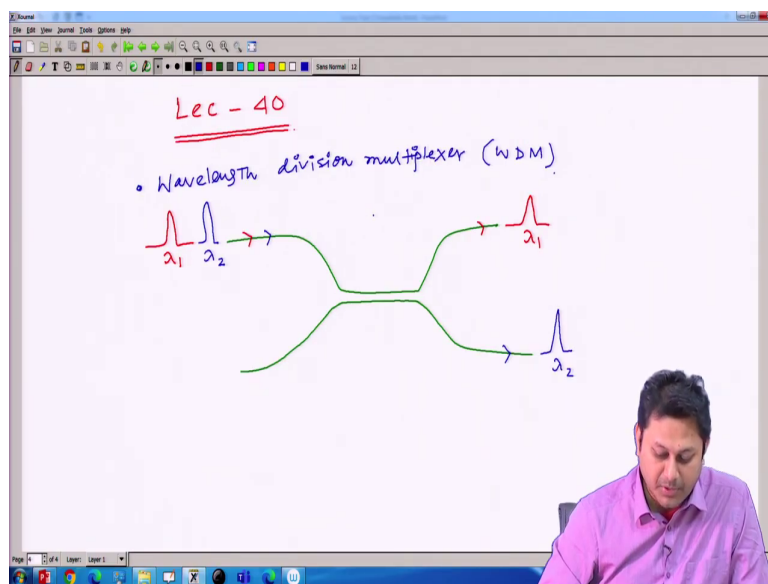


**Physics of Linear and Non Linear Optical Waveguides**  
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**Module - 04**  
**Fiber Optics Components**  
**Lecture - 40**  
**Working Principle of WDM Coupler**

Welcome student to the course of Physics of linear and non-linear optical waveguides. Today, we have lecture number-40. And in this lecture number-40, we will going to learn about the Working principle of WDM coupler.

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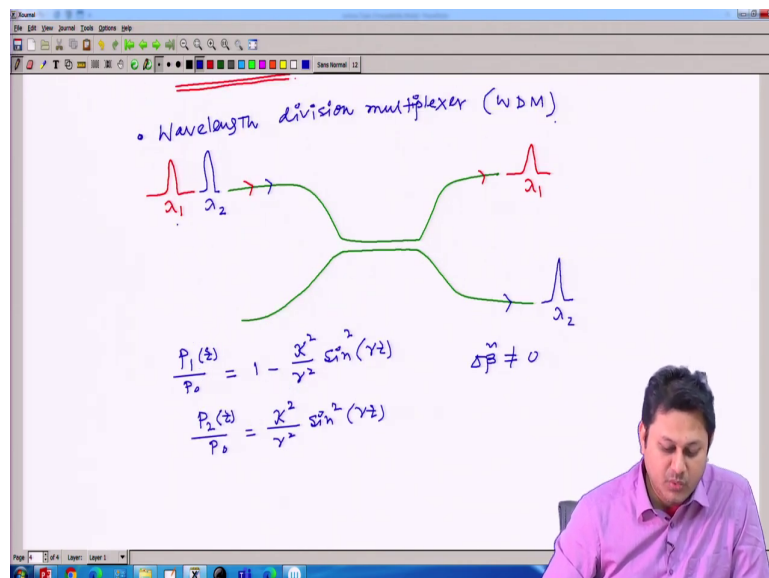


So, today's topic we will going to learn WDM coupler so which is wavelength division multiplexer or WDM. So, what is the WDM coupler? So, let me draw that. So, this is the

standard coupler system we are drawing every day. So, here what happened that if I launch two wavelengths, say one wavelength is  $\lambda_1$ , and another wavelength is  $\lambda_2$ .

So, I launch two wavelength  $\lambda_1$  and  $\lambda_2$ . In the output, what I want that two wavelength should separate. So,  $\lambda_1$  wavelength will come to this branch, and the wavelength  $\lambda_2$  should be here in this branch, that we can do also using the principle of coupler.

