Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber optics Components
Lecture - 45
Reflectivity Calculation of FBG (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we will going to continue the calculation that we have started in the last class, which is the Reflectivity of the Fiber Bragg Grating ok.

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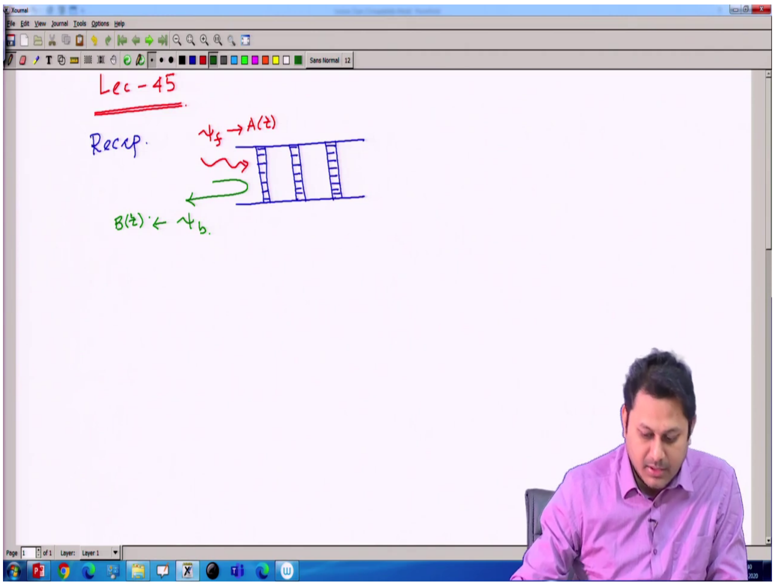

So, quickly let me recap what we have done. So, we had a grating structure like this, a periodic refractive index variation over the core of a optic of an optical fiber; and then we had incident light moving in the forward direction and because of this grating some portion of the light will reflect back. And if the amplitude related to this wave is A function of Z and the amplitude related to this backward wave is considered B function of Z.

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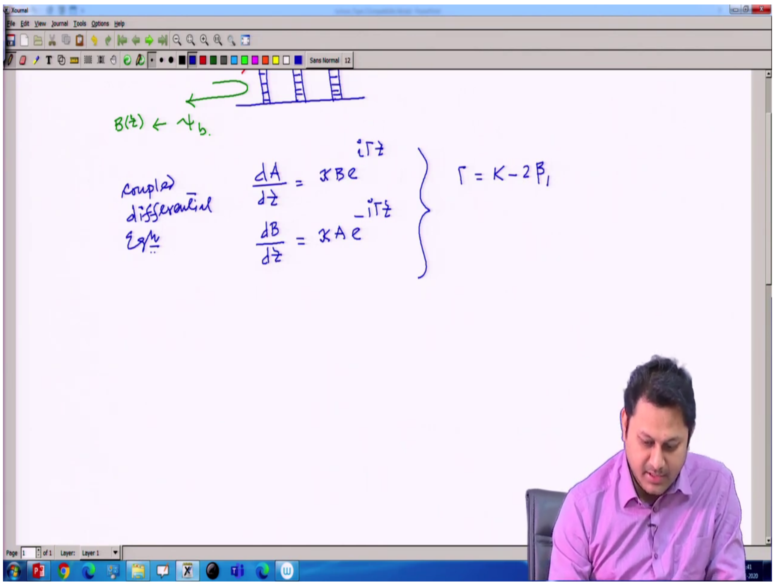

Then, we had a differential equation of A and B. And the coupled differential equation of A and B was this, where gamma was this all the definitions has already mentioned in earlier classes.

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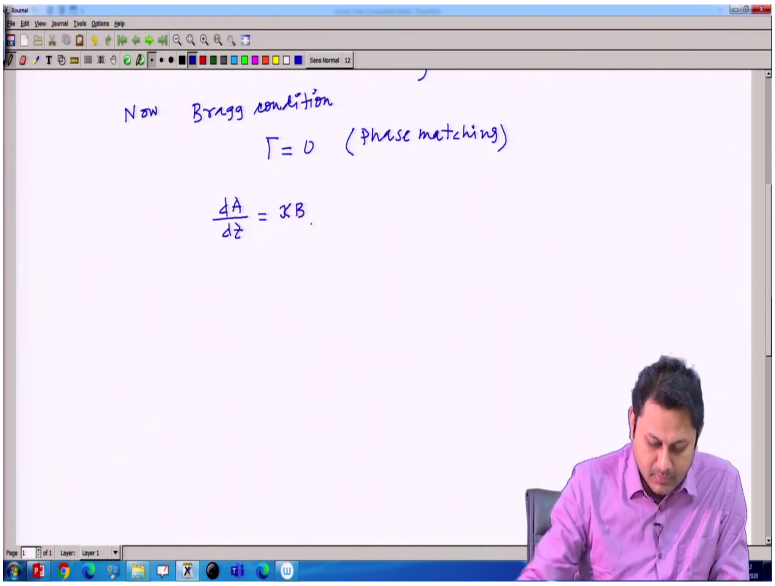

