Physics of Linear and Non-Linear Optical Waveguides

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Module - 04
Fiber optics components
Lecture - 46
Reflectivity Calculation of FBG (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 46 and we will going to continue the Reflectivity Calculation.

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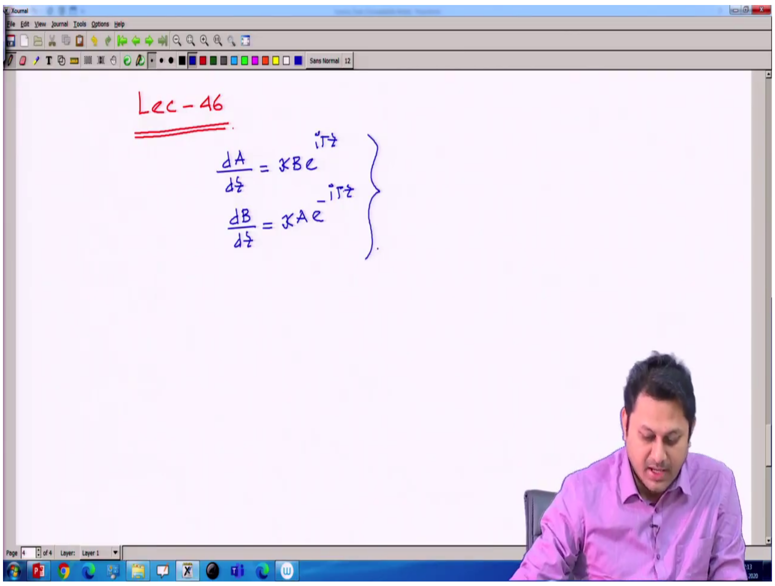

So, let me write down the expression that we have done in the last class. And we try to solve the coupled differential equation and this coupled differential equation we try to solve, let me write it once again dA d Z is equal to kappa B e to the power of i gamma Z and d Z is equal to kappa A e to the power of minus i gamma Z. We try to solve this differential these two coupled differential equation.

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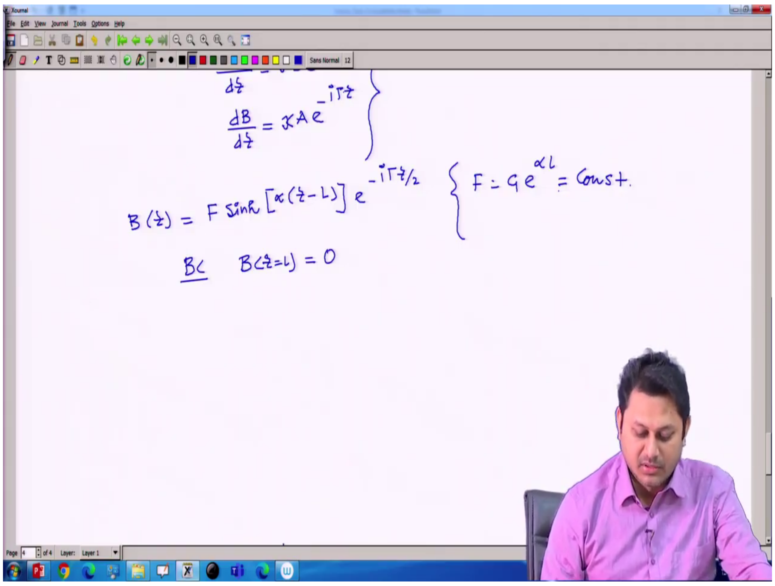

And we were almost there we figure out what is the what is the expression of B and B was something like F sin hyperbolic alpha Z minus L with a phase term e to the power of minus i gamma Z by 2.

When F is a constant, F which was c 1 e to the power of alpha L it was a constant and the boundary condition was this is the this is based on the boundary condition that B at Z equal to L is equal to 0 and from this expression if somebody put the value of Z equal to L you can readily find that the equation this B value it will go to vanish.

