Physics of Linear and Non Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 04
Fiber optics components
Lecture - 44
Reflectivity Calculation (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 44 and in this lecture we will going to continue the Reflectivity Calculation ok.

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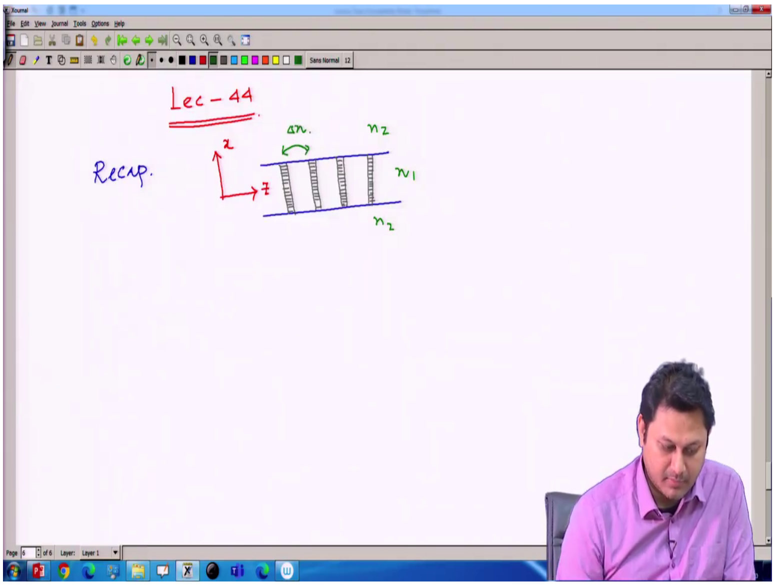

So, we have done many things learned in last two classes. So, let me recall what we have done, so recap. So, we started with a grating structure. So, let me first draw the grating structure I have a periodic refractive index variation inside the core of a fiber. So, I am drawing that part only.

So, these are the refractive index variation, periodic refractive index variation, and I am having a coordinate here along this direction it is x, this is z and the refractive index here is n 1, this is n 2, this is n 2 and this is the refractive index variation, there is a modulation with the amplitude delta n that we have.

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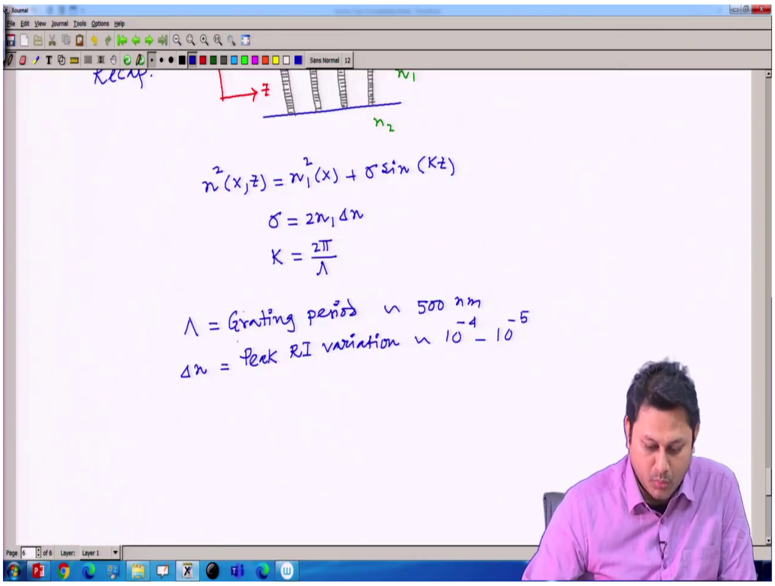

If I write the expression of the refractive index that we derive, it was n square which is a function of x and z is equal to n 1 square which is a function of x plus sigma sin is not square sorry it is sigma sin then, this case it where sigma is equal to 2 of n 1 delta n and K equals to 2 pi divided by this.

This is called the grating period. This is typically of the order of say 500 nanometer. And delta n is the peak reflective index variation, which is typically of the order of say 10 to the power of minus 4 to 10 to the power of minus 5.

