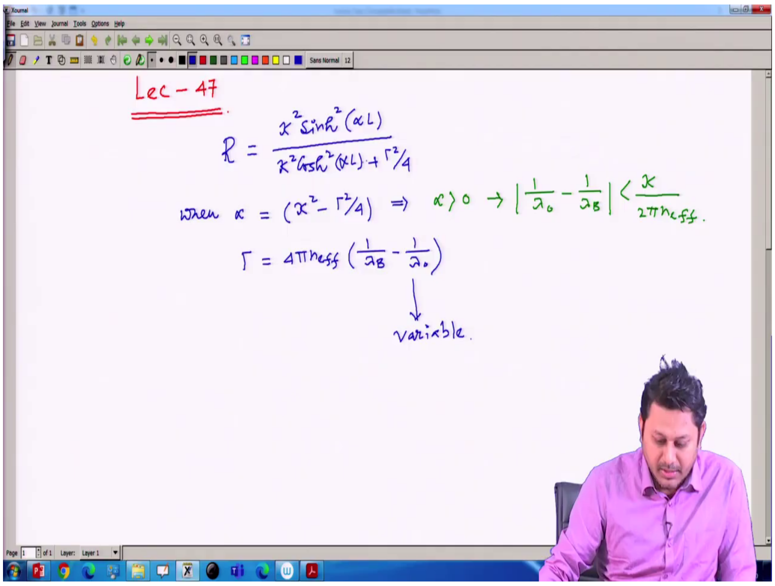

So, if you remember we find the reflectivity expression of the reflectivity R for the Bragg grating as kappa square sin hyperbolic square alpha L, where L is the grating length and then divided by kappa square cos hyperbolic square alpha L then plus gamma square by 4 that was the expression we derived, when alpha is defined as kappa square minus gamma square divided by 4.

And also in this condition we have alpha greater than 0 and for that we had an expression like that ok, fine and also the gamma was something like 4 pi n effective 1 divided by lambda B divided by 1 minus lambda 0. Now this 1 by lambda 0 term that is the variable parameter this is variable.

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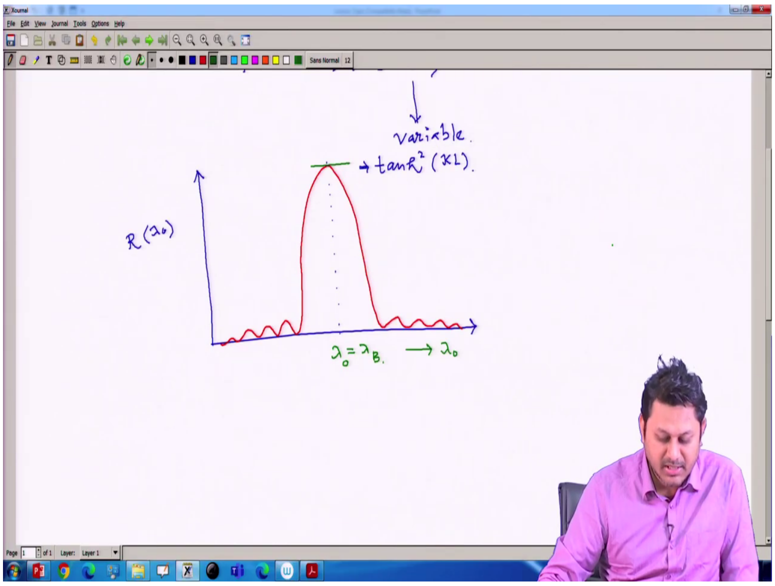

Now, using this expression if I now able to if I now plot this reflectivity curve R as a say function of lambda 0 the operating wavelength it should be something like this. The reflectivity curve will be in general something like this where in this axis I plot the operating wavelength lambda 0 and this point is when lambda 0 matches with the Bragg wavelength lambda B.