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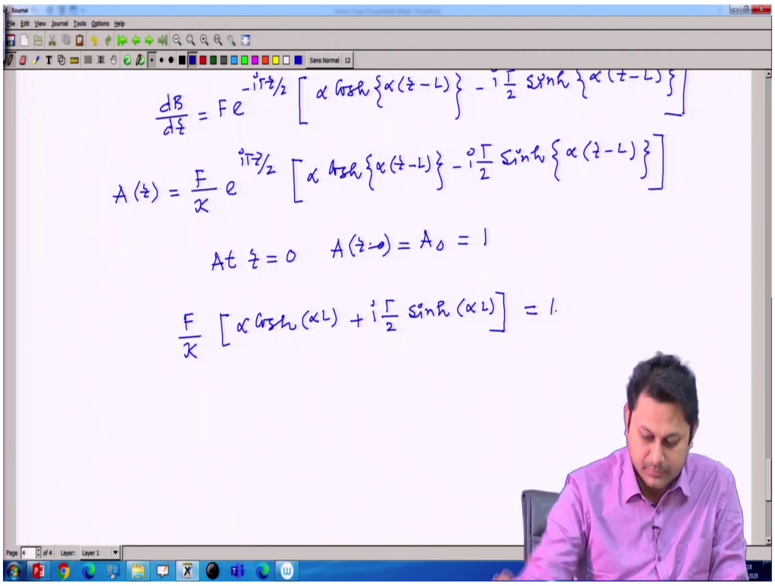

Well, after that the next thing we need to find is A. So, now A which is a function of Z we can write this as simply the derivative with respect to Z B and then e to the power of i gamma Z from the first equation from the second equation, from this equation I can extract the value of A. So, B I know. So, del B del Z will be simply F e to the power of minus i gamma Z by 2.

And I can have derivative like cos alpha cos hyperbolic of alpha Z minus L and for this I have 1 gamma divided by i gamma divided by 2 sin hyperbolic alpha Z minus L bracket close. Well then A Z will be simply F divided by kappa e to the power of i gamma Z divided by 2.

Because here we have already 1 i gamma Z and here I have minus of i gamma Z divided by 2. So, eventually I have e to the power of i gamma Z by 2 and then the rest of the term, alpha cos hyperbolic of alpha Z minus L and minus of i gamma by 2 sin hyperbolic of alpha Z minus L.

Well the boundary condition again at say Z equal to 0, I can have the value A Z 0 is equal to A 0 and without any loss of generality I can put this as 1.

So, I can have mod of. So, I can have if I if this is the condition then from this equation I can have that F by kappa and this term if I put Z equal to 0, then I simply have alpha cos hyperbolic of alpha L and here I have plus i gamma by 2 sin hyperbolic of alpha L this is equal to 1.

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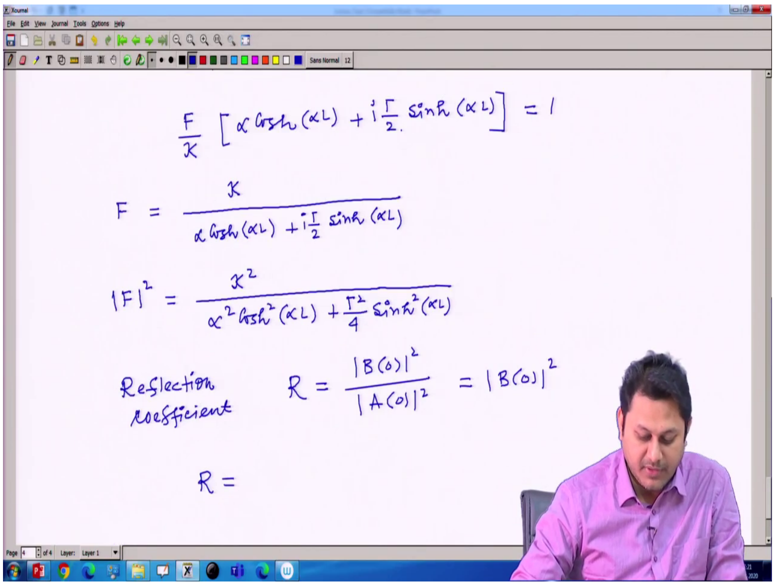

So, what is my F which is a constant. So, F is simply kappa divided by this quantity alpha cos hyperbolic of alpha L plus i gamma by 2 sin hyperbolic of alpha, this is the value of F. So, what is mod of F square? Because this is a complex term so, if I want to find mod of a square F square, then it should be kappa square divided by alpha square cos hyperbolic square alpha L plus gamma square divided by 4 sin hyperbolic square, then alpha L.