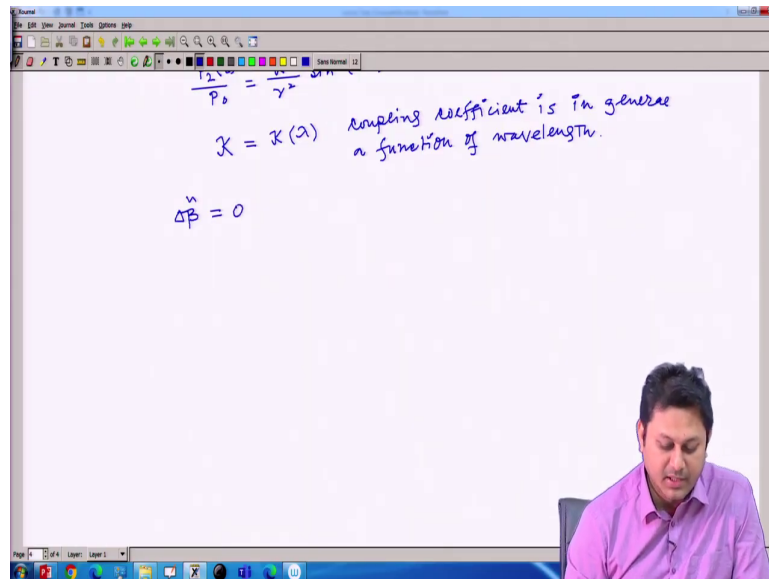
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So, we should write the power. What is the expression of the power? This is the expression of the power transfer from one branch to another branch. So, let me write it once again. So, this is a square associated with that. This is the expression with the condition when  $\Delta\beta$  is not equal to 0.

Well, when I am launching a light having two different wavelengths say  $\lambda_1$  and  $\lambda_2$ , then I must say the coupling constant  $\kappa$ , whatever the  $\kappa$  so far we are using should be a function of  $\lambda$ , so that we need to take account now.

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So, coupling coefficient, so that means, coupling coefficient is now is in general is in general a function of function of wavelength, it is a function of in general function of wavelength. So, that means, for  $\lambda_1$  and  $\lambda_2$ , these values are different. Now, consider let us consider to make a life simple these is equal to 0,  $\Delta\tilde{\beta}$  is equal to 0.

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$$\Delta\beta = 0$$

$$\frac{P_1(z)}{P_0} = \cos^2 \kappa z$$

$$\frac{P_2(z)}{P_0} = \sin^2 \kappa z$$

For  $\lambda_1 \rightarrow \kappa(\lambda) = \kappa_1$   
 $\lambda_2 \rightarrow \kappa(\lambda) = \kappa_2$

$$P_1 = P_0 \text{ when } \kappa_1 L = m\pi$$

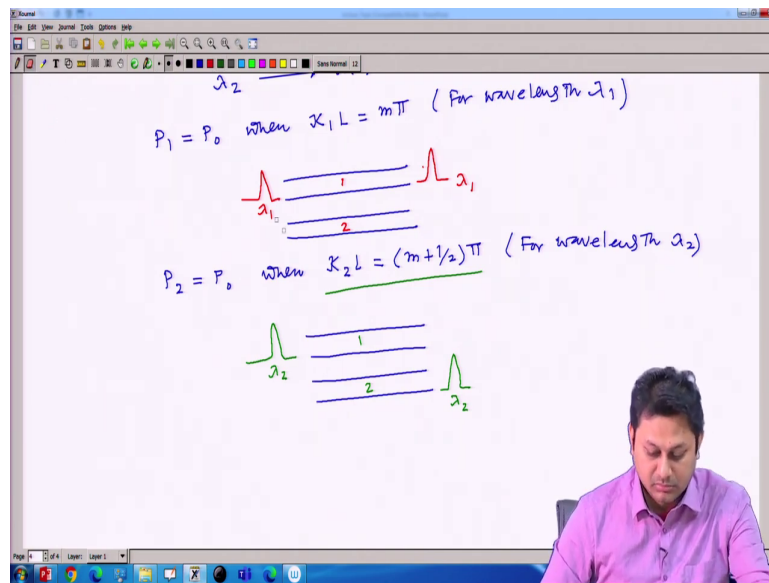
If I put delta beta equal to 0, then P 1 the expression will be simpler. So, P 1 divided by P 0 should be equal to cos square of kappa z, and P 2 divided by P 0 which is a function of z is equal to sin square of kappa z. Now, I mention that kappa is a function of lambda. So, for lambda 1, for lambda 1, kappa should be equal to kappa 1; and for lambda 2, kappa should be equal to kappa 2.

