Physics of Linear and Non-Linear Optical Waveguides
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Module – 01
Basic Optics
Lecture – 04
Total Internal Reflection (TIR), Evanescent Wave

Hello student, welcome to class number lecture number 4. In this particular lecture, we will going to cover the Total Internal Reflection that we started in the last class and the concept of Evanescent Wave.

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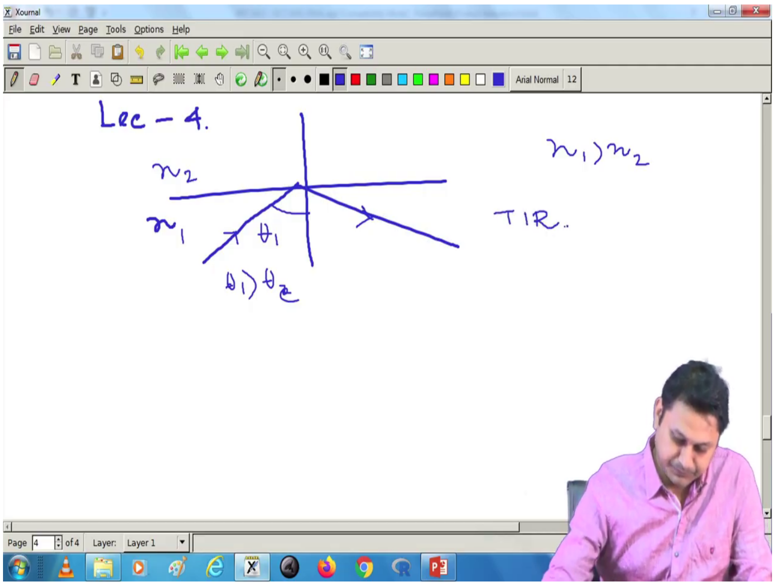

In the last class, it is lecture number 4. So, in the last class we find that a total internal reflection can happen. So, this is n 1 and this is n 2, where n 1 is greater than n 2 and this angle theta 1 is greater than the critical angle theta 2 critical angle theta c, then we have the total internal reflection. So, that is the phenomena we discussed in the last class it is called the total internal reflection.

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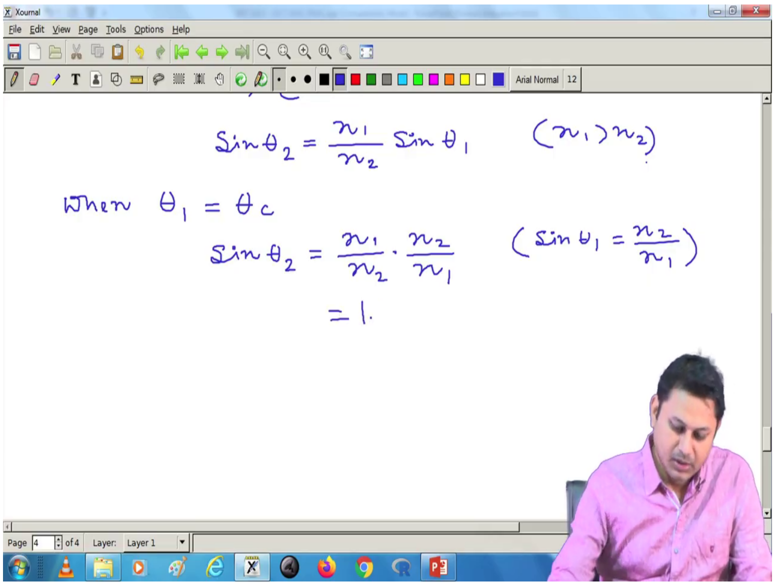

So, we have sin theta 2 is equal to n 1 divided by n 2 sin theta 1. The condition that n 1 is greater than n 2. When theta 1 is equal to theta c; that means, at the critical condition I can write sin theta 2 as n 1 divided by n 2 multiplied by n 2 divided by n 1. Because theta 1 in that case sin theta 1 was simply n 2 divided by n 1 because theta 2 was pi by 2. So, I can write in this way. So, it is eventually 1.