Why I take the mod squares? Because, at the end of the day I need to calculate the reflectivity. So, the reflection coefficient so, I just define the reflectivity or the deflection coefficient as R which is mod of B 0 square divided by 0 square. So, which is simply mod of B 0 square, because mod of A 0 square is nothing, but 1 as per this condition; as per this condition.

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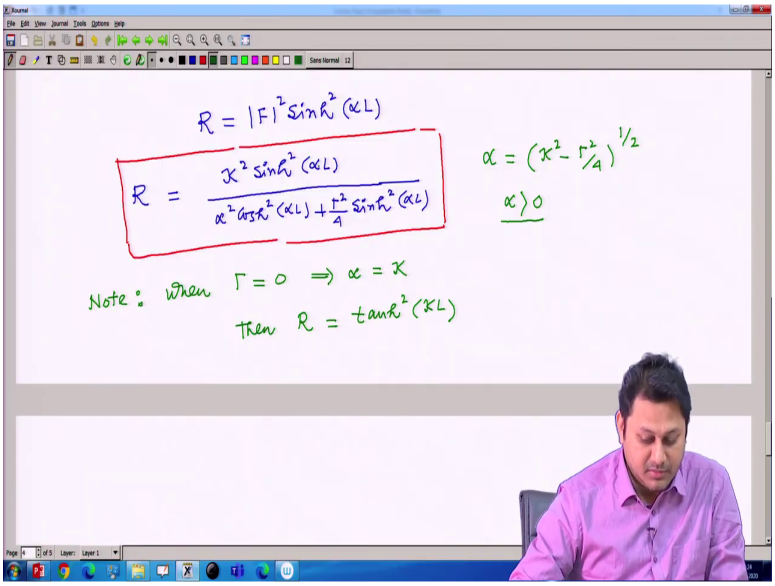

So, R is simply R is simply B I already figured out here my B is F sin hyperbolic alpha Z minus L e to the power of minus i Z by 2. So, if I want to find out what is B 0 square mod of B 0 square, it should be simply mod of F square then sin hyperbolic square alpha L and mod of x that is why the mod of x F square term I already calculated here it was necessary.

So, eventually my reflectivity is kappa square sin hyperbolic square alpha L divided by alpha square cos hyperbolic square alpha L plus by 4 sin square alpha L. So, this is the value of the reflectivity I find. So, this is finally, the reflectivity which I wanted to find is coming in this,

this particular form. So, here few things we need to note one thing we can readily find that. So, I should also write what is alpha so, that because I am going to use these things.

So, alpha here is. So, alpha is kappa square minus sigma big gamma square divided by 4 whole to the power half that was my alpha. With the condition that alpha is real and it is greater than 0; that means, it is positive that was the condition. So, based on that condition I get this result, but also we calculate a condition where this kappa square is less than this one, then I get a different solution for that.

So, note when gamma is equal to 0, this basically tells us that gamma equal to 0 basically tells us alpha is equal to kappa. And the reflectivity expression simply becomes tan hyperbolic square kappa L, which already I we calculated that what should be the case, what should be the value of the reflectivity when the Bragg condition is satisfied.

So, here this is a general expression, in this general expression if we put gamma equal to 0, when you put gamma equal to 0 then this term will not be there the second term here in the denominator and this alpha will be replaced by gamma because at when alpha because alpha will should be replaced by kappa as the gamma is 0. So, eventually we have R equal to tan hyperbolic square then kappa L which is basically the reflectivity under the Bragg condition, well.