So, better to write; better to write in this way. Better to write in this way. Kappa as a function of lambda 1 at lambda 1 is this; kappa at lambda 2 is this. So, I can have two different kappa, because now my wavelength is different in the system it is lambda 1 and lambda 2 two wavelengths are there. So, my kappa's are different.

So, now, if the kappa's are difference, so I can this conditions are different for different lambdas. For example, P 1 will be equal to P 0 when kappa 1 L is equal to m pi. So, for

wavelength  $\lambda_1$ , I can have  $P_1$  equal to  $P_0$ . And if I use the  $k$  for wavelength  $\lambda_1$ , then this condition at  $m\pi$ , so the power will totally converted from branch 1. So, the power will come to branch 1.

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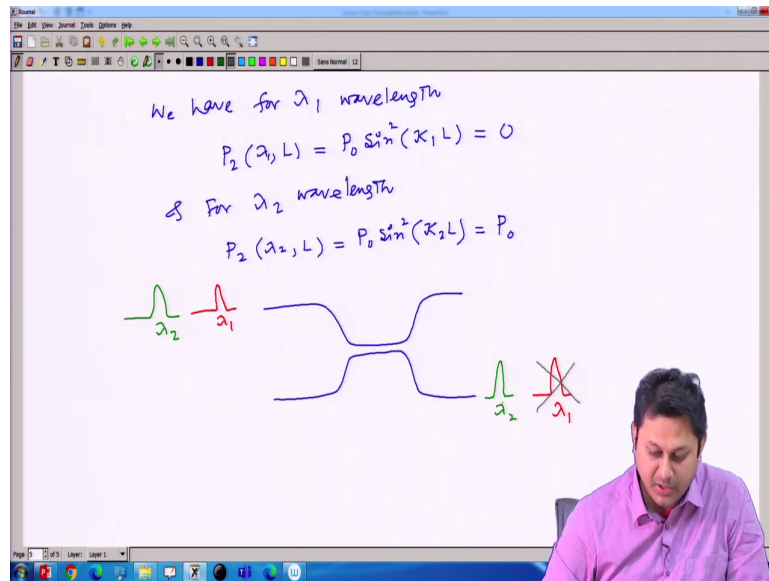


So, this is for wavelength  $\lambda_1$ , so this is the value at which in the branch 1, I have entire power. So, this is the coupler, and I have  $\lambda_1$  here. So, there will be transfer, but the output I can have my  $\lambda_1$  here, because this is branch 1, and this is branch 2.

However, when  $P_2$  equal to  $P_0$  is this condition one can have when  $k_2 L$  is equal to  $m + \text{half } \pi$ . And this is for wavelength  $\lambda_2$ , so that means, for  $\lambda_2$  what happened, this is the waveguide we have, this is the waveguide we have. I launch a  $\lambda_2$  here in waveguide 1, but this  $\lambda_2$  will come here; this is 1, this is 2. So, I want the  $\lambda_2$  in branch 2.

If I want the  $\lambda_2$  at branch 2, this condition has to follow. Well, with this idea, with this idea, we can find out the length at which both the condition should satisfy simultaneously. What is the condition?

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So, let me write it here. So, we have, so we have for  $\lambda_1$  wavelength,  $P_2(\lambda_1)$  at some distance  $L$  is equal to  $P_0 \sin^2(\kappa_1 L) = 0$ , that means, I do not want any kind of wavelength at output at second  $P_2$  means at second branch. And for  $\lambda_2$  wavelength, I can have  $P_2$ . So, this is  $\lambda_1$ . This is  $\lambda_2$ ,  $L$  equal to  $P_0 \sin^2(\kappa_2 L) = P_0$ .

So, in the second branch, so what I am trying to do is this. So, this is the second branch, I have I want to have my so this is  $\lambda_1$  say and this is  $\lambda_2$ . So, in this branch I only

have lambda 2. But at the same time, I do not want, I do not want lambda 1. So, lambda 1 if I write, so I do not want this so, these things should I cross. So, I do not want.