Now, Bragg condition was gamma is equal to 0 that is the Bragg condition or phase matching. So, equation under phase matching was simply written as this.

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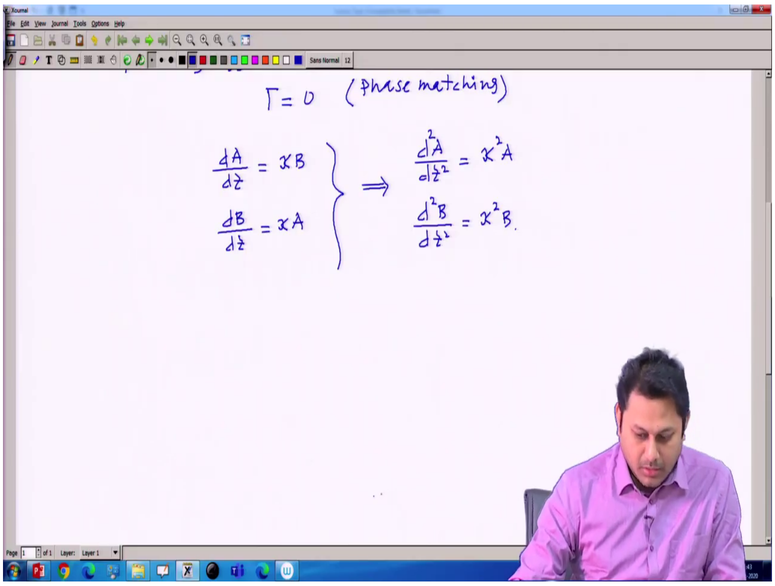

And also from these two equations coupled equation and decoupled and we had a differential equation for A and B individually, which is this.

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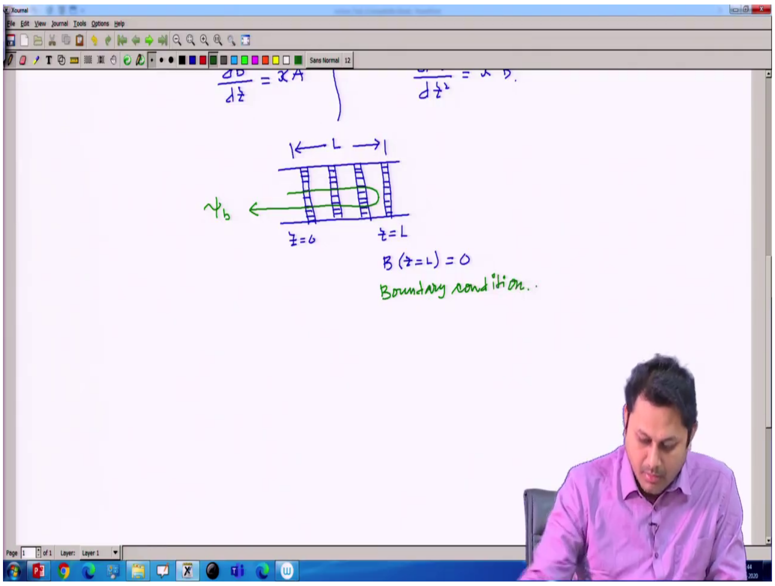

Now, the boundary condition which is important here, we apply for the equation of b and figure out the expression. So, this is the structure of the grating, this is Z equal to 0 and this is Z equal to L this is the grating length L. So, at this point B at Z equal to L is 0 because, when the waves are reflecting back it will reflect back from this point. So, this was the backward wave. So that means, the amplitude of the backward wave at Z equal to L is equal to 0. After putting this boundary condition so, this is the boundary condition. After putting this boundary condition we managed to find out the value of B.