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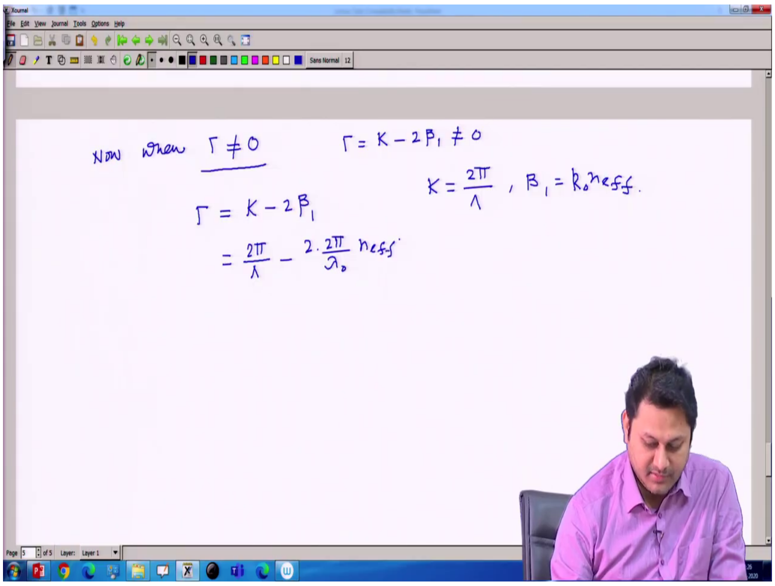

For now, when gamma is not equal to 0 what does it means? That gamma which is equal to K minus 2 beta 1 so; that means, this quantity is not equal to 0. What is K by the way? So, gamma let me write once again it is defined by kappa K minus 2 beta 1 big K minus 2 beta 1. Big K is 2 pi divided by the grating period. So, and beta 1 is k 0 n effective. So, if I write it should be 2 pi divided by this minus 2 K 0 I can write as 2 pi divided by lambda 0 and then n effective. I just rewrite what is gamma with the known terms.

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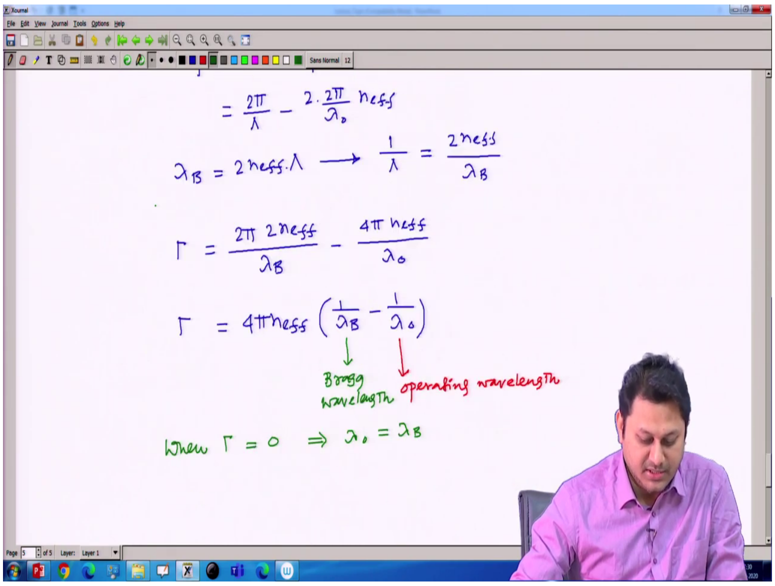

Lambda B we know is equal to 2 n effective multiplied by the grating period lambda. So, grating period from here I can write 1 by grating period because it is here is simply 2 n effective divided by lambda B. So, my gamma in terms of Bragg wave length lambda and the operating wavelength the wavelength I am launching into the system lambda 0.

I can write as 2 pi here 1 by lambda I write 2 n effective divided by lambda B minus 4 pi n effective divided by lambda 0. So, eventually this value is 4 pi n effective 1 by lambda B minus 1 by lambda 0. So, this, this is a expression of gamma in terms of whatever the wavelength I am launching which is this one. So, this is the wavelength I am launching, this is operating wavelength, this is Bragg wavelength.