So, I have both the conditions here. The same condition for lambda 1 and lambda 2 I put the equation which is same. In one case I have lambda 1; in one case I have lambda 2. And in other case the equation is same, but I do not have lambda 1. This equation is for lambda 1, and this equation is for lambda 2.

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Handwritten notes on a digital whiteboard:

Equation:  $P_2(x_2, L) = P_0 \sin^2(k_2 L) = P_0$

Diagram: A potential well diagram with two horizontal lines representing energy levels. The top line is labeled 1 and the bottom line is labeled 2. On the left, there are two wave function plots: a green one labeled  $x_2$  and a red one labeled  $x_1$ . On the right, there are two wave function plots: a red one labeled  $x_1$  and a green one labeled  $x_2$ . The red  $x_1$  plot on the right is crossed out with a red 'X'.

Equations:

$$\begin{cases} k_1 L = m\pi & (m = 1, 2, 3, 4, \dots) \\ k_2 L = (m - 1/2)\pi \end{cases}$$

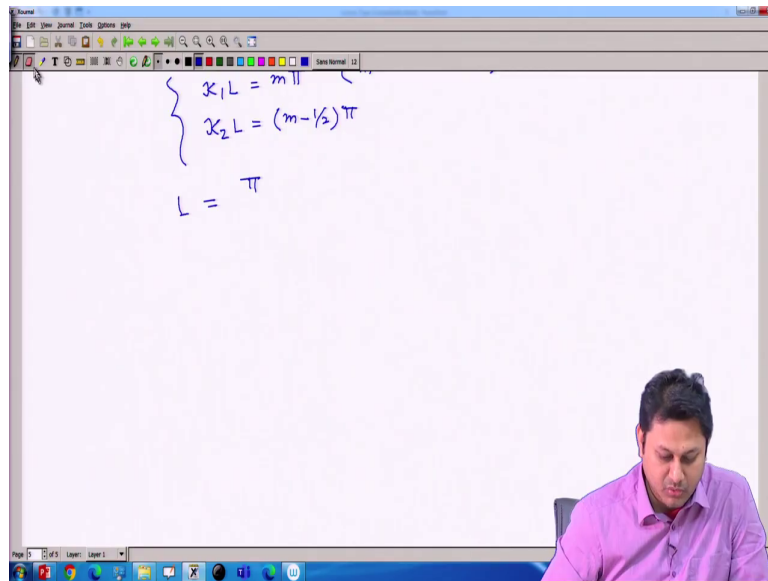
$$(k_1 - k_2)L = \frac{\pi}{2}$$

$$L = \frac{\pi}{2(k_1 - k_2)}$$

critical coupler length corresponds to (well)

So, this condition if I look carefully satisfied when kappa 1 that length L is equal to m pi say m equal to 1, 2, 3, 4 so on. And simultaneously kappa 2 for the same length it has to be m minus half pi. So, from these two expressions, one can extract what is the value of the L.

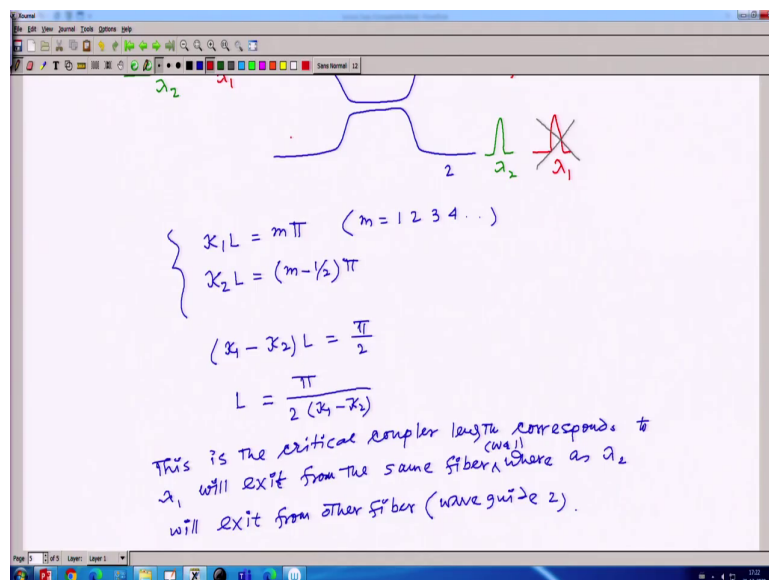
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$$\begin{cases} \kappa_1 L = m\pi \\ \kappa_2 L = (m - 1/2)\pi \end{cases}$$
$$L = \pi$$