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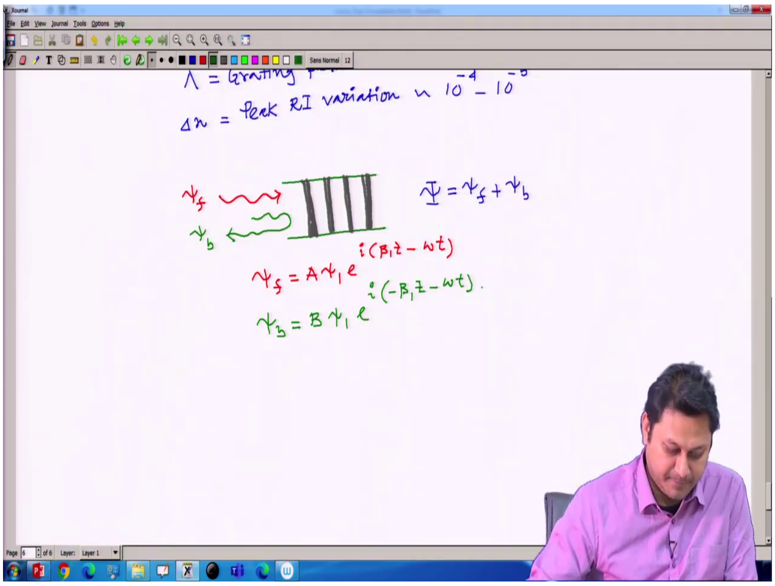

Now, when we have this structure; when we have this structure, the Bragg grating structure these are the gratings. And when you launch a light here there is certain light that is reflecting back, I call this as forward propagating light psi f field and this is psi b. We defined a big psi, which is a total field as psi f plus psi b, where psi f was something like A psi 1 e to the power of i beta 1 z minus omega t. And psi b is B psi 1 e to the power of i minus beta1 because, it is moving in the opposite direction omega t that was the structure.

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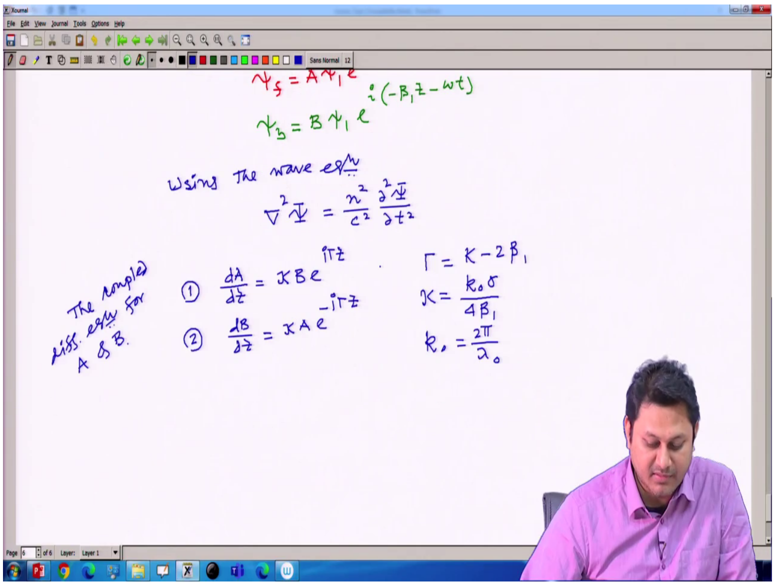

And after that using the wave equation, which was this and we managed to find out the coupled equation for A and B. So, 1 equation was dA dZ is equal to kappa B e to the power of i gamma z and another equation is dB dZ is equal to kappa A e to the power of minus i gamma z. So, this is the coupled equation for; the coupled differential equation for the amplitude A and B.

Where sigma is K minus 2 beta 1 where kappa is equal to K 0 sigma divided by 4 beta 1; and k 0 is 2 pi divided by lambda the lambda 0. The wavelength in the free space that you launch into the system, well I can so, this is roughly the structure this is roughly the calculation the key part of the calculation I just write as a recall.