So, in terms of operating wavelength and Bragg wavelength I can now; I can now have the value of gamma. So, this gamma is now greater than not equal to 0. So, under not equal to 0 condition, we can have this. So, when gamma is 0, we have lambda 0 is equal to lambda B from this expression also we can find that.

So, that means, if I change the value of gamma then the reflectivity will going to change because this expression inside this expression the gamma is hidden and this big gamma is here in the alpha. So, if I change the wavelength then what happened this reflectivity will going to change.

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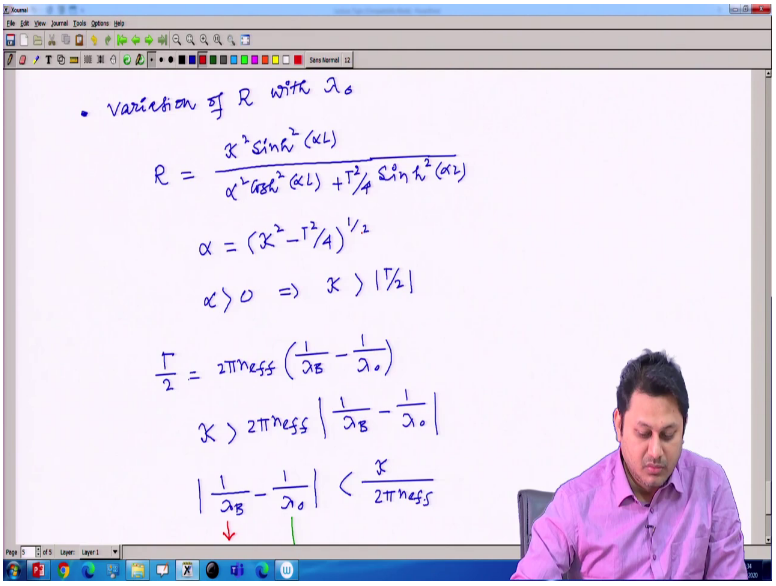

So, let me write it here. So, the variation of R with lambda 0. So, we need to realize that that the reflectivity whatever the reflectivity we figure out here, we will going to change. So, let me write it once again. So, reflectivity R is simply kappa square sin hyperbolic square then alpha L whole divided by alpha square cos alpha square alpha cos hyperbolic square alpha L plus gamma square by 4.

We will also write a compact form of that this is the form which we derived, but we can also make a simplification for that form. So, that we will do, but let us understand. So, alpha is equal to kappa square minus gamma square. So, to write it properly gamma square by 4 whole to the power half.

Alpha is real positive. So, alpha greater than 0 so, alpha greater than 0 means this means kappa is greater than mod of gamma by 2 that we need to understand. Kappa is the coupling coefficient so, that is always positive. So, this condition is greater than 0 means this condition holds now we find that, what is gamma? So, gamma by 2 is essentially 2 pi n effective 1 by lambda B minus 1 by lambda 0.

So, when alpha is greater than 0; that means, kappa is greater than gamma divided by 2, gamma divided by 2 is this quantity. So, kappa is essentially greater than 2 pi n effective mod of 1 by lambda B minus 1 by lambda 0 or in other word 1 divided by lambda B minus 1 divided by lambda 0, this has to be less than kappa divided by 2 pi n effective.