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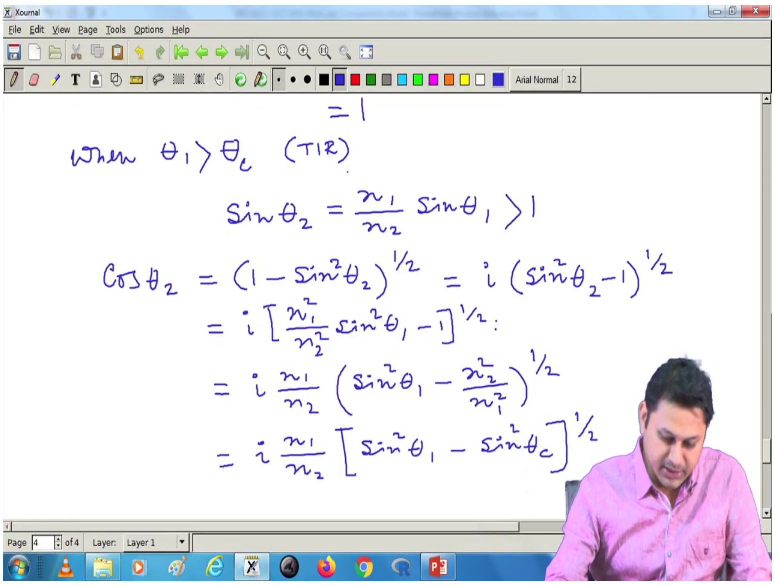

Now, what happened when theta 1 is greater than theta c, that is the condition for total internal reflection total internal difference this is the condition. Under this condition; obviously, sin theta 2 which is equal to n 1 in general into sin theta 1 will be greater than 1.

Because now sin theta 1 will have more value whatever the value it has in the previous case. So, this value should be greater than 1 if it is greater than 1 then we can have a complex value for cos theta 2, if I calculate cos theta 2 or sin theta 2. So, some complex value is coming here.

So, cos theta 2 will be 1 minus sin square theta 2 whole to the power half, which is I can write it as i n 1 square divided by n 2 square, because sin theta 2 is greater than 1. So, I just replace this thing as I sin square theta 2 minus 1 whole to the power half. And now I put the value of sin square theta 2 whatever I have here. So, it is simply sin square theta 1 minus 1 whole to the power half.

I rearrange few things like n 1 divided by n 2 I take this n 1 into common, then I can have here sin square theta 1 minus into square divided by n 1 square whole to the power half. Now, I can write these again like i n 1 divided by n 2 sin square theta 1 minus into square divided by n 1 square I can write as in terms of critical angle to the power half. Because at critical angle sin theta c is equal to n 2 by n 1, that we already calculated; so, at critical angle when theta sin theta n 2 by n 1 is basically sin theta c.

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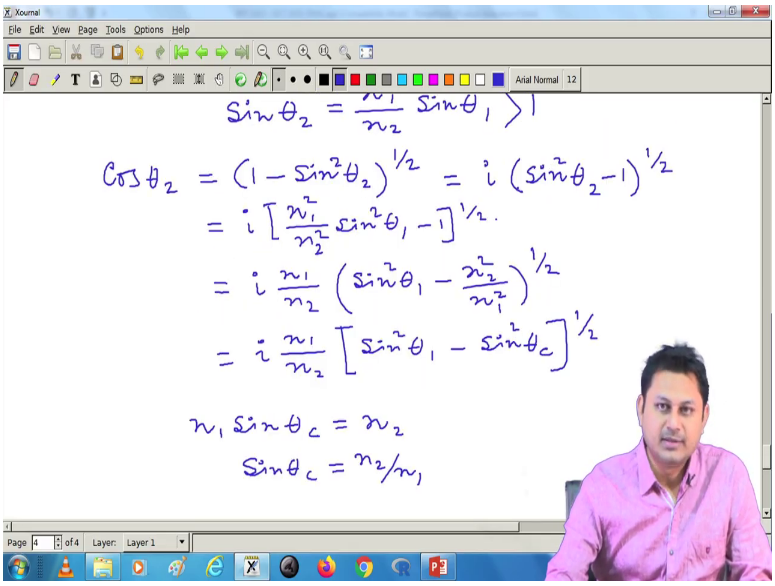

