

Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber Optics Components
Lecture - 39
3 d B power splitter

Hello student to the course of Physics of linear and non-linear optical waveguides. Today we have lecture number 39 and today we will going to learn the working principle of 3 d B power splitter.

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Lec-39

$$\frac{P_1(z)}{P_0} = 1 - \frac{k^2}{\gamma^2} \sin^2 \gamma z$$

$$\frac{P_2(z)}{P_0} = \frac{k^2}{\gamma^2} \sin^2 \gamma z$$

$$\gamma^2 = k^2 + \frac{\delta \beta^2}{4}$$

When $\delta \beta = 0 \rightarrow$ we have $\gamma = k$.

$\delta \beta = (\beta_2 + k_{22}) - (\beta_1 + k_{11})$

So, in the last class, let me remind what we have done in the last class so, these are two optical waveguides place very close to each other and we call is a coupler and if I launch a

light here, then at certain distance the light can come here and there is a periodic variation of that.

If this is say channel 1 and this is channel 2. So, if the power of the channel 1 if I define these as $P_1 Z$ and if I defined is $P_2 Z$ for power at channel 2, then we derive a differential equation we derive a differential equation based on the coupler mode theory and the equation was something like that $P_1 Z P_0$ where P_0 is initial power here.

This is equal to $1 - \kappa^2 \sin^2 \gamma Z$ and $P_2 Z$ divided by P_0 is equal to $\kappa^2 \sin^2 \gamma Z$. You should remember that γ^2 is defined as $\kappa^2 + \tilde{\Delta\beta}^2$ divided by 4, where $\tilde{\Delta\beta}$ was the mismatch between the propagation constant of these two wave guides.

Now when, $\tilde{\Delta\beta}$ is equal to 0 we can have with this condition we can have γ is equal to κ and $\tilde{\Delta\beta} = 0$ means, $\tilde{\Delta\beta} = 0$ means $\tilde{\Delta\beta}$ it was defined like $\beta_2 + \kappa - \beta_1$ plus κ . What this is a correction term κ and κ we already mentioned.

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When $\delta\beta = 0 \rightarrow$ we have $\gamma = \kappa$.

$\delta\beta = (\beta_2 + \kappa_{22}) - (\beta_1 + \kappa_{11})$

$= 0$

we consider the mismatch of the propagation constant is small.

So, this thing equal to 0 that means, we consider the miss match of the propagation constant propagation constant is small. So, eventually this miss match is considered to be 0 so, there is no difference between the propagation constant of these two.

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Then.

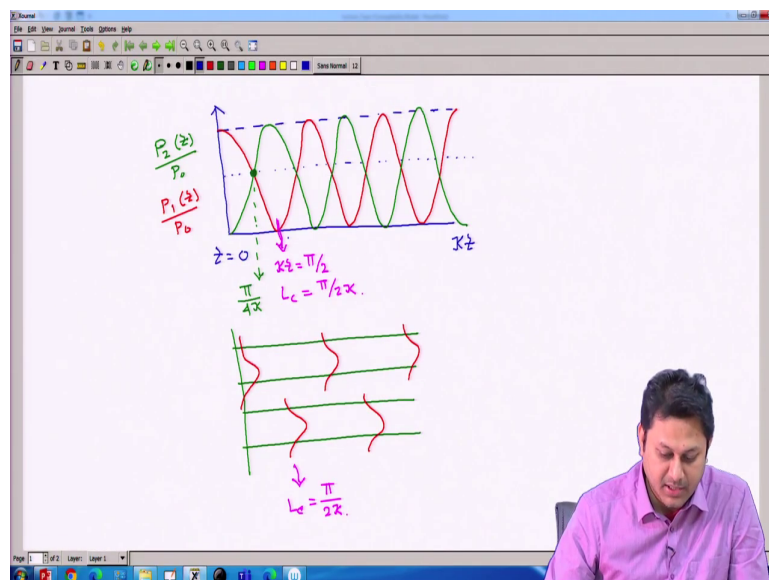
$$\frac{P_1(z)}{P_0} = 1 - \sin^2 \gamma z \equiv \cos^2 \gamma z = \cos^2 \kappa z$$

$$\frac{P_2(z)}{P_0} = \sin^2 \kappa z$$

$$\left\{ \begin{array}{l} \Delta \beta = 0 \\ \gamma = \kappa \end{array} \right.$$