So, at this point I have a maximum reflectivity and we know the maximum reflectivity what is the value of the maximum reflectivity and from here I can see that when it is 0 then it should be simply that value should be simply tan hyperbolic square then kappa L that was the maximum value of the reflectivity or the peak reflectivity at the condition of when lambda 0 is equal to lambda B; well it looks B so let me rewrite it once again this lambda 0 ok.

So now another condition so you can understand that when lambda 0 is changing there is a possibility that this condition does not hold anymore; that means, 1 divided by lambda 0 minus 1 by lambda B mod of that may not be less than equal to less than 2 kappa divided by 2 pi n effective.

If another condition is possible that is this quantity is now greater than this one, then obviously the solution will going to change and exactly using the earlier technique we can find out what is the expression and let me do that in detail.

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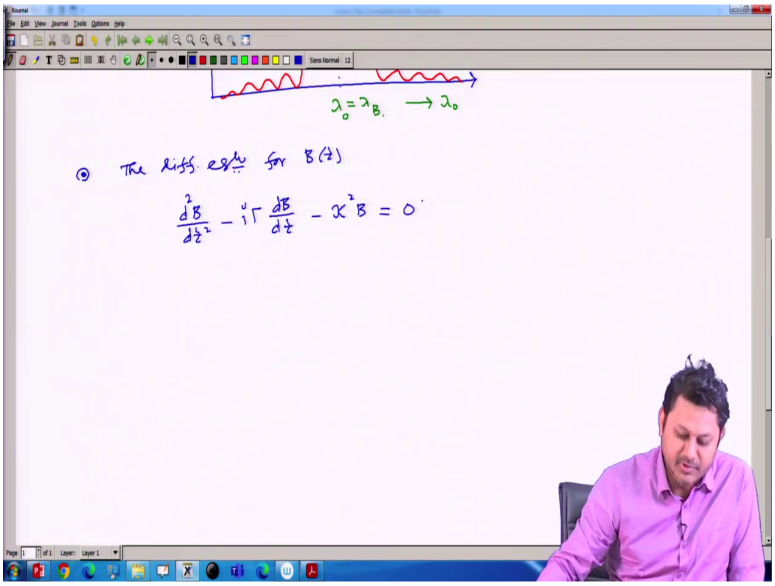

So, the differential equation let me once again write the differential equation for B which is a function of z as this. So, that was the differential equation we solve that is the first differential equation we take and then solve and put some condition to find out what is the reflectivity. So, here we are going to do the same thing, so minus kappa square B equal to 0 that was the differential equation B.

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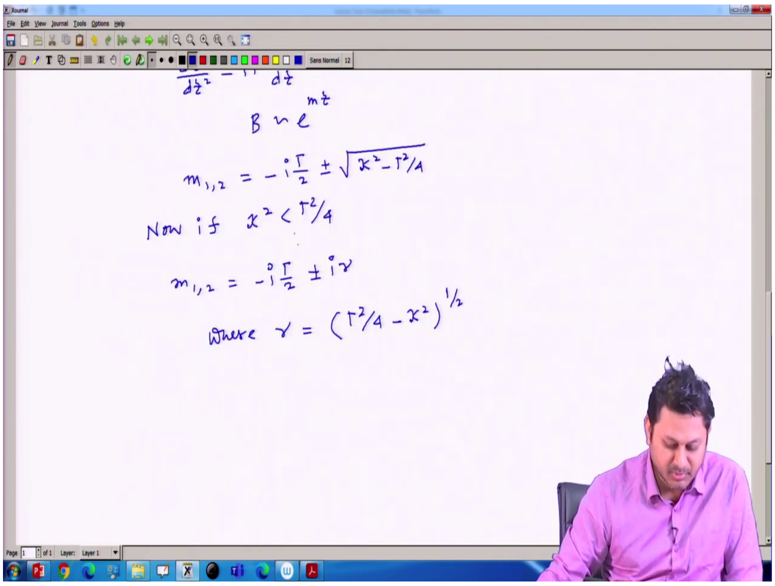

Now I am going to solve that so B I am expecting a solution something like this, then I know I can find out the value of this m and let me write down quickly. So, my m 1, 2 will be minus of i gamma by 2 plus minus of root over kappa square minus gamma square by 4.

o, kappa square minus gamma square by 4 this condition appears here and now if kappa square is less than this quantity then what happened I should have a solution something like m 1, 2 is equal to minus of i gamma by 2 plus minus of i small gamma; where this small gamma is equal to gamma square by 4 minus kappa square whole to the power half.