So, n 1 sin theta c is equal to n 2. So, sin theta c is equal to n 2 by n 1 and I replace this n 2 by n 1 as sin theta c. So, I can see that the value of cos theta is become imaginary, which is expected, but because of that we will going to find something interesting which is called the evanescent wave. So, that we need to understand what is the meaning of that?

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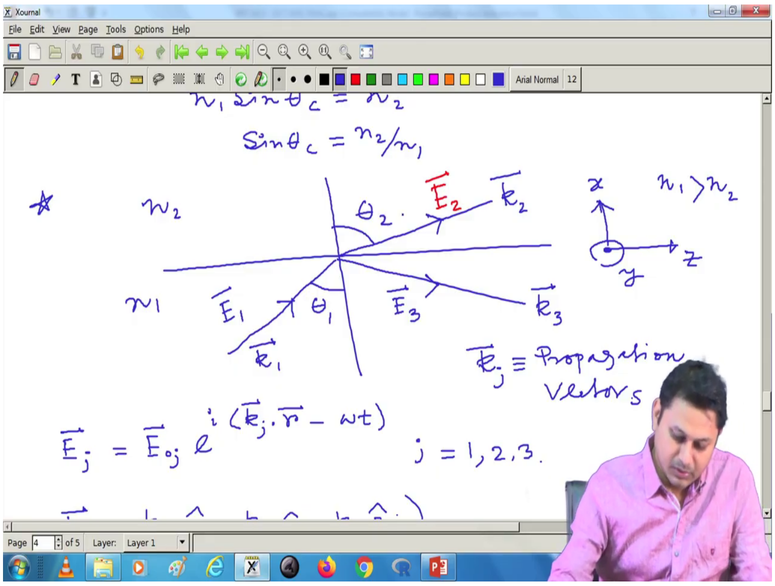

So, some wave that is passing through the medium I mean that is leaking through the medium is decaying down so, that we need to understand. So, let us draw this way. So, this is the interface we have. We have n 2, n 1 two medium with refractive index n 1 and n 1. Where my n 1 is greater than n 2 and if I have a wave like this it one portion is reflected and another portion is transmitted. That is generally that is the case we have one is transmitted one is reflected and this is incident.

If this has a vector propagation vector k 1, I can write this as a propagation vector k 2 and the reflected wave as a propagation vector k 3. So, three propagation vectors associated with these three waves, where these vectors are the propagation vector. Propagation vectors for three waves incident, transmitted and reflected.

Now, the electric field for these three waves can also be represented I write as E j is a general electric field like E 0 j E to the power of i vector j dot r minus omega t, where this index j is for 1, 2, 3 where 1 for incident wave, 2 for transmitted wave and 3 for reflected wave.

Now, the k vector so, now I need to put one coordinate system here to make because I am now dealing with vectors. So, I should divide this vector into 3 components. So, I should have a coordinate system. Suppose this is my coordinate system along these I have z along this I have x and y is perpendicular to the plane of z and y. So, this is if this is a plane then y is perpendicular to the to that plane.