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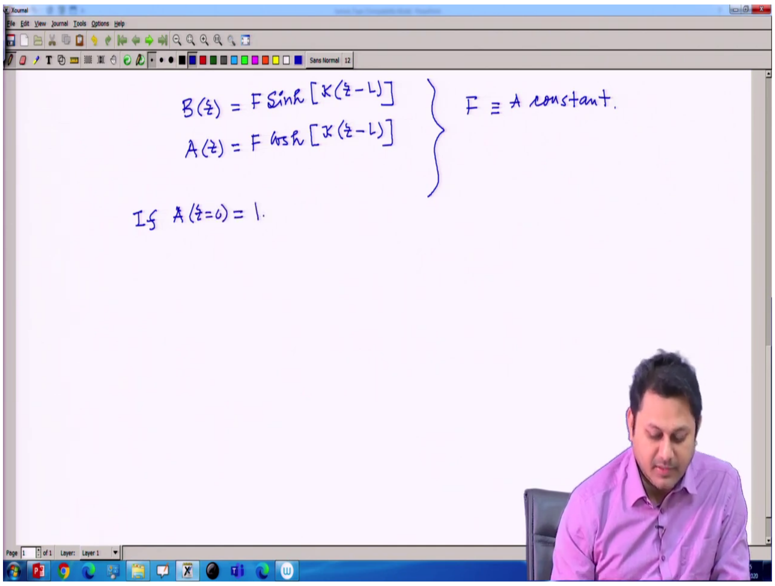

Which is F sin hyperbolic then, kappa multiplied by Z minus L and A Z was F cos hyperbolic kappa Z minus L, where F is a constant. Now, if A Z equal to 0 that means at the input if I consider this value as 1.

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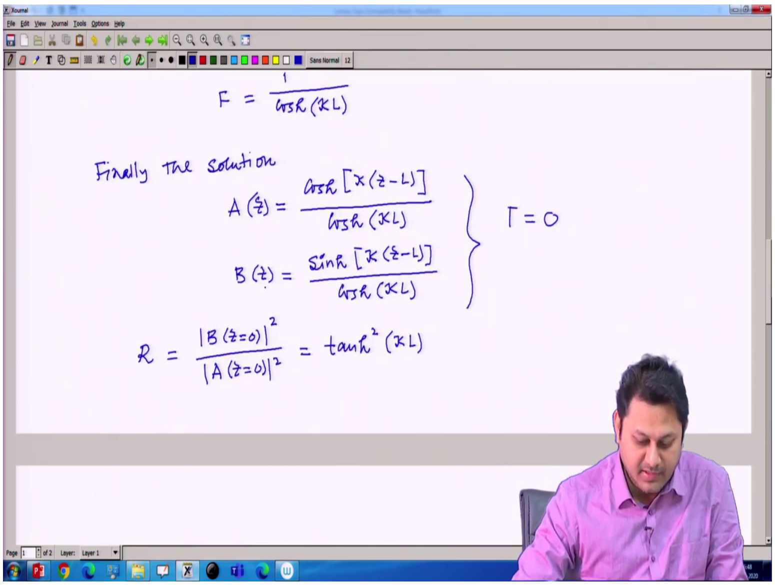

Then, the value of F I can write simply 1 divided by cos hyperbolic of kappa L. So, cos hyperbolic a kappa L is a constant. So, I can write the value of this constant F in terms of the coupling coefficient coupling constant kappa and the grating length L. So, finally, the solution that we had was A Z as cos hyperbolic of kappa Z minus L divided by cos of hyperbolic of kappa L.

And B Z was sin hyperbolic of kappa Z minus L divided by cos hyperbolic of kappa L, this condition is precisely for the phase matching situation that means, when gamma is equal to 0. You need to remember these things we will today we will going to calculate when gamma not equal to 0, what should be the case when gamma is not equal to 0.

So, this is the solution and then, we find the reflectivity which we defined as this we defined at B at Z equal to 0 whatever the value square divided by mod of A at Z equal to 0 this. So this value was tan hyperbolic square of kappa L so, this is the peak reflectivity we have under the phase matching condition.

There is a peak reflectivity we have under the phase matching condition. Well today we will going to plot further so, we have the A and Z in this way cos hyperbolic of kappa Z minus L divided by cos hyperbolic of kappa L and B Z is sin hyperbolic of so, this is the evolution of A and B.