If we assume that then, $P_1 Z$ divided by P_0 is equal to 1 minus sin square gamma Z which is equivalent to cos square gamma Z and I can write also is that cos square kappa Z because, if delta beta is equal to 0 gamma and kappa are same. In a similar way $P_2 Z$ divided by P_0 should be equivalent to or equal to sin square kappa Z so, this will going to evolve as cos square kappa Z and sin square kappa Z a very straightforward equations.

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Now, if I plot these two equations, it readily tells us that there should be a variation sinusoidal variation of the power in two branches of the coupler. So, let us draw that so, in one branch is the power will vary like this and for other branch is will vary like that. So, this green one is $P_2(z)/P_0$ because, at the beginning when z is equal to 0 we do not have any kind of power if I look to this figure we do not have any kind of power so this is z equal to 0 at branch 2 so, there is no power, the power is launched only in branch 1.

So, we have the full power at branch 1 and gradually what happened so, the red one is this evolution of $P_1(z)/P_0$ because, at the beginning when z is equal to 0 we do not have any kind of power if I look to this figure we do not have any kind of power so this is z equal to 0 at branch 2 so, there is no power, the power is launched only in branch 1. So, the power is gradually moving from one branch to another branch, this specific this specific figure basically tells us how the power is periodically changing from one branch to another branch.

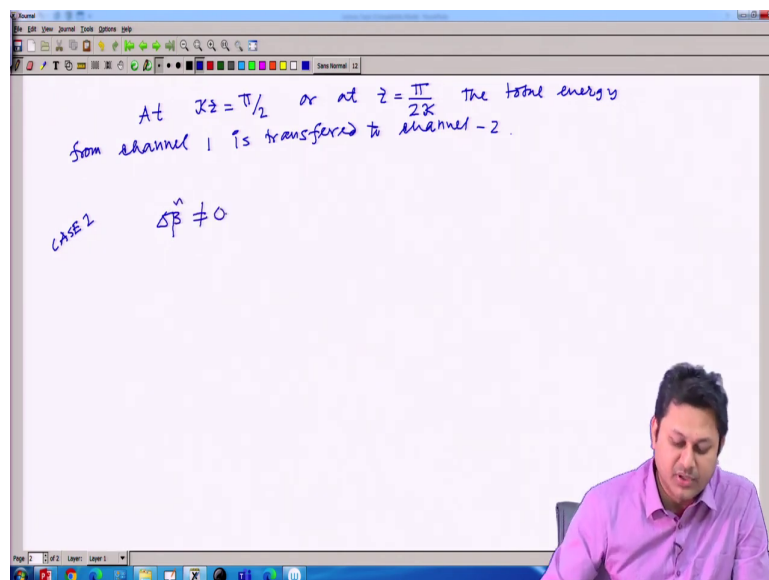
So, if I want to draw that, these are supposed two channels initially I launch power here there is no power here, but at certain instance the power will come here. After that it will again go back to this one and again it will come back to this one and so on. So, there will be a periodic exchange of power, there will be a periodic exchange of power.

And from this curve, one can readily find out the value at which the total so, I should also draw the x axis so, x axis eventually I plot κZ . So what happened at certain point specific point say here, total power will going to come so this is for branch 1 so, branch 1 the power becomes 0 and the branch 2 the power is maximum.

So, this is the point this is the point say Z I can call this as so this is the point where κZ is equal to $\pi/2$ or I can have Z specific length say L_c , which is $\pi/2\kappa$. So, this is a specific length or the coupling length at which the entire power is coupled to another branch.