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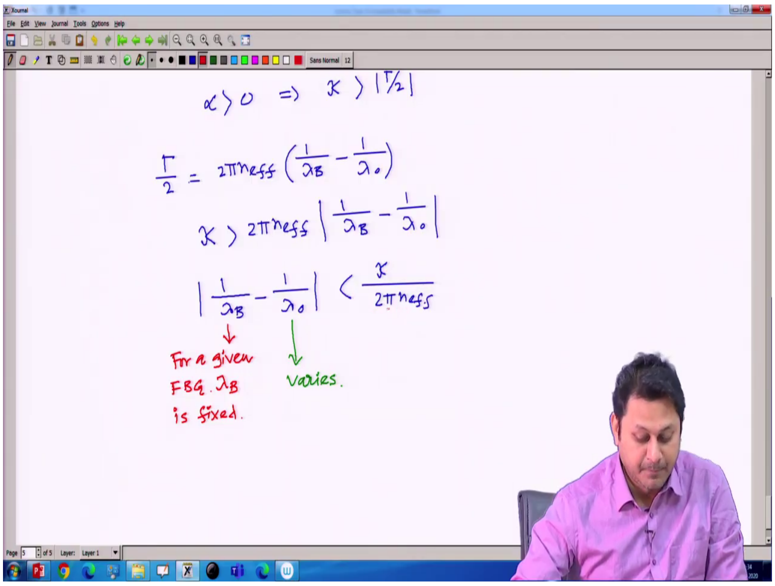

So, that means, this is the variable this basically varies. So, when this quantity varies, then there should be a condition this is fixed because for a given for a given FBG lambda B is fixed and I am launching a light for which; I am launching a light for which this quantity I mean lambda 0 is not equal to lambda B. So, there is a mismatch of these things.

Now if this quantity is less than 0, this quantity is less than this quantity kappa divided by 2 pi n effective, then we have the reflectivity expression this. Now, there is a possibility that if I keep on changing lambda 0 what happened that, this quantity may be greater than kappa divided by 2 pi n effective.

At some point it should be 0 and then again it will start changing. So, when it is changing and it is greater than kappa divided by 2 pi n effective, we have a different condition and for that case again this reflectivity expression will going to change. Now, let us figure out what is the compact form of reflectivity few minutes ago I was talking about these things. So, let me derive that. So, one thing we understand that reflectivity the expression of the reflectivity is going to change with respect to the condition that whether these is less than this quantity or this is greater than this quantity.

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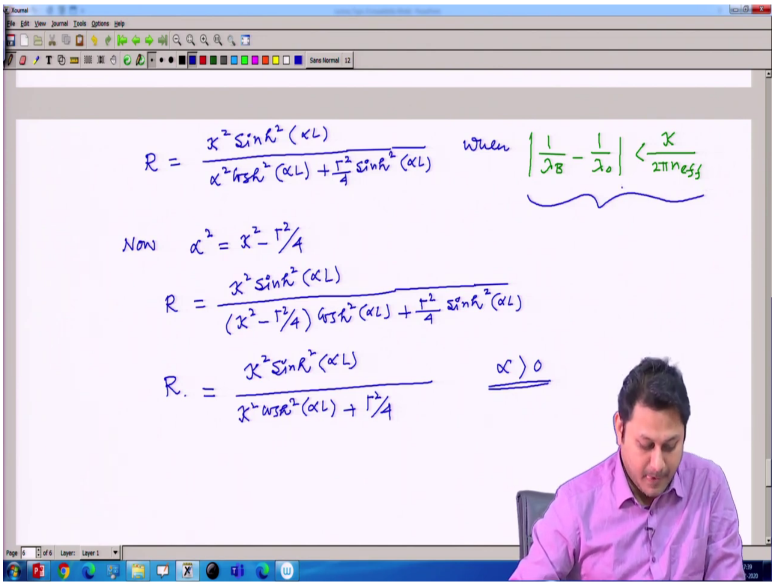

Well, when R I can write it as kappa square sin hyperbolic square alpha L divided by alpha square cos hyperbolic square alpha L plus gamma square divided by 4 and sin hyperbolic square alpha L. This value we find when this condition is satisfied mind it.

When mod of 1 by lambda B minus 1 by lambda 0, this quantity is say less than kappa divided by 2 pi n effective. This is a specific condition for which this reflectivity is written in this form, but anyway I can write this reflectivity in a in another convenient form using the relation, we have now alpha square is equal to kappa square minus gamma square divided by 4.