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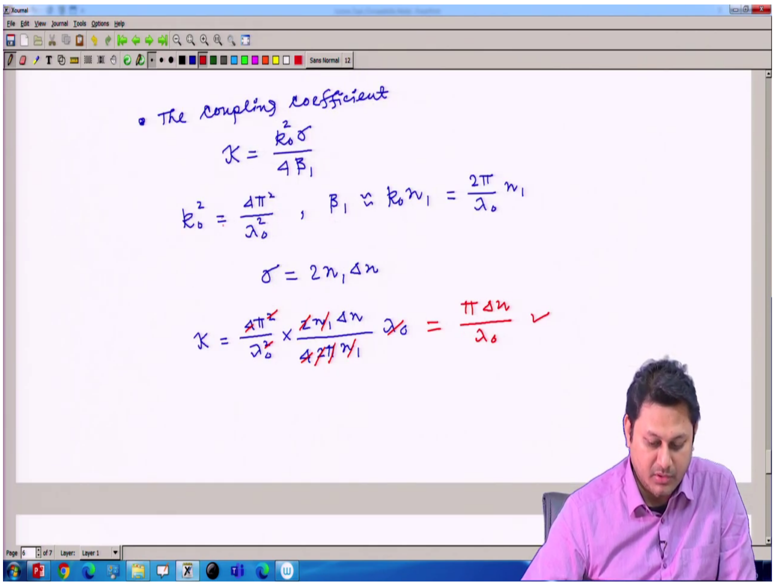

So now, we try to understand more on this kappa, which I call the coupling coefficient because, it coupled the two amplitude A and B through these two coupled equation. So, the coupling coefficient, kappa is defined as K 0 sigma divided by 4 beta 1. Now, sorry here I have a k 0 square because, it was a square.

So, now k 0 square is simply 4 pi square divided by lambda 0 square beta 1 the propagation constant is roughly k 0 n 1 the refractive index of the core without any perturbation, which is equal to 2 pi lambda 0 n 1 and sigma is 2 n 1 delta n. So, if I want to find out kappa with using all this expression, then my kappa will be something like, 4 pi square divided by lambda 0 square multiplied by 2 n 1 delta n divided by 4 beta 1 is this so, 2 pi n 1 and I have lambda 0 here.

So, few terms is going to cancel out this 4 term, 4 term will cancel out 1 pi 1 pi will be gone so this term will cancel out here, n 1 n 1 will cancel out 2 and 2 will cancel out. So, eventually I have a term, which is pi multiplied by delta n whole divided by lambda 0. So, my coupling constant my coupling coefficient can be determined by the term by this expression.

So, if I know delta n then I can roughly understand what is the value of the coupling coefficient? So, that is important in order to execute these two differential equation. Now, let us directly jump to the, to those two differential equation that we already derived so, coupled differential equation of A and B.

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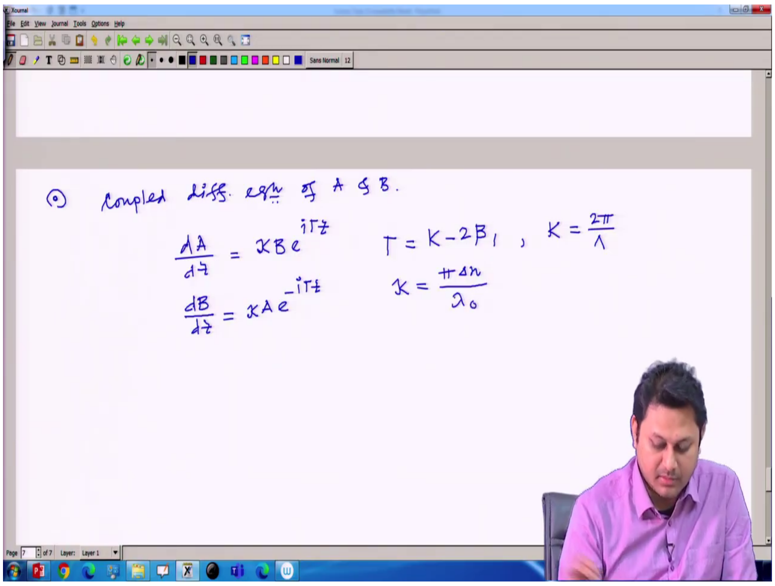

So, we already have dA dZ is equal to kappa B e to the power of i gamma z and del B del Z is kappa A e to the power of i gamma z, let me define once again gamma, gamma is K divided by 2 beta 1 every time you need to remember that is why I am writing so many times the same thing; and this is equal to 2 divided by the grating period.

Now, kappa is pi delta n divided by lambda 0 that we derived last just here. Well now, I need to solve this differential equation so let us take the simple one so, Bragg condition.