So, if I try to understand in this figure at this point my L_c will be equal to $\pi/2\kappa$. So, this is this length of the coupler at which the minimum length of the coupler at which the total power is transferred to another branch. So, I need to define this so let me write it here.

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So, at kZ equal to π by 2 or at a distance Z equal to π by 2 k the total energy from channel 1 is transferred to say channel 2. So, the total energy from channel 1 is transferred to channel 2 when if I have a distance when the distance of the coupler is π by 2 k .

Well, this is important also you can see we had a point here in between I just mark this point here, where we have the 50-50 power coupling. So, you can see that 50 percent of the power is here and 50 percent of the power in channel 1 and 50 percent of the power in channel 2. So, this length is a length where I can distribute the power equally to two coupler so, that is the important thing today. So, this kind of coupler is eventually called the 3 d B coupler where we can divide the power.

So, we will do that we will find the length. So, this length so for so here I can write this length also is nothing but pi divided by 4 kappa. So, if I know the coupling coefficient of a waveguide then pi divided by 4 kappa length where we can have the equal power distribution.

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Handwritten mathematical derivation on a whiteboard:

$$\frac{P_1(z)}{P_0} = 1 - \frac{\kappa^2}{\gamma^2} \sin^2 \gamma z$$

$$\frac{P_2(z)}{P_0} = \frac{\kappa^2}{\gamma^2} \sin^2 \gamma z$$

Case 1 When $\delta\beta = 0 \rightarrow$ we have $\gamma = \kappa$.

$$\delta\beta = (\beta_2 + \kappa_{22}) - (\beta_1 + \kappa_{11}) = 0$$

We consider the mismatch of the propagation constant is small.

Then.

$$\frac{P_1(z)}{P_0} = 1 - \sin^2 \gamma z \equiv \cos^2 \gamma z = \cos^2 \kappa z$$

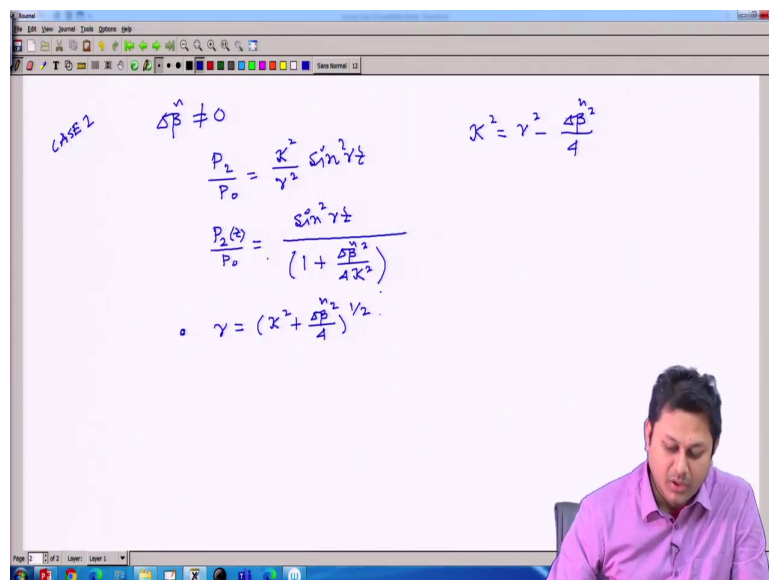
$$\frac{P_2(z)}{P_0} = \sin^2 \kappa z$$

$$\gamma^2 = \kappa^2 + \frac{\delta\beta^2}{4}$$

$$\left. \begin{array}{l} \delta\beta = 0 \\ \gamma = \kappa \end{array} \right\}$$

Now, I can have a condition say so it is case 1 suppose I call it as case 1 when delta B equal to 0, but I can also have another case, case 2. When delta beta tilde is not equal to 0 that means, we consider that there is a mismatch between the propagation constant of these two waveguides, waveguide 1 and waveguide 2.