So, this is a quantity gamma square divided by 4 minus kappa square previously it was kappa square minus gamma square divided by 4. So now, it will going to change because the condition is this kappa square is now less than this 1, so that gamma is positive.

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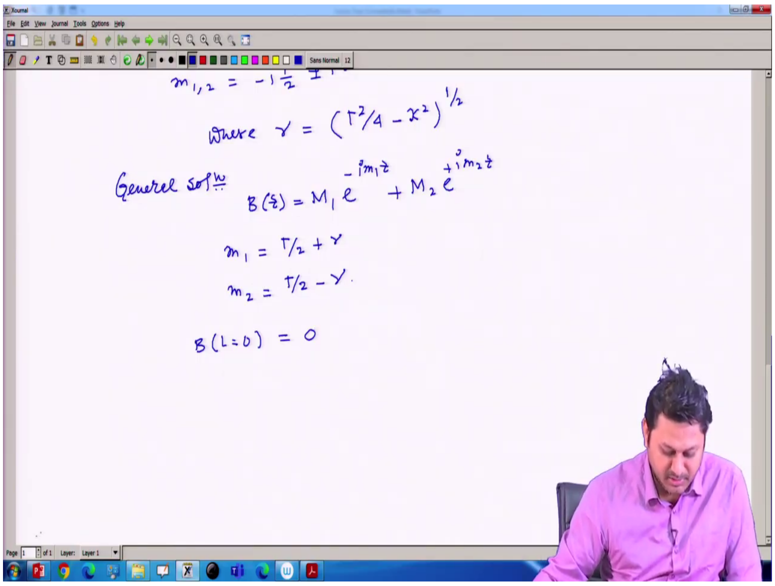

So, the general solution for that if I want to find out the general solution it should be simply equal to some constant M 1 e to the power of say minus i m 1 z plus big M 2 e to the power of minus i say this 1. So, now I put M 1 and M 2 if I put M 1and M 2 then I have s m 1 is say gamma by 2 plus gamma and m 2 is gamma by 2 minus gamma.

And the boundary condition tells us that at B at L equal to 0 is 0 that is the boundary condition that is the boundary condition we have. So, if I use this boundary condition because at the end of the grating this quantity is 0.

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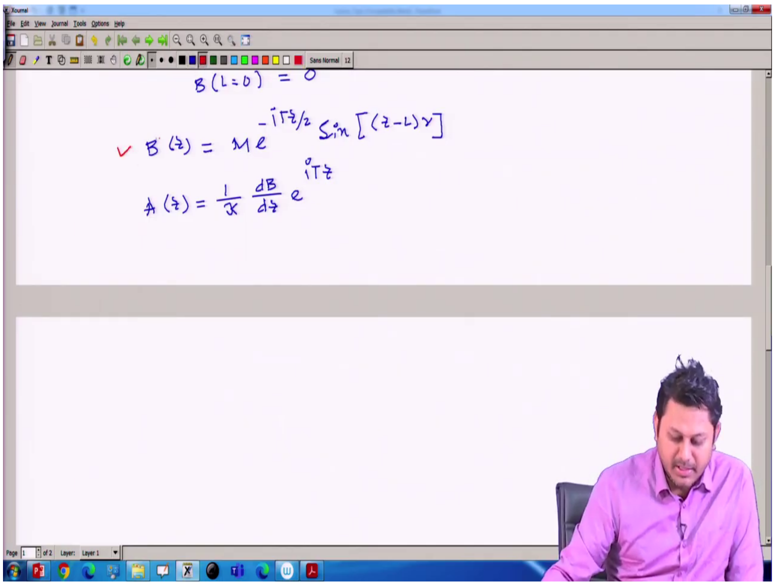

If I use this boundary condition and do the calculation I will simply have B z, because the calculation is straight forward and I have already done this calculation earlier. So, I am not repeating the same calculation with some constant M e to the power of minus of i gamma z by 2. Now instead of a hyperbolic function now I will going to have a sine function like that sin of z minus L sorry sin of z minus L.