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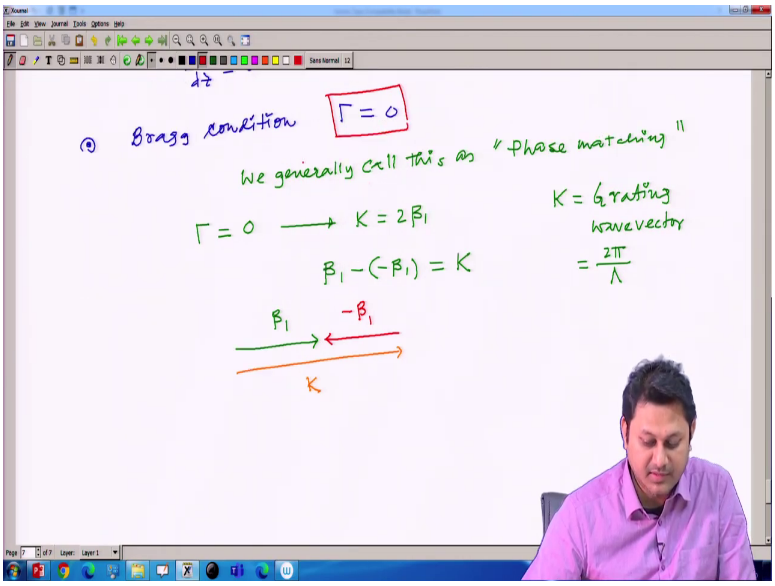

So, the Bragg condition; the Bragg condition suggests that when gamma is equal to 0 this is a very special condition, when we have no phase term. So, gamma is equal to 0 so this we generally called as phase matching, we generally called as a phase matching. So, gamma equal to 0 eventually tells us b K is equal to 2 beta 1 right, 2 beta 1 I can write as beta 1 plus sorry beta 1 minus of minus of beta 1; which is equal to K mind it K is the grating K is the grating wave vector because, it is 2 pi divided by the grating period.

So that means, if I have a propagation constant like this for forward and for backward. So, if I make a sum over difference over these two then, I can have this case so this is the phase matching curve phase matching diagram. So, this is my K this is say beta 1 and this is beta 2 minus beta 1.

So, if I have beta 1 minus of minus of beta 1 then it has to be equal to K. So, this is the phase how the phase is related so if the phase is 0 phase matching condition means gamma is 0. So, I can represent these propagation constant forward and backward propagation constant in this way.

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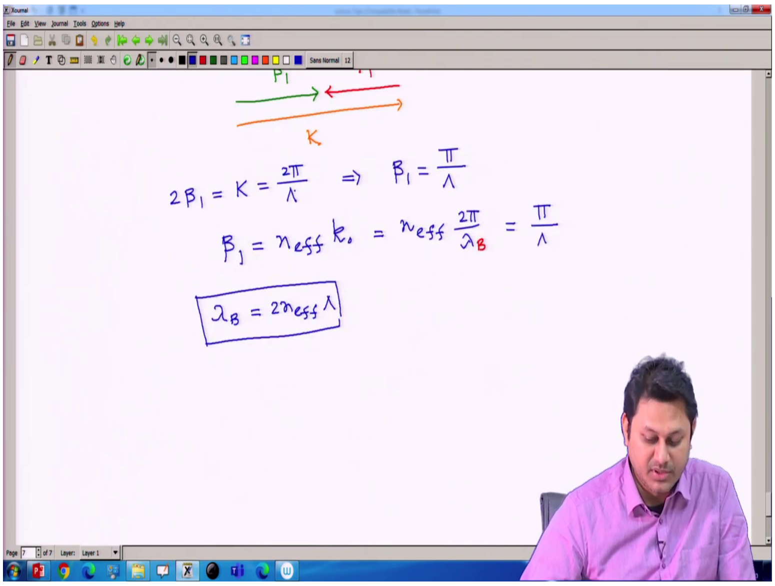

Now, 2 of beta 1 is K is equal to 2 pi divided by this one, which suggests that beta 1 is equal to pi divided by big lambda. Now, beta 1 is n effective if I write critically what is the beta 1 it should be n effective multiplied by k 0, that is n effective means the effective refractive index the wave will going to experience when it is propagating under the perturbation. 2 pi divided by lambda 0.

So, this lambda 0 should be a specific lambda 0. So, that should be equivalent to this is equivalent to pi divided by this. So, this lambda 0 should be spatial lambda 0, which basically satisfy this condition. So, now on we should write this lambda 0 as lambda b the Bragg wavelength this will not going to happen for all the wavelengths then, there is a specific wavelength I should write lambda B.