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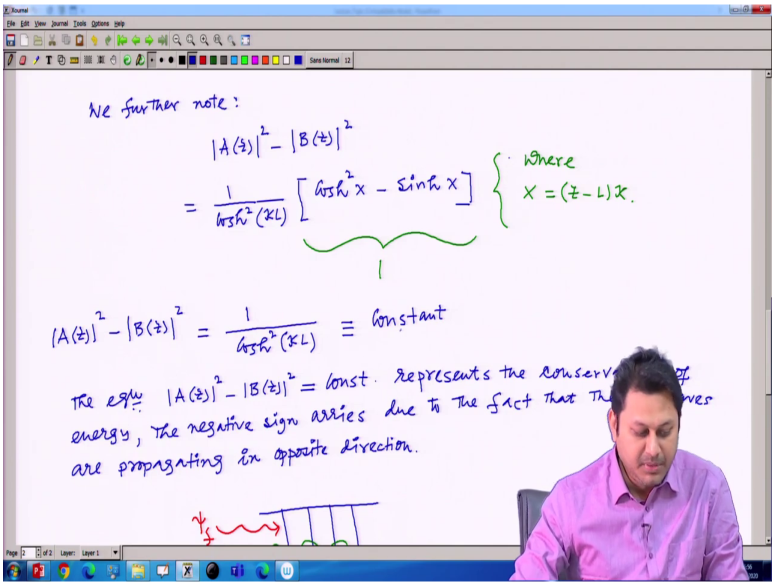

So, what today we do is to check this value. So, we further note: mod of A Z square minus mod of beta Z square this quantity if I want to calculate then, you can see this value is simply cos hyperbolic of say square of x minus sin hyperbolic of big X, where X is; where X is Z sorry Z minus L multiplied by kappa which is this one, kappa multiplied by Z minus L I just write it here in terms of X. So, this quantity is 1.

So, eventually we have mod of A as a function of Z square minus mod of B function of Z square is equal to 1 divided by cos hyperbolic square kappa L, which is a constant. So that means, the difference between mod of A Z square minus mod of B Z square is constant so, it is essentially this equation essentially the equation, which equation I am talking about this one represent represents the conservation of energy.

The negative sign, because normally mod of A Z square plus mod of B Z square is equal to constant, this is normally the way one can write equal to constant this, this is normally the way one can write the energy conservation but here, this negative sign basically arises the negative sign arises due to the fact that the two waves are propagating in opposite direction.

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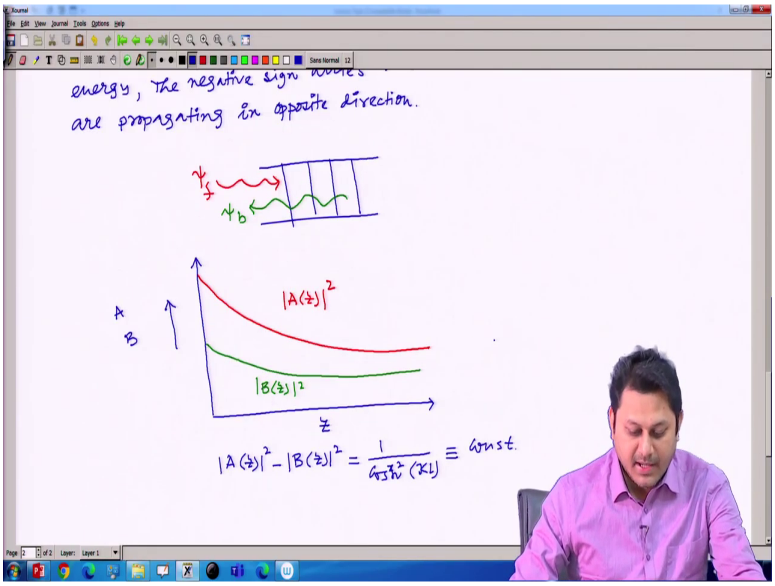

So, in the this we already see so, the if this is the Bragg grating structure, then the one wave is propagating along this direction which is our psi f, but another wave, which is reflecting back from that. So, another wave is moving actually in this direction psi b. So, since two waves are moving in opposite direction this negative sign arises for that. But, when we calculate mod of A Z square minus mod of B Z square, it is it come it is coming as a constant so, it basically gives that energy is conserved.

Well, if I plot these two functions, I can also plot the function say A it will be like that. So, this is mod of A Z square and if I plot mod of B Z square it should be something like that. Along this direction I have Z, along this direction I plot either A or B. And every time at every distance if I make this difference mod of A Z square minus mod of B Z square at any Z point, it has to be the value like this, which is a constant that is all.

So, we extract the information of A and B using the boundary conditions and then we find that what is the reflectivity, reflectivity is coming out to be tan hyperbolic square of kappa L. And then, we find out that what is the value of A Z square minus B Z square, which essentially give us the energy conservation condition and it comes to be a constant, it is coming as a constant value. Now, the reflection coefficient we already calculate.