So, let me erase it and write it once again sin of z minus L gamma that is the solution of B. In a similar way you can find out the solution of A I am writing the solution because A is let me first write what is a it is 1 by kappa del B del z e to the power of i gamma z that was A.

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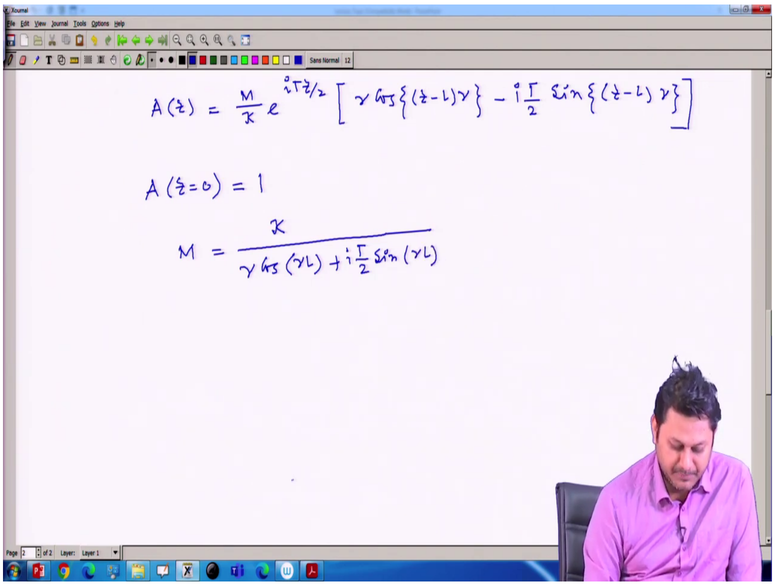

So, from this B whatever the value I find as b I can write the solution for A and the expression of the A should be something like M by kappa then e to the power of i gamma z by 2 and then I have gamma cos of z minus L small gamma minus 1 by gamma by 2 sin of z minus L gamma bracket close, so that is my A.

Here another condition we also have that A at z equal to 0 is unity, without any loss of generality we consider that from the very beginning and then M should have a form like this cos of gamma L plus i big gamma divided by 2 then sin of gamma L that is the condition of that is the value of M.

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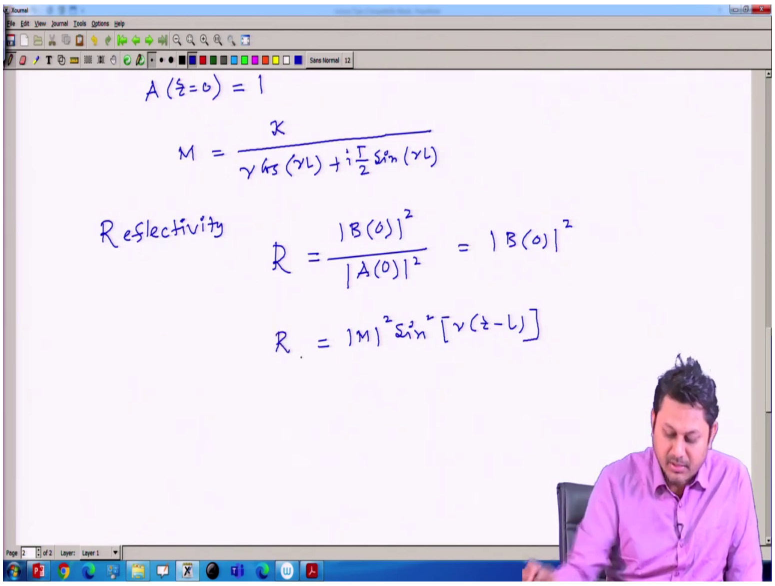