So,  $L$ , I can extract which is  $\pi$  divide if I just make this minus this, so it should be simply. So, ok let me do that clearly. So, I can have  $\kappa_1 L - \kappa_2 L$  is equal to simply  $\pi$  by 2. So, my  $L$ , the critical length at which these two wavelengths will be separate it out should be 2 of  $\kappa_1 L - \kappa_2 L$ .



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So, this is, so this is the critical coupler length corresponds to the fact that lambda 1 will exit from the same fiber where as lambda 2 will exit from other fiber. Other fiber means waveguide 2; same fiber means waveguide 1. So, I already draw that, that if this is waveguide 1, and if it is waveguide 2, so lambda 1 will going to come here, and lambda 2 will be here.

Where there is no lambda 1 here in waveguide 2. So, I just put this condition. And after putting this condition I can have the length L equal to pi divided by 2 multiplied by kappa 1 by kappa 2. This is a critical length at which this condition this thing may happen.

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This is the critical coupler length <sup>(cm)</sup> corresponds to  $\lambda_1$  will exit from the same fiber where as  $\lambda_2$  will exit from other fiber (waveguide 2).

Example

$$n_1(\lambda_1) = 1.4525 \quad \lambda_1 = 1.55 \mu\text{m}$$

$$n_2(\lambda_2) = 1.45 \quad \lambda_2 = 1.3 \mu\text{m}$$

$$\left. \begin{array}{l} \text{for } \lambda_1 \Rightarrow \kappa_1 = 6.496 \text{ cm}^{-1} \\ \text{for } \lambda_2 \Rightarrow \kappa_2 = 4.872 \text{ cm}^{-1} \end{array} \right\}$$

$$L =$$

So, we quickly write down the example. So, one example, so consider a coupler with identical fiber such that say  $n_1$  for  $\lambda_1$  is equal to 1.4525, and  $n_2$  which is for  $\lambda_2$  is equal to 1.45. So, the refractive index of  $\lambda_1$  is this, and refractive index of the  $\lambda_2$  is this.

Where  $\lambda_1$  is say 1.55 micrometer,  $\lambda_2$  is say 1.3 micrometer. And the coupling coefficient  $\kappa_1$  is equal to 6 point say 496 centimeter inverse, and  $\kappa_2$  this is for  $\lambda_1$ , and this is for  $\lambda_2$ , 4.872, this is point centimeter inverse. So,  $\lambda_1$  and  $\lambda_2$  is given for these two  $\lambda$ ;  $\kappa_1$  and  $\kappa_2$  is given for these two  $\lambda_1$  and  $\lambda_2$ .

And I can calculate readily I can calculate what is my critical length at which this separation of the wavelength happens, and that we already calculated it is  $\pi$  divided by 2  $\kappa_1$  minus

kappa 2, kappa 1 and kappa 2 value is given. So, I can calculate that. So, L is equal to pi divided by 2 of kappa 1 minus kappa 2 which is equal to 9.67 nanometer. If you calculate that, it should come like this. Well, once we know the value of l then I can find out one thing. So, for this interaction length, what happened?

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for  $\alpha_2 \Rightarrow k_2 = 4 \times 10^7$