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Handwritten notes on a whiteboard:

CASE 2 $\Delta\beta \neq 0$

$$\frac{P_2}{P_0} = \frac{\kappa^2 \sin^2 \gamma Z}{\gamma^2}$$

$$\frac{P_2(\gamma)}{P_0} = \frac{\sin^2 \gamma Z}{\left(1 + \frac{\Delta\beta^2}{4\kappa^2}\right)}$$

$$\gamma = \left(\kappa^2 + \frac{\Delta\beta^2}{4}\right)^{1/2}$$

$$\kappa^2 = \gamma^2 - \frac{\Delta\beta^2}{4}$$

Then my P_2 divided by P_0 is κ^2 square divided by expression is κ^2 square divided by γ^2 square then $\sin^2 \gamma Z$ and κ^2 square is γ^2 square minus $4 \Delta\beta^2$ square divided by 4. We know γ^2 square is equal to κ^2 square plus $4 \Delta\beta^2$ square divided by 4 so, I can write κ^2 square equal to γ^2 square minus $4 \Delta\beta^2$ square divided by 4.

So, I can have from this relation I can have these is equal to $\sin^2 \gamma Z$ whole divided by $1 + \Delta\beta^2$ square divided by $4 \kappa^2$ square. So, I can now find that my P_2 divided by P_0 this ratio is having some denominator which is greater than 1 so that means, the total power will not going to convert here. So, previously what happened I converted the entire power from this branch to this branch, but here that will not be the case.

So, gamma is equal to a kappa square plus delta beta square divided by 4 whole to the power half that we always need to remember. So now, from these two expression P 2 divided sorry this is P 2 Z divided by P 0 is this 1. When delta beta equal to 0 we have simply sin square kappa Z divided by 1 that is all, which we already derive here in this expression.

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$$\gamma = \left(\kappa^2 + \frac{\delta\beta^2}{4} \right)^{1/2}$$

• When $\delta\beta = 0$ The period $\kappa L_p = 2\pi$
 $L_p = \frac{2\pi}{\kappa}$

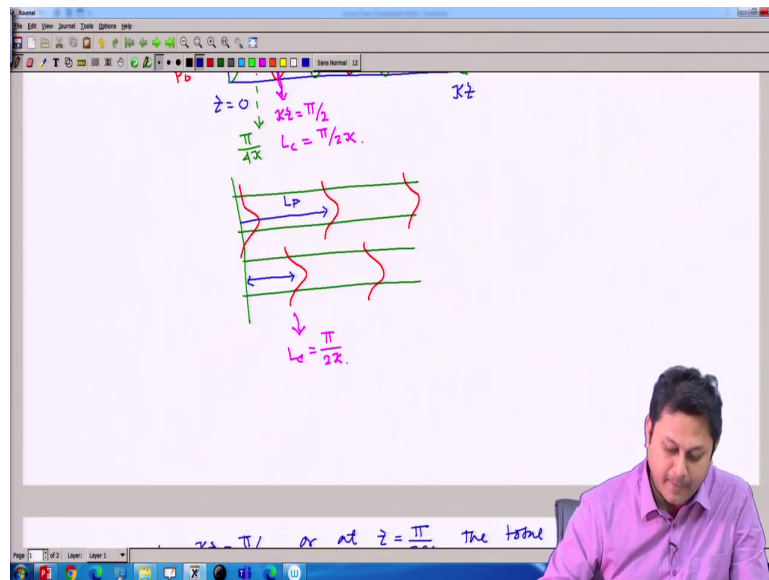
When $\delta\beta \neq 0$ The period $\gamma L_p = 2\pi$
 $L_p' = \frac{2\pi}{\gamma}$
 $= \frac{2\pi}{\kappa \left(1 + \frac{\delta\beta^2}{4\kappa^2} \right)^{1/2}}$

So, from these two expression so we can find few things, when delta beta tilde equal to 0 we find the period the period of power transfer was kappa if L c is a distance at which there is a complete exchange of power so, it should it was like that.