Now, the reflectivity which I wanted to calculate are which is mod of beta 0 square divided by mod of A 0 square, so that means at the output at the input what is the value of the propagating and forward propagating and backward propagating amplitude ratio of the forward and backward propagating modes and this value is essentially mod of square because this quantity is 1, that value is simply mod of M square and then sin square then gamma z minus L.

So, that is the value of the reflectivity I calculate under the condition that my condition here is this I should. So, this is the condition based on which I am calculating right now previously the condition was opposite that kappa kappa square was greater than this one. So, I have a reflectivity and the value of M is also there.

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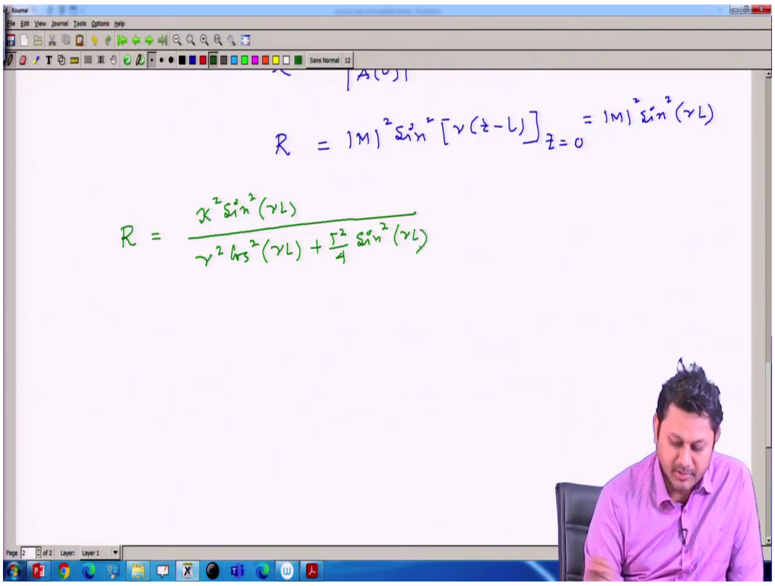

So, I can directly write the reflectivity here as kappa square then sin hyperbolic square ok. So, I make a mistake here. So, this value should be at z equal to so this value should be at z equal to 0, so this is the expression. So, that is at z equal to 0. So, this quantity is eventually mod of M square sin square gamma L.

So, then it should be sin square of gamma L divided by gamma square cos square gamma L plus gamma square by 4 and then sin square gamma L that is the full form of reflectivity.

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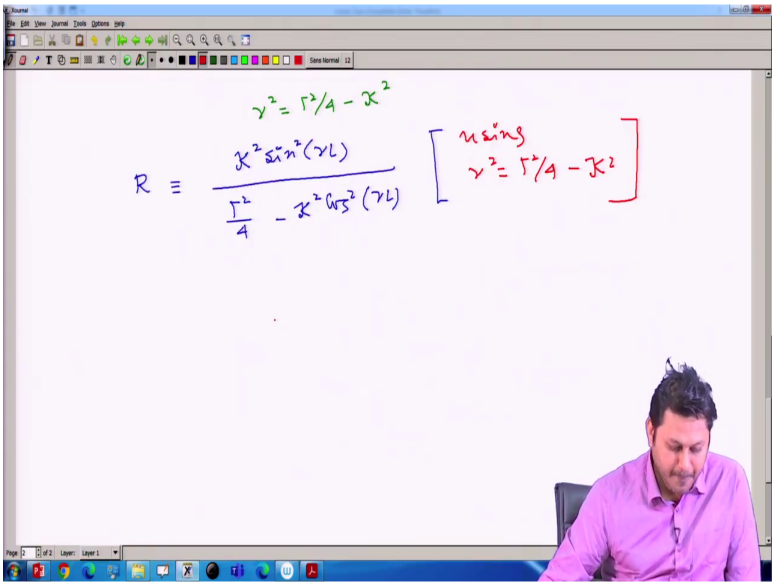