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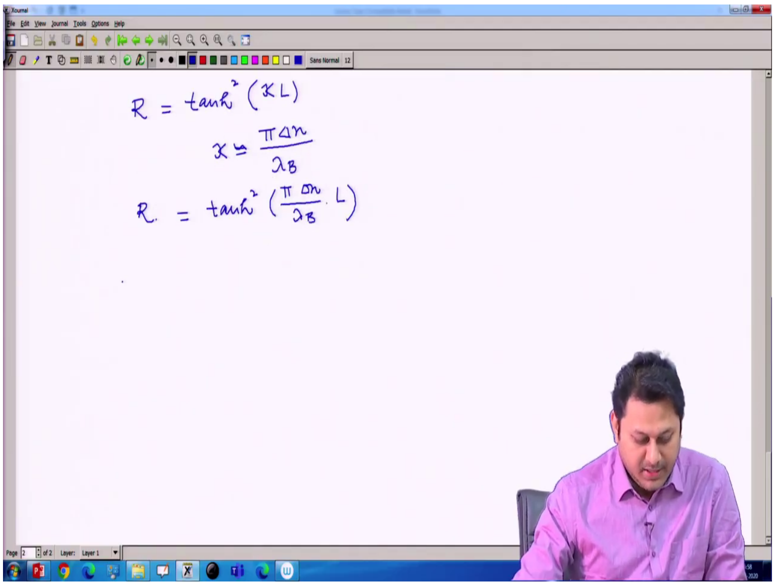

So, R which we figure out as tan hyperbolic square then kappa L, it can be written as so, kappa so kappa we also find out that it is this roughly nearly equal to when the operating wavelength is very close to the value of the brag wavelength then, this reflectivity can be simply written as tan hyperbolic square pi delta n divided by lambda B multiplied by L.

If all these values are known then we can calculate the peak reflectivity. Well, next we start a we should start a calculation when these Bragg condition is not satisfied. So, this all these calculation you should note that this condition is there so, that is why our calculation was little bit simple. But now, we consider a case a more general case say case 2.

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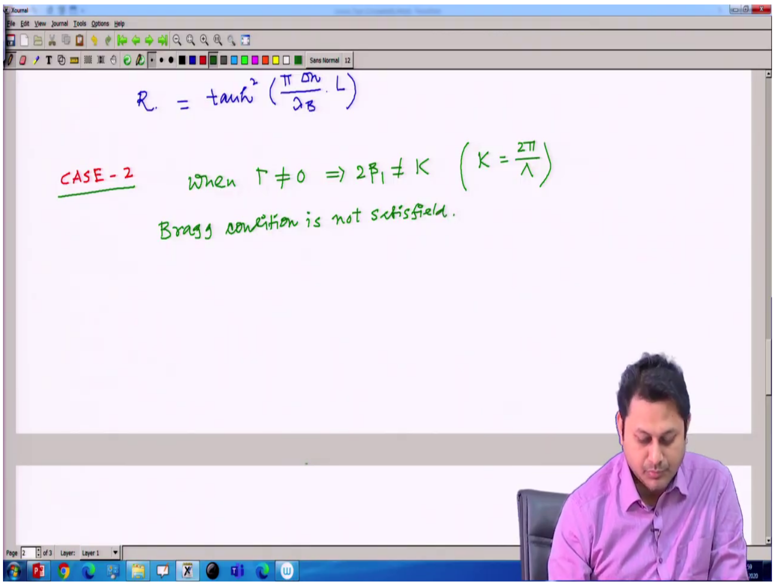

When so, this case 2 is the case, when this is not equal to 0 that means, the phase matching condition is not there anymore so, 2 beta 1 is not equal to big K mind it big K is 2 pi divided by lambda the grating period. So, that means, the Bragg condition is not satisfied Bragg condition is not satisfied.