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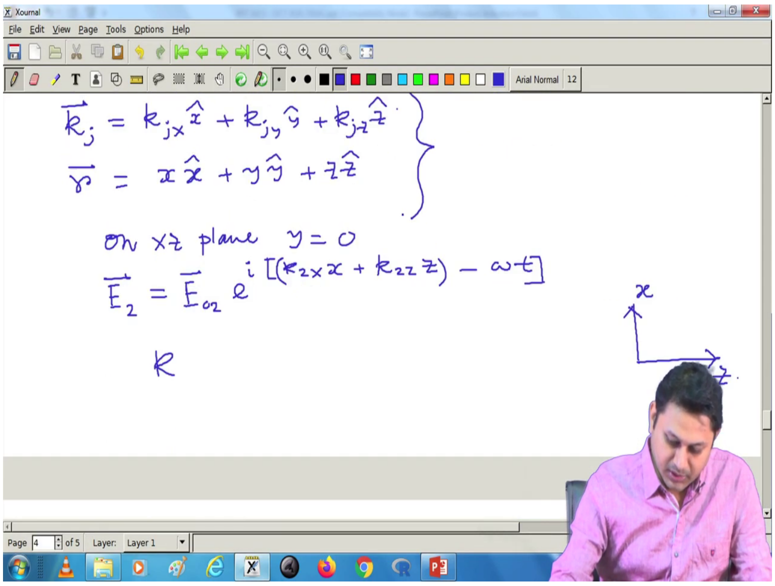

In that case whatever the vector I have k 1, k 2, k 3 in general k j. I have three components: one is k j x is the x component of k j with unit vector x plus k j y of unit vector y plus k j z of unit vector z this is 1. And r vector is simply the position vector. So, I can write is according to our coordinate system I have this plus y cap z plus z z cap. So, these two are the vectors and I write in a component wise.

Now, if I consider that these three are in the plane of y z then the y component is not there. So, on x z plane y is equal to 0. So, if y equal to 0 I do not have any kind of y component. So, entirely the system is now became a 2D system and if I calculate E 2, which is the transmit transmitted wave. Specially now calculate E 2 which is this one, which is this one, this is my E 2 where this is my E 1 and this is my E 3 the electric field associated with this waves.

Now, if I now I want to concentrate only on the E 2 waves that is which is transmitted and we will going to put the total internal reflection condition to find out what is the fate of this E 2 wave? What is going to happen to this E 2? So, E 2 field is represented explicitly according to our notation is this. It should be k 2 x x plus k 2 z z mind it y component is 0. I should have a bracket here and then minus of omega t.

Now, what is k what is my; what is my k k x and k z component. So, this is my x and this is my z and if I decompose my k vector then simply and this is the angle by the way this is the angle. So, whatever the angle I have let me now put my angle then only I can equal to. So, suppose this is my theta 1 and this angle is theta 2. So, the k is making an angle theta 1 and theta 2 with this axis which is y which is x.

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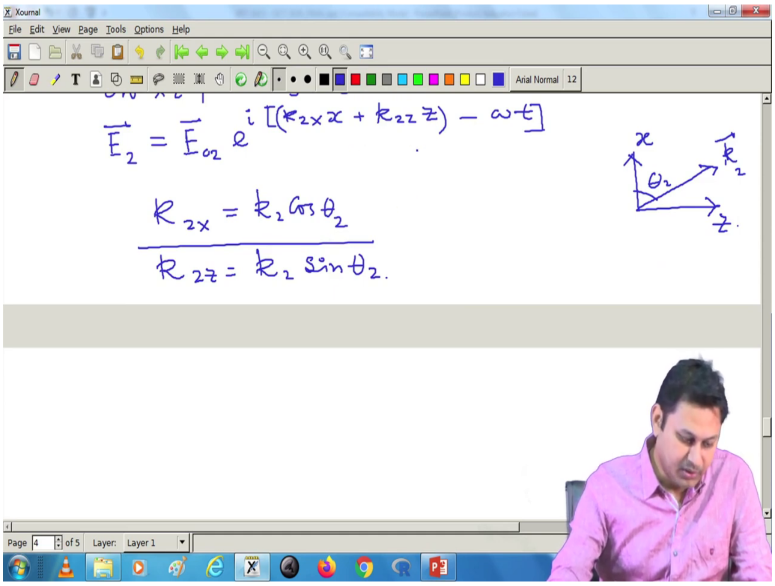

So, that means, k 2s x should be simply k 2 cos theta cos theta 2, because this wave is moving in this way, this is my k 2 vector and it is having an angle theta 2 here. So, if I decompose this component to k k x and k z. So, k 2 x component will be simply the magnitude of k 2 multiplied by cos theta, in a similar way simply k 2 sin theta 2.