Now we can use the relation that gamma square is equal to big gamma square divided by 4 minus kappa square that is the condition we know that is the equation we know. And if I use this here then R can be represented as R can be represented as kappa square. I insist the student to please calculate this by your own exactly the procedure I used in the earlier cases here also I am using the same thing.

So, I can have a compact form like this kappa square cos square gamma L. So, here I should write using gamma square equal to big gamma square divided by 4 minus kappa square, if I use this condition then I can get this equation ok. So, I now I have almost all the so now I should summarize.

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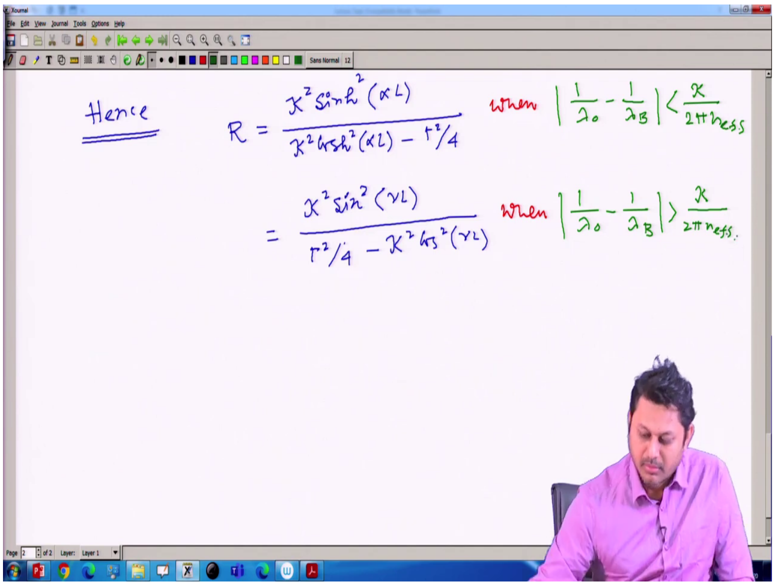

So, if I summarize so hence I have several representation of R. So, I need to write down clearly hence R the reflectivity is kappa square some sin hyperbolic square then alpha L divided by kappa square cos of hyperbolic square alpha L minus gamma square by 4, that is the; that is the condition we derived earlier. When this is important when this condition is valid 1 by lambda 0 minus 1 by lambda B mod of that is less than kappa divided by 2 pi n effective, that is the condition for which this equation is valid.

The same R will going to change the expression of the R is going to change and this expression will be kappa square sine square gamma L divided by kappa square sorry it is not kappa square there is a slight change here you need to be careful enough, it should be gamma square by 4 minus kappa square cos a square gamma L not alpha L gamma L. This is when this condition is reversed that means 1 by lambda 0 minus 1 by lambda b, if it is greater than kappa divided by 2 pi n effective then we have this condition.

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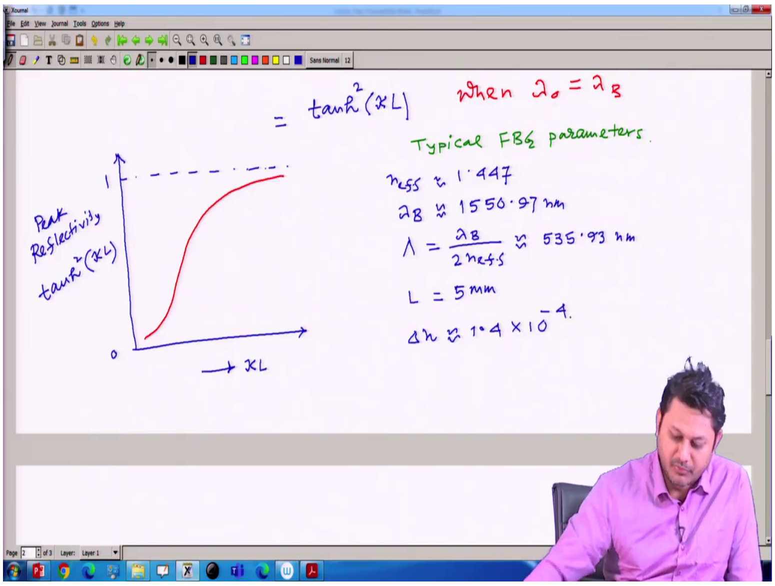