So, lambda B is a specific wavelength for which this condition will going to satisfy and if I write from this expression lambda B is equal to from this expression is simply 2n effective and then the grating period.

So, this is a very specific wavelength for which the phase matching condition is happening and if the phase matching condition is happening, we can see we will going to calculate that how A and B will going to evolve. So, that differential equation we will going to solve.

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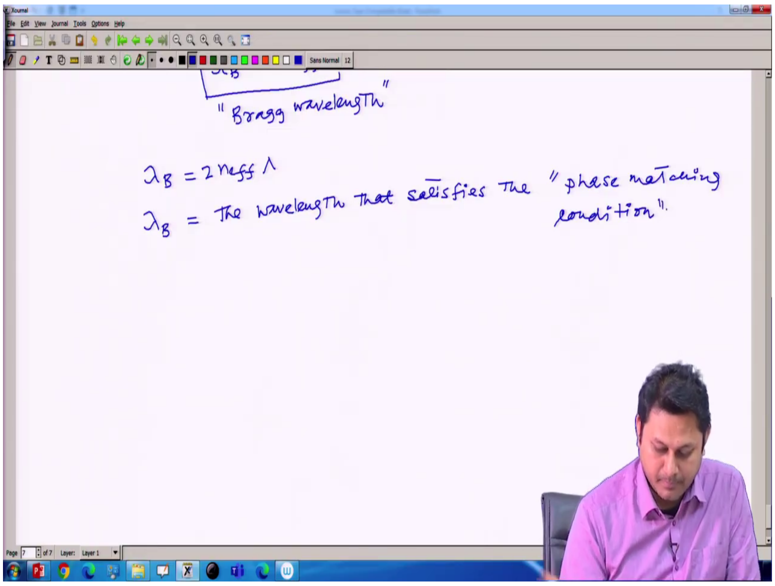

So, this is called the Bragg wavelength this is a special name called Bragg wavelength. So, this is a spatial wavelength, which will going to be reflected back from the grating structure. Well. So, lambda B is equal to 2 n effective lambda this is so, lambda B is the wavelength that satisfies the phase matching condition, typical phase matching condition ok.

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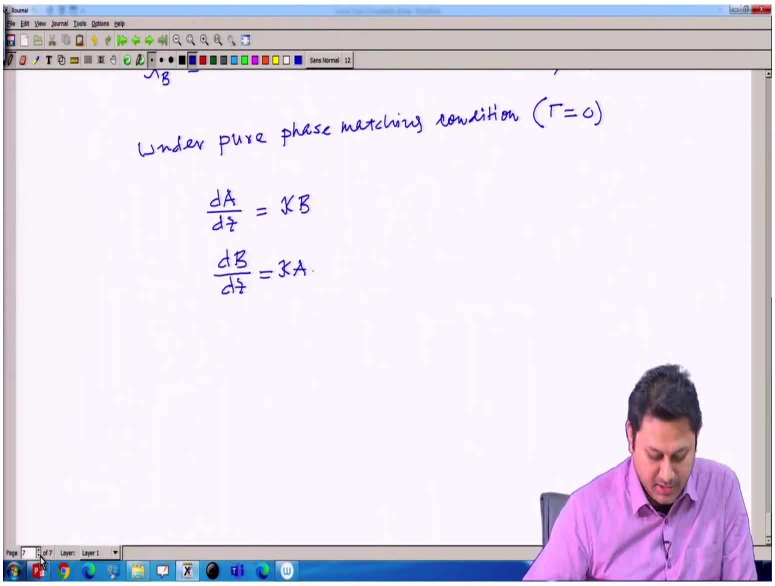

So, under pure phase matching condition I have so under pure phase matching condition, that is when these is equal to 0. We can have the equation dA dZ is equal to kappa B and dB dZ is equal to kappa A. So, this simple coupled equation we have under this condition.