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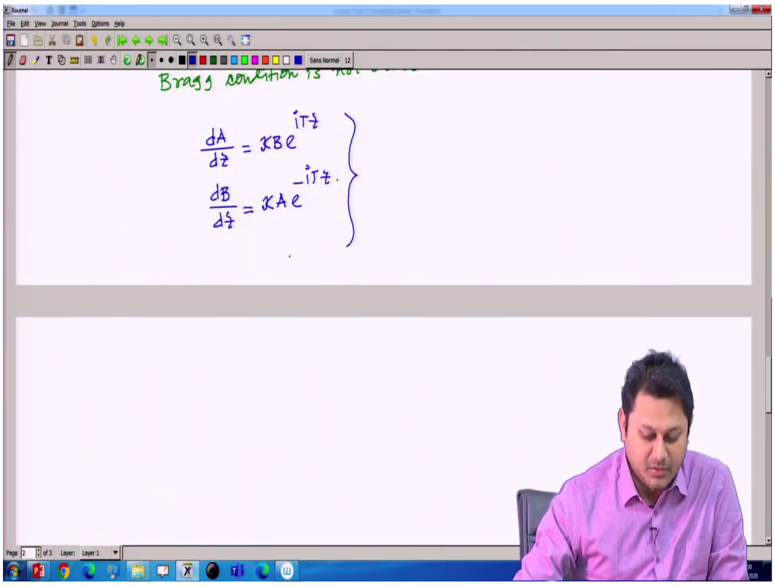

Under that condition we have two equation general equation and these two general equation I need to write once again and del B del Z is equal to kappa A e to the power of minus i gamma Z. So, these two equation I write several time. So, these are the two equations.

Now, we need to solve this, the solution of this kind of equation we already done in the Coupled mode theory during understanding the coupler, optical coupler, directional coupler. So, here we will going to follow the same treatment.

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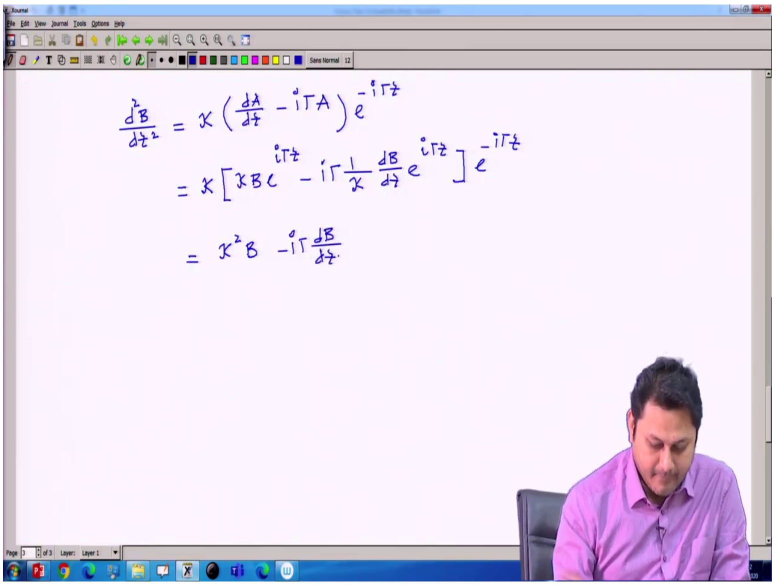

So, let us make a derivative with respect to Z of the second equation the right hand side I should have kappa then the derivative with respect to Z of A then, I have another term I should write it as this with e to the power of minus of i gamma Z. I just make a derivative of this equation, this one. Now, dA dZ I know so, I will just put all the values here.

So, dA dZ I will put from here and A I will again put this, I can use this equation and I should have a differential equation for that for B only so I can decouple that. So, this dA dZ I can write kappa B e to the power of I gamma Z minus i gamma I can write a as 1 by kappa del B del Z then e to the power of i gamma Z; and e to the power of minus i gamma Z will be here.

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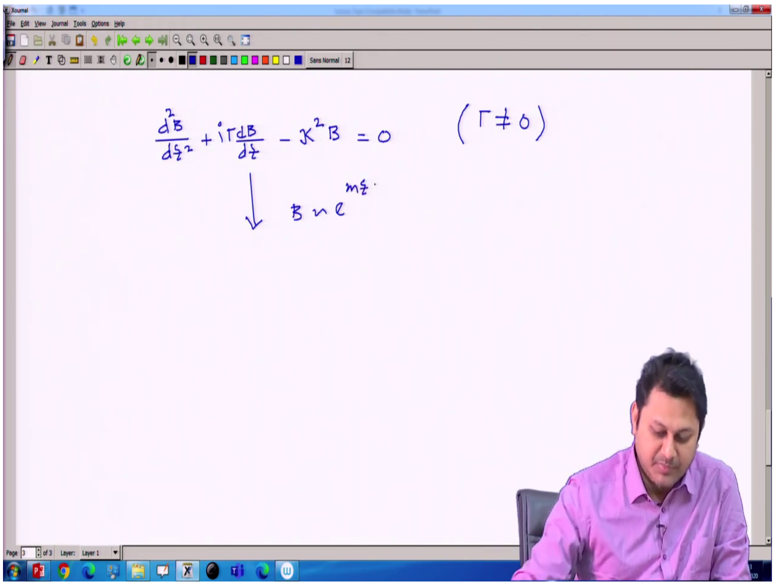

So, this simplify the equation as so, eventually I have a differential equation, and then minus kappa square B is equal to 0. After that so, this is the this is the condition and definitely when gamma is not equal to 0 goodness to say. Now, I should use this technique that I want to have a solution of the form say e to the power of mz.