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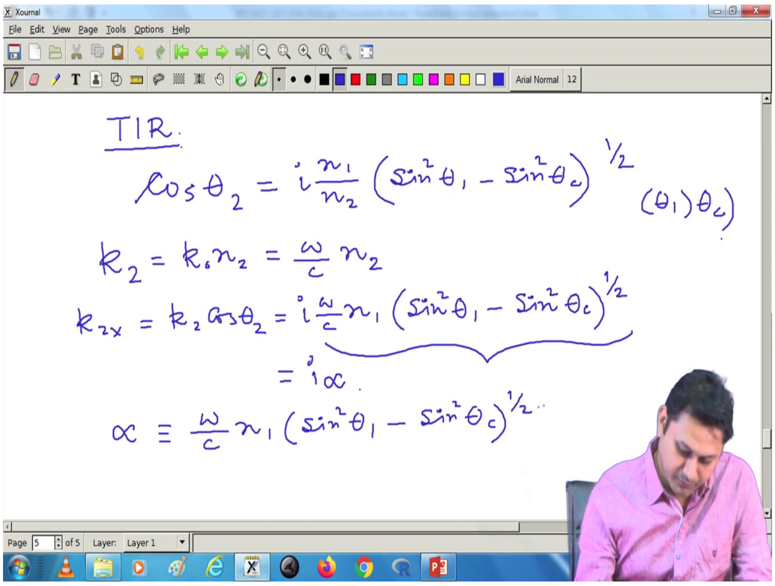

Now, I will going to put these value here into the equation, but before that I need to put the condition of total internal reflection. Under total internal reflection what happened we just seen in the last class. That cos theta 2 should be equal to n 1 divided by n 2 this not in the last class, I think it is just today's class, no this in the yeah this is in today's class this one.

So, I am going to use this equation and it is simply i into n 1 sin square theta 1 minus sin square theta c equal to the power of half. This it should be a first bracket like this with the condition mind it theta 1 is greater than theta c. Well, we know that k 2 is a propagation vector in the medium 2 having the refractive index n 2. So, I can write it as free medium wave vector k 0 multiplied by the refractive index n 2.

So, it should be simply omega divided by c n 2, this is the way we write the propagation this amplitude of the propagation vector k 2 in the medium 2 where the refractive index is n 2. So, k 2 x which is k 2 cos theta 2 is now written as i omega c n 1 I just replace this value of k 2 here. So, n 2 going to cancel out because it is omega c n 2 and here we have n 1 multiplied n 1 divided by n 2. So, it should be omega c multiplied by n 1.

And then sin square theta 1 minus sin square theta c whole to the power half a term like this which we already derived. Now, I write this entire term whatever the term I have as alpha, it is i alpha. So, my alpha is something like omega c n 1 then sin square theta 1 minus sin square theta c whole to the power of half. So, I have k 2 a complex kind of term. So, whenever I have a complex kind of term in k 2 then it should be; it should be; it should be a given some sort of loss. So, that I will going to check.

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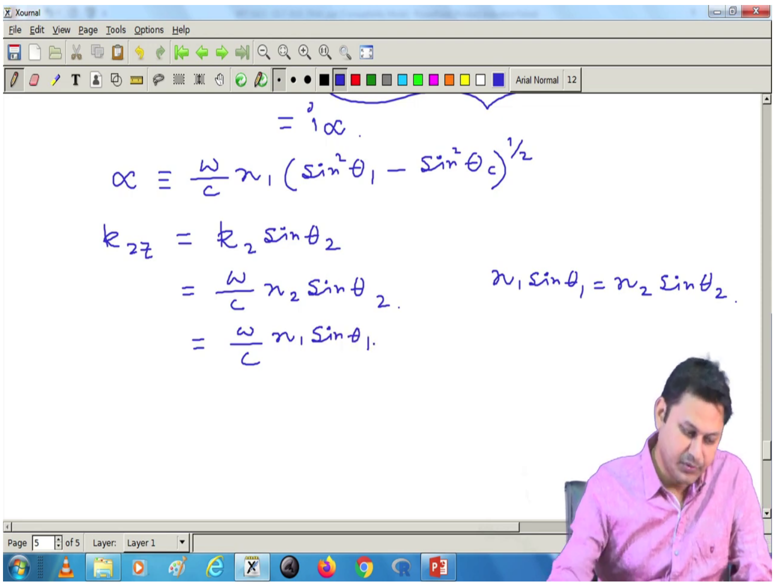