I can make use of this equation and I can write R is equal to kappa square sin hyperbolic square alpha L whole divided by alpha I can write it as kappa square minus gamma square by 4, cos hyperbolic square alpha L plus gamma square by 4 sin hyperbolic square alpha L. This quantity is simply kappa square sin hyperbolic square alpha L divided by kappa square cos hyperbolic square alpha L and here this quantity is simply gamma square divided by 4 because sin hyperbolic square plus minus cos hyperbolic square will be 1.

So, this is the form of the reflectivity we have and this form we find when the condition gamma is not equal to 0. And gamma is not equal to 0 tells us I mean and this expression is basically coming from the fact that when alpha is greater than 0, I can have the solution when alpha is greater than 0 that basically tells me that this is the condition in terms of wavelength.

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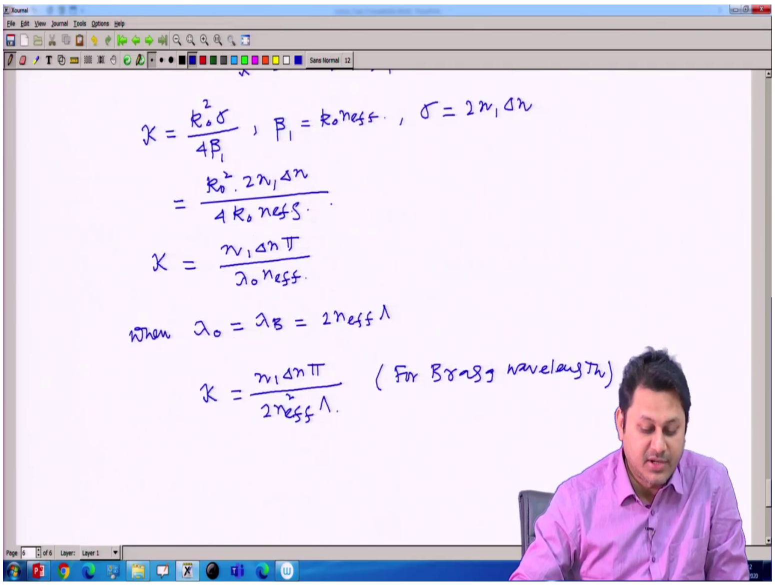

By the way kappa is equal to k 0 square sigma divided by 4 beta 1, beta 1 is k 0 n effective sigma these are the values we defined. So, let me write everything so, that I can finally, write the coupling coefficient in terms of some well known form. So, this is k 0 square sigma I can write as 2 n 1 delta n divided by 4 of k 0 n effective and this k 0, k 0 will going to cancel out 1 k 0 will be remaining there.

And if I replace this k 0 by 2 pi divided by lambda 0. Eventually, I will going to have n 1 delta n pi divided by lambda 0 n effective this is a form of kappa. So, normally this delta n, n 1, delta n, n 1 lambda 0 operating wavelength and n effective for the system these are given. If these terms are given then we can have a value of kappa.

So, when by the way when this lambda 0 is equal to lambda B; that means, at the condition Bragg condition then this quantity we know it is n effective by lambda. So, under the Bragg condition; when the Bragg condition is satisfied. So, this quantity kappa is n 1 delta n pi divided by 2 n effective square gamma. So, this is the condition when that the wavelength is equal to lambda B. So, for Bragg wavelength this condition is there for Bragg wavelength we have this. So, we already figured out the expression of reflectivity for the condition.

So, in the next class again we will try to find out the reflectivity for another condition because this as I mentioned here if lambda 0 is changing, then what happened that there is there might be a condition that this left hand side will no more less than to the right hand side. So, it should be greater than this right hand side, it should be 0 at some point when lambda 0 is equal to lambda B.

So, when it is greater than this kappa divided by 2 pi n effective, we should have a different expression in during that in that case alpha will be a complex quantity and from this we can readily understand that if alpha is a complex, this sin hyperbolic will be replaced by sin and cos that we will calculate in detail. And try to find out the expression for the reflectivity in that situation. So, with that note I like to conclude today's class, in the next class we will start our investigation from this point. So, see you from and.

Thank you for your attention.