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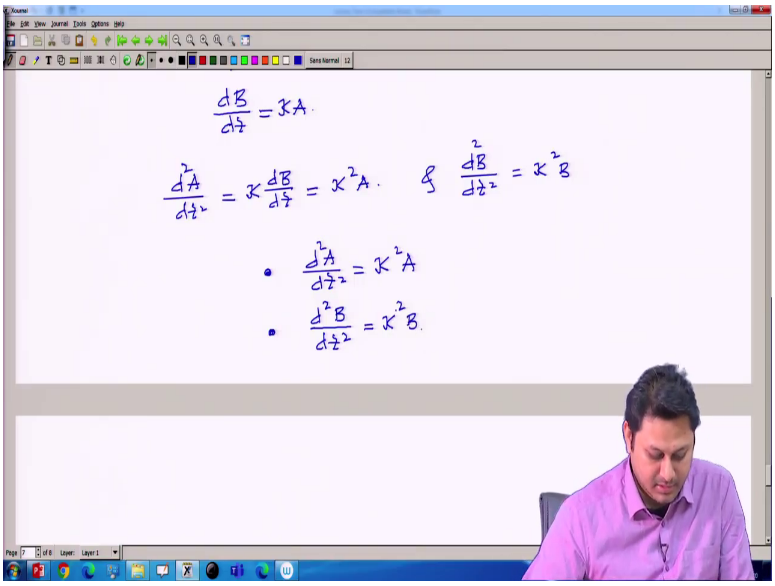

So, now we are going to quickly try to solve it. So, if I make a derivative of the first equation. So, it should be double derivative with respect to Z and that will be kappa del B del Z and it should be equal to kappa square A. In the similar way I can have del 2 B del Z square will be equal to kappa square B.

So, I have now two equation one is this one so, let me write it once again d 2 A dZ square is equal to kappa square A that is one equation. And another equation is d 2 B dZ square dZ square is equal to kappa square B two identical equation. Now, I need to solve this differential equation, but in order to solve this differential equation I need to put certain boundary condition that is important.

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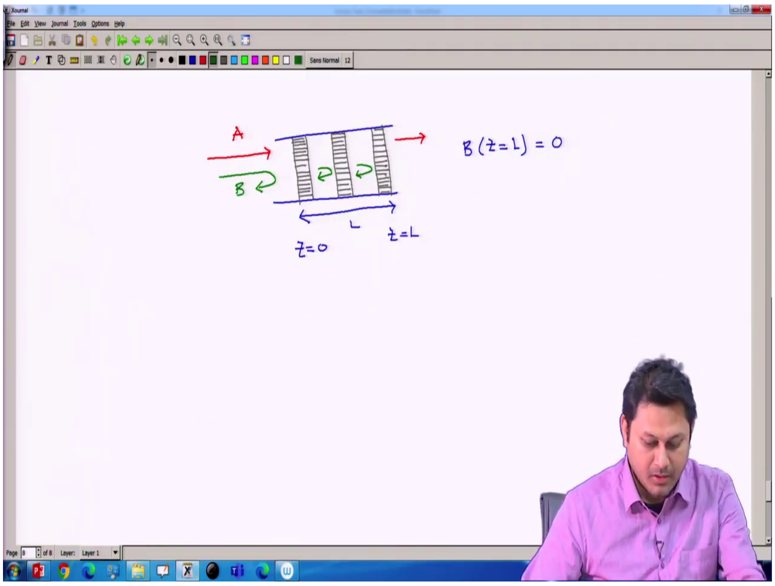

So, let us quickly done, quickly do this part. So, I had a grating structure like this I am drawing this structure several times so, that you understand it should be in your mind that what is going on. So, this is the incoming wave amplitude A and this is the reflected wave amplitude B. At this point this is the grating length suppose big L this is the point at Z equal to 0 and this is at Z equal to L.

So, this is the structure so mind it when the light will going to reflect this B when the light will going to reflect that means, this B at the boundary at least that means, at Z equal to L there should not be any kind of B or backward wave. So, one condition I can write here, that B at Z equal to L should be 0 that is one boundary condition I can write. So, this is a very important boundary condition, that when this wave is moving here there should not be any kind of reflection.

Because, it will be gradually reflected and we should not have after that we should not have. So, the output only we have this red line there will be no blue line, no green line so, everything will be reflected. It will be reflected gradually like these and like these and there will be no part here. So, everything will be reflected at the end of the grating.