So, L c was 2 pi divided by kappa the length at which the entire power so, this is the period basically at which so I should write here in different notation so, I should write L say L p

because L_c I write a length at which the entire power is converted from one channel to another channel, but here I am try to find out this length.

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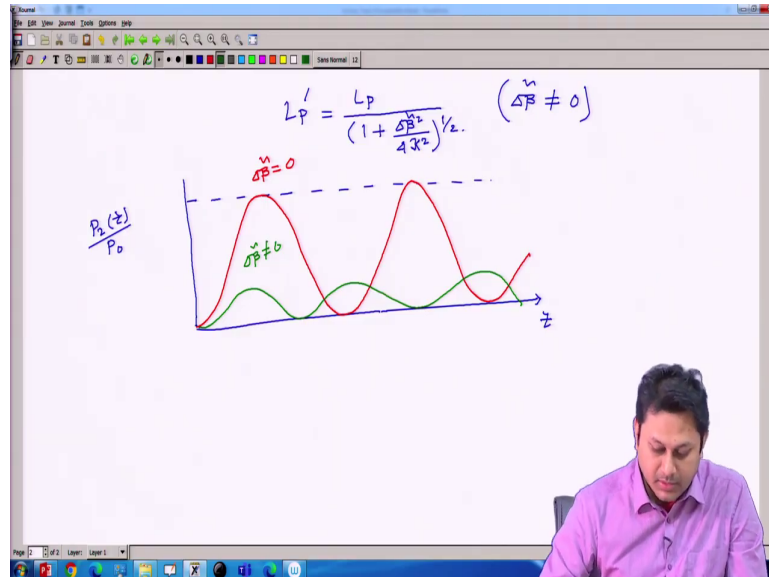


So, this length is L_c from here to here is L_c and from here to here this length is say L_p . But when, $\delta\beta$ is not equal to 0 then from this expression we can find that the period κL_p is equal to 2π , I should not write κ here because $\delta\beta$ when $\delta\beta$ equal to 0 it should be γ from the expression.

So, it should be γ this is equal to 0 because, in this expression I have γ and here γ and κ not equal because of this β term. So, L_p is 2π divided by γ . So, I can readily find out that L so which is let me write it here so, this is 2π divided by I know what is this γ . γ is κ^2 plus this quantity so, I can write it as

$\kappa^2 (1 + \Delta\beta^2)$ divided by $4\kappa^2$ and then, whole to the power half.

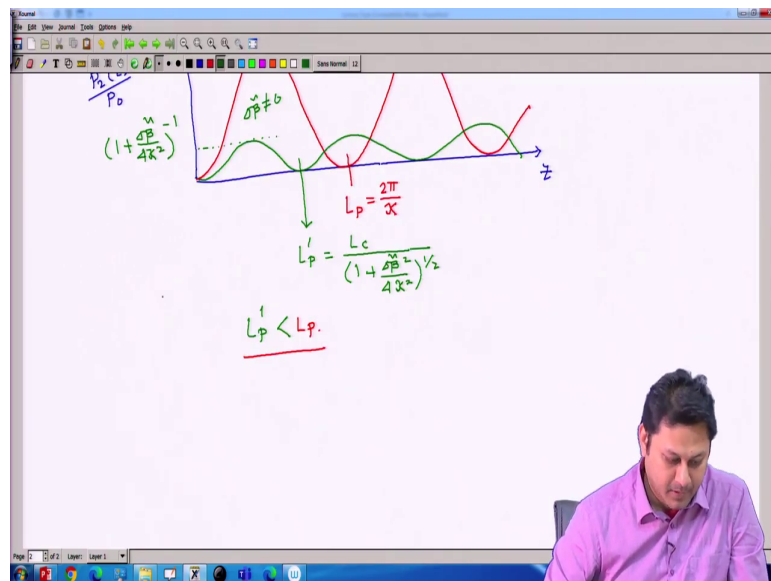
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So, 2π by κ so L_p' is nothing but L_p divided by this quantity, $1 + \Delta\beta^2 / (4\kappa^2)$ whole to the power half $\Delta\beta \neq 0$ condition. If $\Delta\beta = 0$ then L_p' and L_p both are same, but here not it is not the case.