$$L = \frac{\pi}{2(k_1 - k_2)} = 9.67 \text{ nm.}$$

$k_1 L \approx 2\pi$   
 $k_2 L \approx \frac{3}{2}\pi$

Note: in general the coupling length  
 $L_c = \frac{\pi}{2k}$

For  $\alpha_1$   $\frac{L}{L_c} = \frac{2\pi}{k_1} \times \frac{2k}{\pi}$

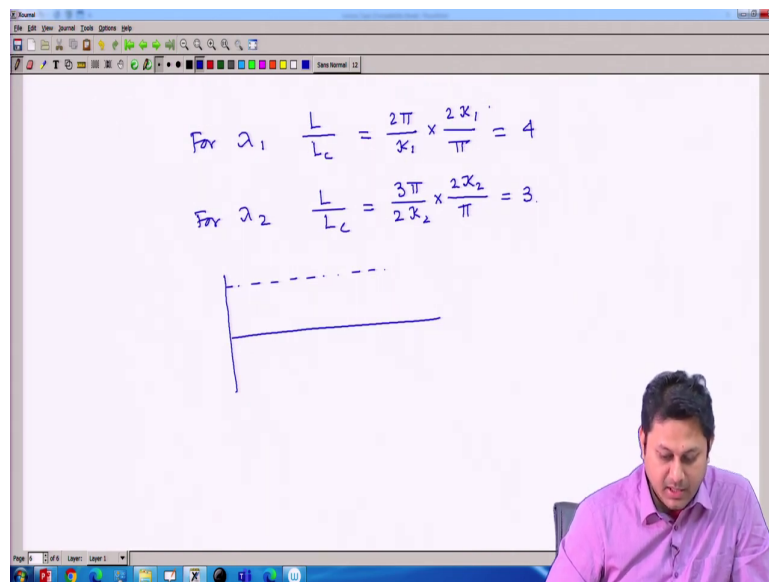
So, for this interaction length, we can have kappa 1 L is coming on to be very close to 2 pi. If we calculate because we know L, we know kappa 1. So, kappa 1 and L should be very close to 2 pi; and kappa 2, L will be around 4 by 2 pi. If you calculate, you will find this. Now, the general coupling, note: in general the coupling length is given by L c is equal to say pi by 2 kappa.

This is the coupling length we know. So, for lambda 1 you can see for lambda 1 what happened, whatever the length is there divided by coupling length. So, this is the length L is

the length of the coupler and I want to find out how many coupling length are there for lambda 1 I can calculate that. So, L I know, this is  $2\pi$  divided by kappa 1 multiplied by 2 kappa 1 divided by pi. So, this is the value I know pi divided by 2 kappa L c.

And L, I can calculate from this calculation which is  $2\pi$  divided by kappa 1. So, I just put this value  $2\pi$  divided by kappa 1 multiplied by 1 by L c. 1 by L c is 2 kappa 1; it should be kappa 1. So, this value is equal to 4. So, the total coupling length total length total couple, total coupler length is equivalent to the 4 coupling length for lambda 1.

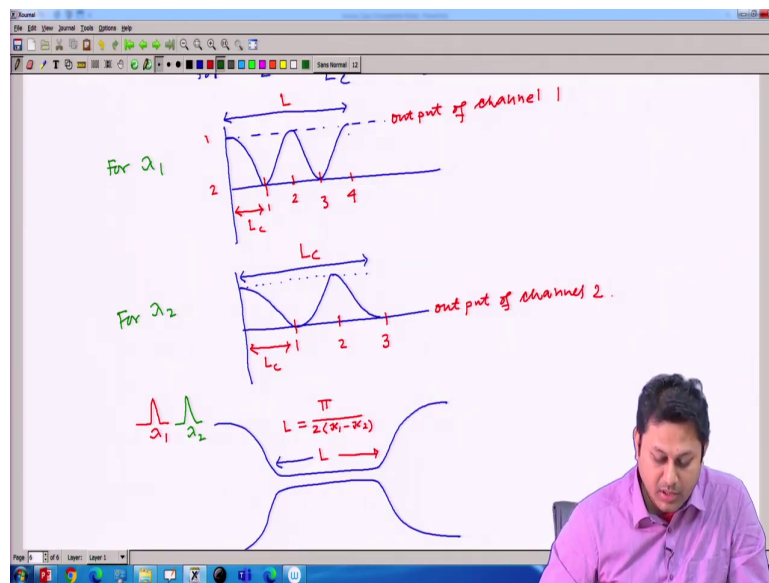
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So, what about for lambda 2? For lambda 2 this value L divided by L c which is equal to 3 pi divided by 2 kappa 2 multiplied by 2 of kappa 2 divided by pi which comes out to be 3. So, that means, if I draw this stuff it should be something like that. So, this is the total coupling length.