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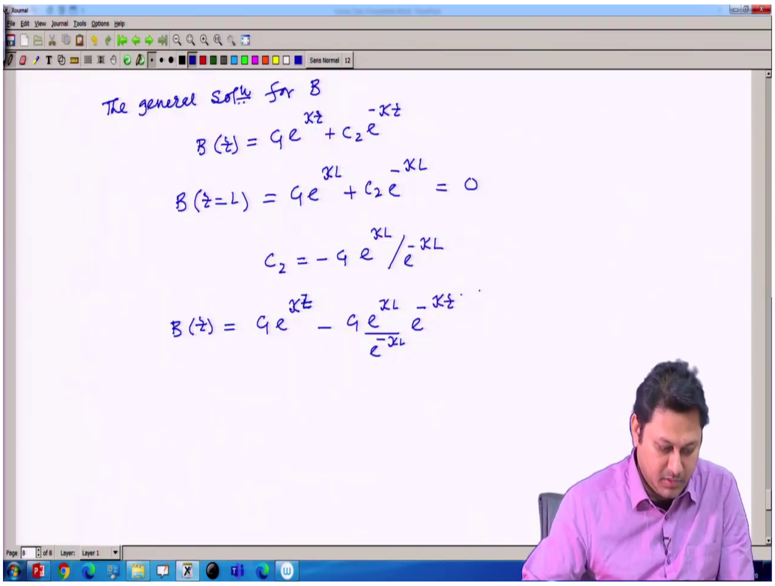

Well, let us write the general solution for B then the general solution for B is B Z because, it is a known differential equations I can directly write the solution for b it should be the exponential solutions exponential kappa z a general solution plus c exponential. So, this is the differential equation del B del z square is equal to kappa square B so, this is the solution for that.

Where c 1 c 2 are two constant so, boundary condition I already mentioned so that means, B at Z equal to L which is c 1 e to the power of kappa L plus c 2 e to the power of minus kappa

L that is equal to 0. So, B Z I can write as c 1 let me write c 2 in terms of c 1. So, I can have from here c 2, which is equal to minus of c 1 e to the power of; e to the power of; e to the power of kappa e to the power of kappa L e to the power of.

So, let me write it here it should be e to the power of kappa L divided by e to the power of minus of kappa L or e to the power of two kappa L, but anyway. So, my B Z will be c 1 e to the power of kappa L sorry kappa Z minus c 2 I will going to replace at c 1 e to the power of kappa L divided by e to the power of minus kappa L and e to the power of minus of kappa Z in this way.

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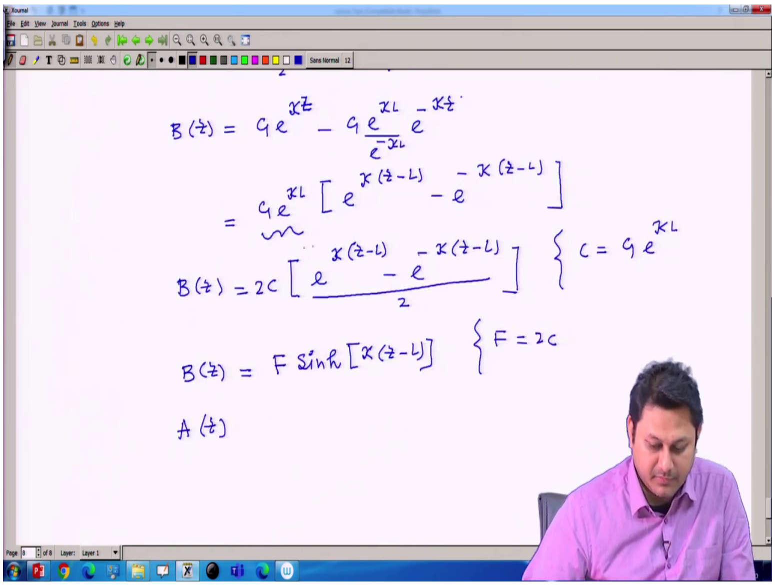

So, these I can write as c 1 e to the power of kappa L then e to the power of kappa Z minus L minus e to the power of minus kappa Z minus L and this is a constant. So, I can write another constant say c. So, my B Z is simply c multiplied by e to the power of so, let us put it as 2c e to the power of kappa Z minus L I put this two for a special reason.

Because, this solution I can write in this hyperbolic form because this is a specific form now I am having this. So, this quantity is nothing but F, I can write it again B of Z is simply F sin hyperbolic of kappa Z minus L, please note that when Z equal to L then B L is 0 so, whatever the boundary condition we put it satisfy.

And F is again equal to 2c and here c is c 1 e to the power of kappa L 1 constant. So, c is constant here and F is again another constant. So, A Z I can also calculate A Z, which is from the coupled equation if I start this coupled equation so, A Z is simply del B del Z in 1 by kappa.