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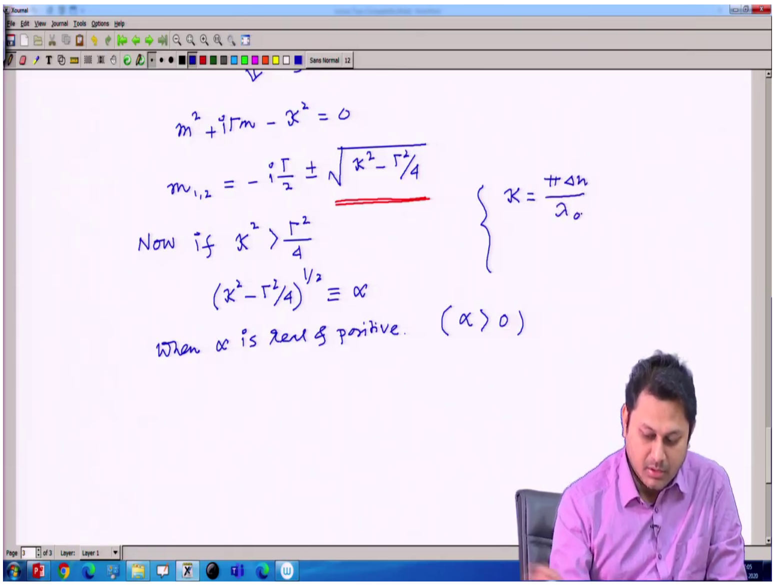

Then, it should be differential equation should be m square plus i gamma m minus kappa square equal to 0. So, the value of m say m 1, 2 there will be two roots of this equation and 1 can write this 2 root as this, which we use several time this technique so, it should be ok. So, this is the two roots this, these are the two roots m 1, 2. Well, now we assume so, we have something here very interesting which is kappa square minus this gamma square divided by 4.

So, now if kappa square is greater than say gamma square divided by 4, we can write this term kappa square minus gamma square divided by 4 whole to the power half equal to alpha. When alpha is real and positive so that means, alpha is greater than 0 it is a real quantity as well as positive; here I should mention quickly what is the value of kappa, kappa is equal to pi delta n divided by operating wavelength.

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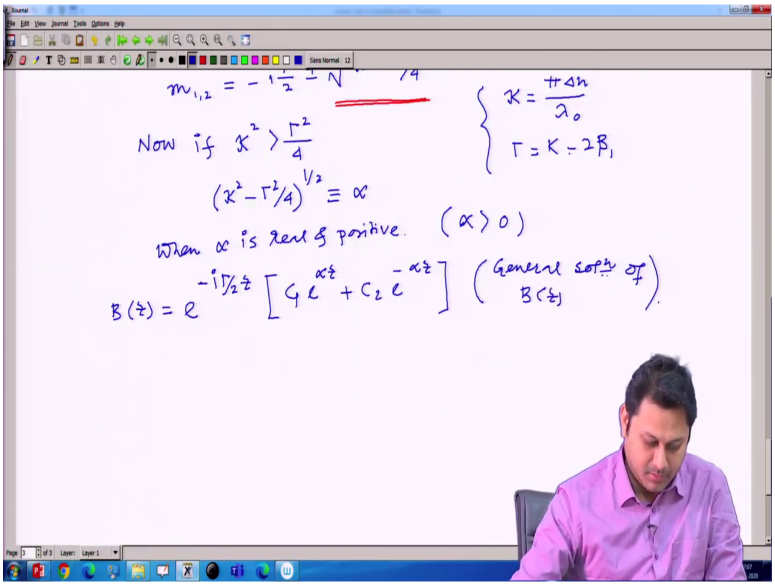

So, it depends on the operating wavelength that whether it this value should be greater than gamma or not and gamma again here big K minus 2 of beta 1 inside the beta 1 mind it; inside the beta 1 also we have lambda. So, we have a if we change lambda 0 then what happened, there is a condition for, which this quantity may be greater than may be real and positive and we are now taking this condition only.