And in total coupling length, I have the total coupler length is  $L$ , and the coupling length  $L_c$  is 4, I mean 4 times coupling length  $L_c$  is my total length. So, this is one coupling length; this is another coupling length; this is another coupling length, and this is another coupling length.

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So, I have this total length of the coupler which is  $L$ . So, this is 1, and this length is my  $L_c$ . This is another length 2, this is another length 3, and this is another length 4. So, you can see this is channel 1 and this is channel 2. So, output, this is the output of channel 1. This is the output of channel 1. What about and for  $\lambda_2$ ; as well because this is for  $\lambda_1$ . So, this is for  $\lambda_1$ .

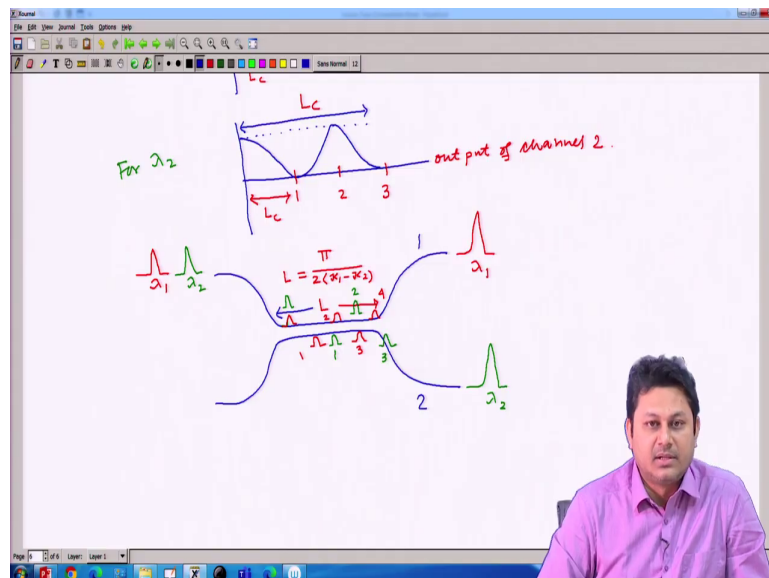
What about  $\lambda_2$ ? The same thing, if I draw the same scale and I do make it in same scale, so my total length  $L$  is same. So, let us first draw my total length. So, this is the length

at which this is the length of the coupler for which I am having this. So, now, if I draw for  $\lambda_2$ , the ratio of total length  $L$  and  $L_c$  we already figured out for these two.

Here we have the curve like this. This is one length, this is second length, and this is third length. So, this length is 1, this length is 2, and this length is 3. What is this length? This length is the coupling length for  $\lambda_2$ . These two are not same. And this is the output, this is the output of channel. This is the output of channel 2.

So, that means, I now have this is the structure, this is the structure this length is  $L$  that I calculate. And what is  $L$ ,  $L$  is equal to  $\pi$  divided by  $2\kappa_1 - \kappa_2$  that I find. And what happened that two wavelengths are launched here. For  $\lambda_1$ , we have four for  $\lambda_1$  we have these four coupling length that we are going to cover in this length.

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So, fourth time the energy will come. So, initially it was here, then it should come here, the second point. Then so sorry initially it was here, then come here, and then again come back to here, again come back to here, and finally, I can have this. So, this is 1, this is 2, this is 3, and this is 4th length. On the other hand, what happened for other wavelength initially it was here.

So, I draw these things together, then it comes to here at some point, then in come to here some point, and finally, it goes back here. So, this is 1, this is 2, and this is 3. So, finally, at the output, I have the wavelength here  $\lambda_1$  and I have the wavelength here  $\lambda_2$  in another branch. So, this is 1 and this is 2 if I write in terms of branches.

So, I hope you understand the working principle of the WDM. So, this is basically the working principle. I try to derive everything from the first principle, so that you can understand what is going on the physics behind that. So, I will conclude here this discussion. So, in the next day, we will start a brand new topic called fiber Bragg grating which is a very, very important topic.

And lot of research is going on in recent times on fiber Bragg grating many applications are there, but in our course we will going to study the physics behind that, the working principle of what is going on in the fiber in terms of fiber Bragg gratings. So, with that note, I like to conclude.

Thank you for your attention. So, see you in the next class.