And finally, there is a condition that this equal to simply tan hyperbolic square of kappa L, I can also have this expression and this is this expression is for peak reflectivity. So, when the Bragg condition is valid. So, when lambda 0 is equal to lambda B we have this condition. So, there are 3 conditions one is this is less than this one, 1 by lambda 0 1 by lambda B mod of that is less than kappa divided by 2 pi n effective.

I have this R I can have a different expression of the R when 1 by lambda 0 minus 1 by lambda B is greater than kappa divided by 2 pi n effective and when this is same lambda 0 is equal to lambda B then I have the peak reflectivity this. So, at exactly at Bragg condition we can have a reflectivity of the form like this.

Now, I can plot the peak reflectivity, now this peak reflectivity tan hyperbolic square is a function of grating length and the coupling coefficient kappa. So, I can plot that so this plot is important. So, if I plot so let me plot it here. So, this is peak reflectivity going from 0 to 1, so I am plotting the peak reflectivity. So, this is 1 peak reflectivity which is essentially the functional form is tan hyperbolic square kappa L that I am plotting and here I am plotting kappa L I am changing this parameter kappa L.

If I do that then the curve will be something like this, this is the formation of the curve it should be something like this. And if I have a typical FBG parameter, so let me write it down. So, typical FBG parameters. What are the typical FBG parameters?

So, let me list here 1 is n effective which is very nearly equal to say 1.447 this is the experimental value we have lambda B very nearly equal to 1550.97 nanometer, lambda which is equal to lambda B divided by 2 of n effective will be nearly equal to some value like 535.93 nanometer this is the value.

Then I have a grating length L which is fixed say 5 millimeter this is a typical grating length we have and delta n is a refractive index variation inside the grating will be of the order of 1.4 into 10 to the power of say minus of 4, kappa I can calculate with all this value kappa which is nearly equal to say pi then delta n then n effective.

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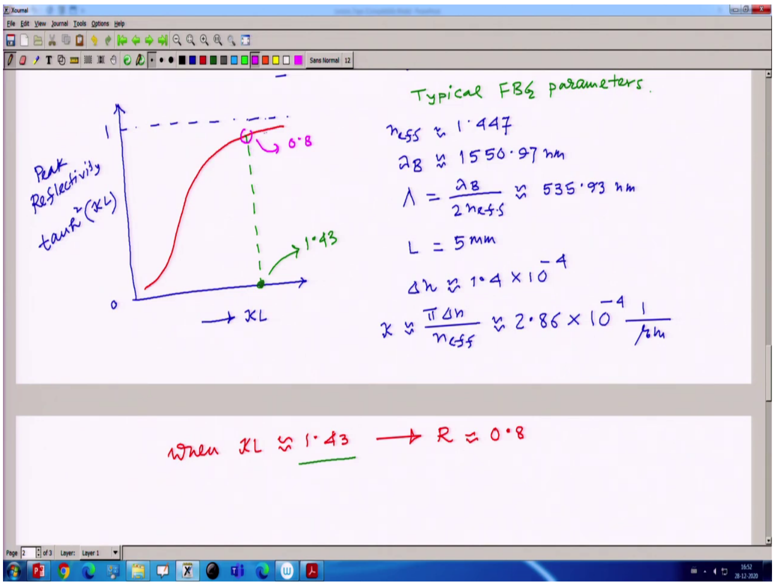

If this is my kappa then this value will be 2.86 into 10 to the power of minus 4 and it is 1 by micrometer. So, these are the typical value and if you use the typical value I can get these values. Now, if I use this value and for this kappa and all these things. So, when kappa L is equal to say when kappa L is nearly equal to 1.43 I can have a reflectivity R nearly equal to 0.8.