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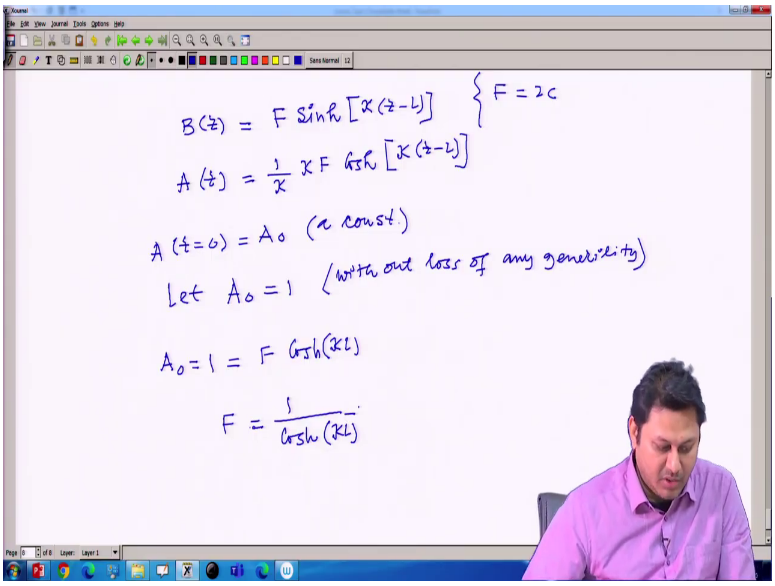

So that means, it is simply 1 by kappa and derivative of this quantity and derivative of this quantity means kappa F cos of hyperbolic then kappa Z by L. So, I can also evaluate the value of A Z. So, now A at Z equal to 0 is some value say A 0 a constant, just entering when this is entering the input is entering, then what we have is we are having some kind of initial amplitude.

So, without loss of generality I can have that A 0 equal to 1 just to put without any loss of generality. We can have this. Then, I can have that 1 is equal to F cos of cos hyperbolic kappa L because at Z equal to 0 this value is A 0. So, I can have A 0 which is equal to 1 as we consider. So, let a 0 is equal to 1 so this value I am having. So, F value is simply 1 divided by cos of cos hyperbolic of kappa of L, where L is a grating length. So, now I execute the value of A Z and B Z.

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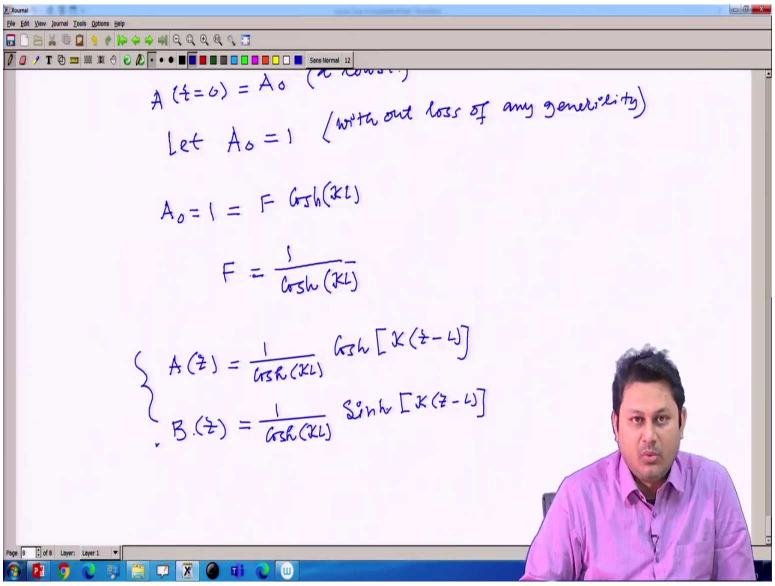

So the forward and backward amplitudes, if I write will going to evolve in this way. So, A Z is equal to 1 divided by cos hyperbolic kappa L and then cos hyperbolic of kappa Z minus L this is 1 solution and another solution for B is 1 divided by cos hyperbolic kappa of L sin hyperbolic; sin hyperbolic of kappa Z minus L, sin hyperbolic of kappa Z minus L.

So, eventually we find these two solutions. So in the next class, we will try to understand what we can do from this solution because, still we need to calculate the reflectivity. So, how one can find out the reflectivity we will going to discuss in the next class. So, with that note I would like to conclude.

Thank you very much for your attention and see you in the next class.