So, I have now k 2 z I need to calculate k 2 z as well, it is k 2 sin theta 2 which we calculated which is omega divided by c because k 2 is omega divided by 2 omega divided by c multiplied by n 2. So, that we are going to calculate put it here sin theta 2.

Now, we have from the Snell's law n 1 sin theta 1 everything, actually we try to put in terms of theta 1 is equal to n 2 sin theta 2. From Snell's law we have this and it is also valid even under total internal reflection. So, omega c n 2 sin theta 2 I replace as n 1 sin theta 1; so, everything in now in sin theta 1.

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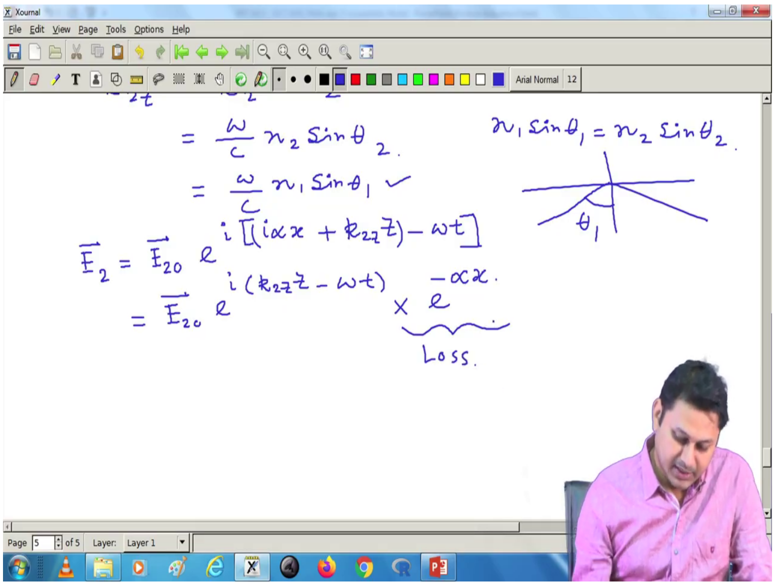

Now, we will going to put this value to E 2 and when we put this value to E 2, my E 2 is E 2 0 vector sin e to the power of i. Now, what value of k I get? k 2 z the this value should be k 2 z; this value should be k 2 z multiplied by x; k 2 z I find as i of alpha.

So, it should be i of alpha multiplied by x and the other term k 2 x k 2 sorry this k 2 z multiplied by z and k 2 z also I find out. And it is purely a real term; k 2 is purely a real term, because omega is a frequency real, c is a velocity of light real, n 1 is a refractive index which is also real.

And sin theta 1 theta 1 in this case real because in total internal reflection this is the case, this angle there is no problem with this angle this is theta 1. So, theta 1 is real here as well. So, I have this quantity and then I have minus of omega t this one. So, finally, I have E 2 0 vector e to the power of i k 2 z z minus omega t into this is a very important term e to the power of minus of alpha x. So, this term gives us some sort of loss and loss over x direction.