So, 80 percent reflectivity one can expect with this value. So, I am talking about this point from this curve I am talking about say some point here. So, I use all these practical values, so after using all these practical values I find what is my kappa and then I find out the value of kappa L because L is there. So, when kappa multiplied by L is some value which is around say 1.43 that basically leads to a value of reflectivity say 0.8.

So, this value of kappa L according to the typical FBG parameter is 1.43 and the value of the reflectivity I can have here is around 0.8. So, with this plot I can have the value of the reflectivity at different kappa L, but if these values are given then I can readily find out what should be the value of the reflectivity and it definitely depends on the value of kappa L. If kappa is high then I can find that or the grating length is high, then one can find that the peak reflectivity is having a higher value ok.

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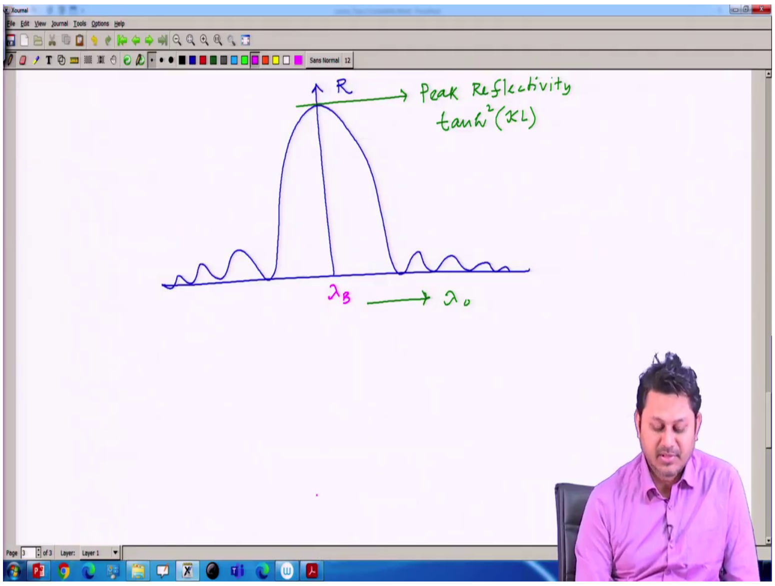

So, after that I will like to draw now the full picture and I like to draw the reflectivity once again and the reflectivity simply look like this. This is a typical picture of reflectivity so I plot R here I already plotted it, but I plot once again these value is the peak reflectivity. So, this value is the peak reflectivity which is tan hyperbolic square kappa L and here I can have a variation of lambda 0.

So, lambda 0 is going to vary and I can have this reflectivity curve. Well from this curve you can readily understand the student you can readily understand that this reflectivity is sharply falling. So, this is the value of lambda B by the way let me remark. So, this is a Bragg wavelength at which we have a peak reflectivity.

Now, if the wavelength is not exactly in the lambda B, but it is either higher or lower value than the lambda B it is around lambda B. Then what happens the reflectivity is falling down very rapidly, but still I can have some kind of reflectivity. So that means, there is certain bandwidth of this reflectivity curve.

So, in the next class we will going to calculate what should be the bandwidth of this reflectivity curve, which is important in many aspects because we need to know that what is the bandwidth of the reflectivity curve, then we can understand what is the sensitivity related to these Bragg gratings.

So, with this note let me conclude. So, next class I will start from this figure and try to understand what is the bandwidth of a given Bragg grating where you know what is the reflectivity curve. So thank you for your attention and see you in the next class.