So, then we have B Z as e to the power of i minus of i gamma by 2 Z and then C 1 e to the power of alpha z plus C 2 e to the power of minus alpha z,this is a general solution of B this is a; this is a general solution of B i is this also general solution of B. Now, again I will going to use the boundary condition.

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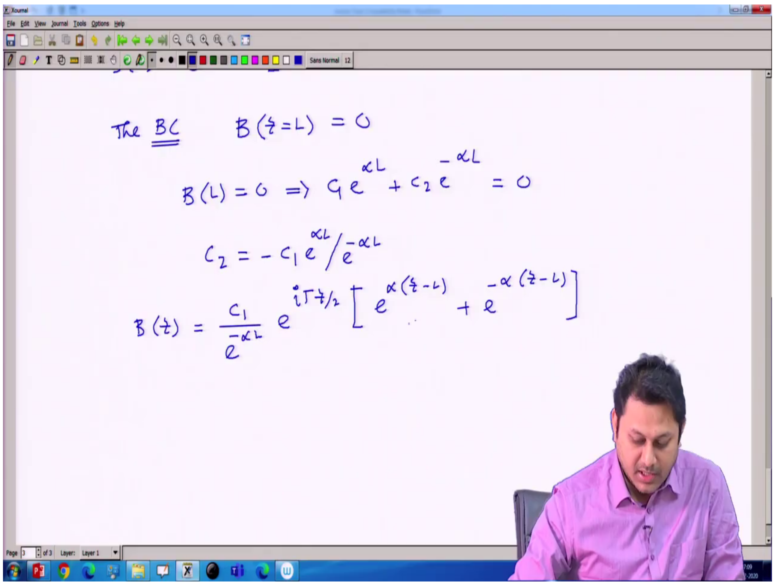

So, the boundary condition is the boundary condition is B at Z equal to L will be 0. So, putting this boundary condition I can write B L which is 0, basically tells us C 1 e to the power of alpha Z plus C 2 e to the power of minus alpha sorry, when I put the boundary condition it should not be Z it now should be written as L, L, L is equal to 0.

So, quickly I can write C 2 as minus of C 1 e to the power of alpha L divided by e to the power of minus alpha L then, B Z will be written as this c 1 I can take common so, it should be e to the power of i gamma Z by 2 and rest of the term e to the power of alpha Z minus L plus e to the power of minus alpha Z minus L bracket close. So, I just put the value of C 2 here and then I take common C 1 divided by this, and whatever the value I get is this one.

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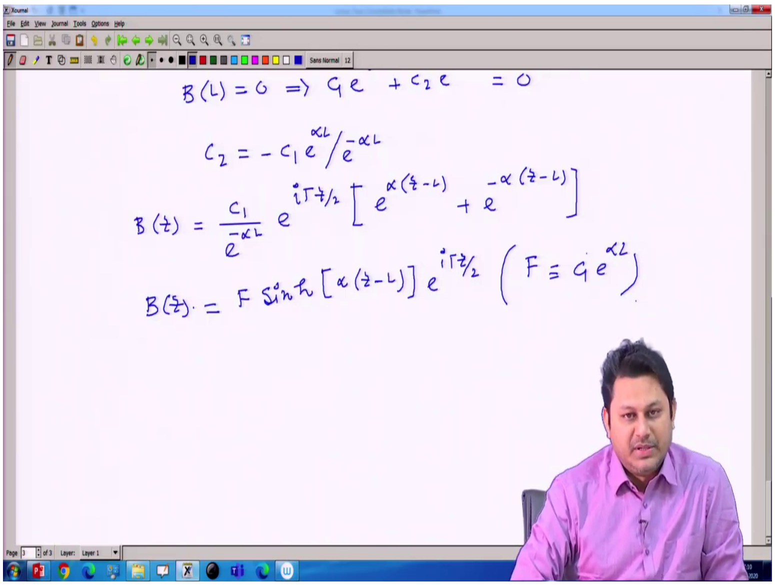

This thing I can write as F of sin hyperbolic alpha Z minus L into e to the power of i gamma Z divided by 2, when F is another constant, which I can write as C 1 e to the power of minus L.

Well I in today's class I derived B Z so, in the next class I will try to find out what is the value of A, which we can find straight forward from B, but today I do not have that much of time to complete this calculation. So, I like to conclude today's class here, so see you in the next class for the further calculations.

So, thank you for your attention and see you.