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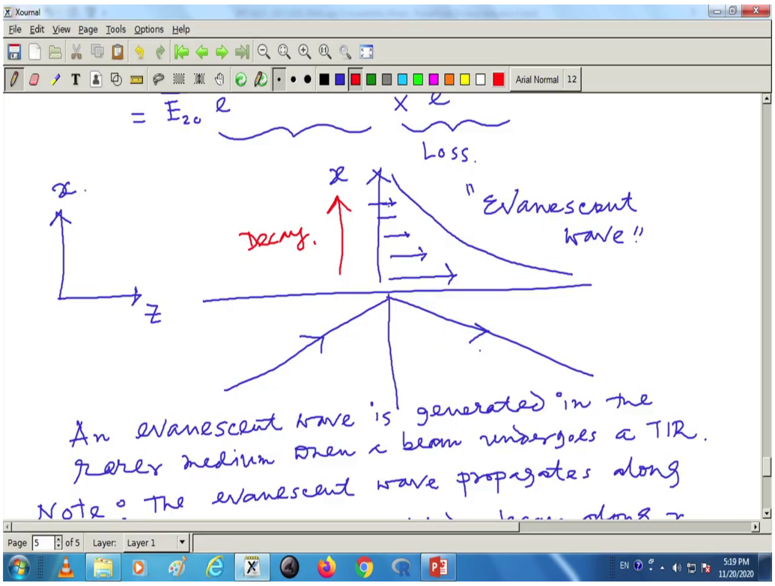

So, if I now try to understand that this is the interface and in this interface I am having a total internal reflection like this. If I have a total internal reflection like this then some wave is in this direction which is called the transmitted wave which is leaking out. But, if I go to y axis x direction according to our notation this is x and this is z direction.

So, wave is moving along z direction over the surface this portion fine, but if I go gradually along y direction along x direction then we find there is a exponential decay of this stuff. So, the energy of this wave is exponentially decay. So, this is called the evanescent wave. This is called; this is called the evanescent wave; evane s c e n t evanescent wave and evanescent wave is some wave which is generated.

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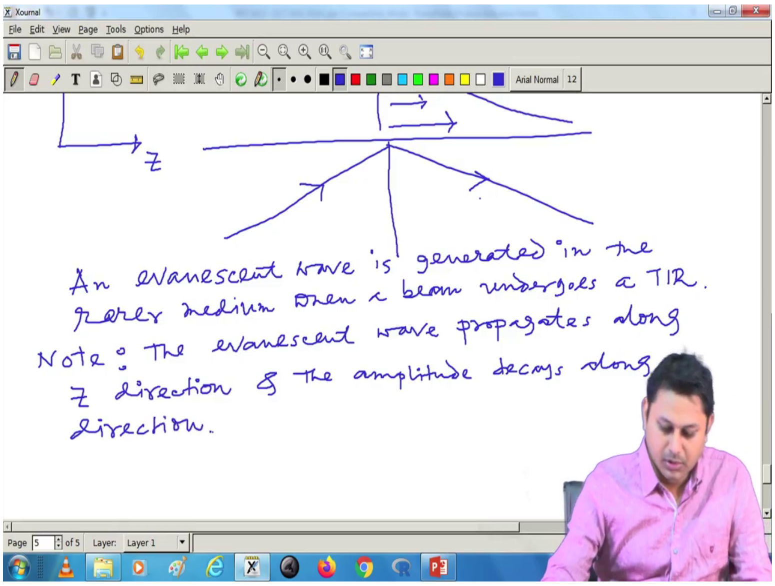

So, an wave is generated in the rarer medium when a beam undergoes a total internal reflection. Note: The evanescent wave propagates along z direction and the amplitude; and the amplitude decays along z x direction. So, I have an evanescent wave that can propagate along z direction. So, this is our z direction and it basically decays in y direction, in these direction it will go into decay. And that is the feature of the evanescent wave, in many cases we will going to encounter an evanescent wave.

So, in the future classes may be we will going to find what kind of structure we have where evanescent waves can generate, it is a very important concept. So, with this note we will like to conclude. In the next class, we will start a very basic understanding of the fiber optics. And, we see how the light can propagate inside an optical fiber. So, with that I like to conclude today's class.

Thank you for your attention.