So that means, if I plot the power transfer for these two conditions where we consider in one case that there is no mismatch between so if I plot this suppose at. So, I am plotting $P_2(z) / P_0$ for two conditions, one when $\Delta\beta = 0$ so I can have a variation like this so, this is for $\Delta\beta = 0$ and other case when it is not zero.

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So, when it is not zero then, their power transfer will be something like that. In this case, this is the value L_p which is 2π divided by κ and in this case I can have the value which is L_p' equal to L_c divided by $1 + \frac{\sigma \beta^2}{4\kappa^2}$ to the power half. So, L_p' will be less than L_p .

So, the energy will go to couple here also, but the period will going to change that is one issue, second thing is amplitude is also going to reduce. So, this is the amplitude and this amplitude will also going to reduce, if $\sigma \beta$ is not equal to 0 and what should be the amplitude, from this expression one can one.

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CASE 1

$$K = \gamma - \frac{1}{4}$$

$$\frac{P_z}{P_o} = \frac{K^2 \sin^2 \gamma z}{\gamma^2}$$

$$\frac{P_z(\gamma)}{P_o} = \frac{\sin^2 \gamma z}{(1 + \frac{\delta \beta^2}{4K^2})} = (1 + \frac{\delta \beta^2}{4K^2})^{-1} \sin^2(\gamma z)$$

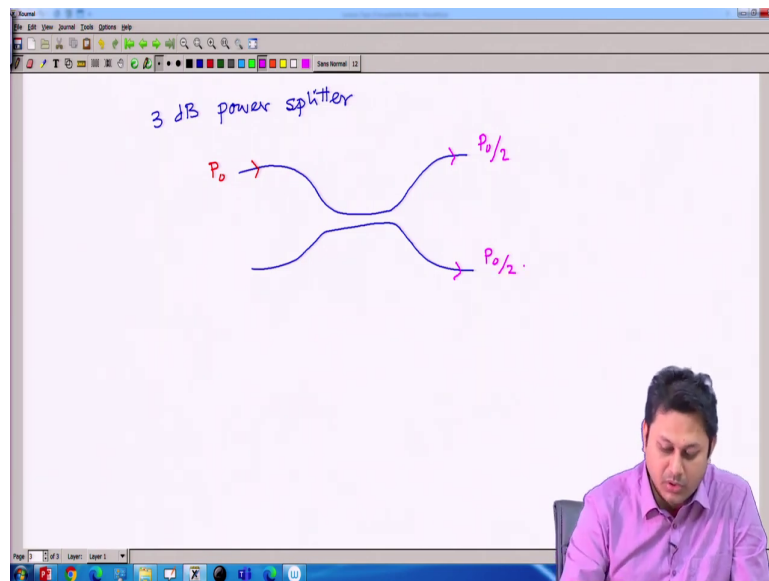
$$\gamma = (K^2 + \frac{\delta \beta^2}{4})^{1/2}$$

- When $\delta \beta^2 = 0$ The period $KL_p = 2\pi$
 $L_p = \frac{2\pi}{K}$
- When $\delta \beta^2 \neq 0$ The period $\gamma L_p = 2\pi$
 $L_p = \frac{2\pi}{\gamma}$
 $= \frac{2\pi}{K(1 + \frac{\delta \beta^2}{4K^2})^{1/2}}$

We can see that this amplitude I can write this amplitude as 1 plus delta beta tilde square divided by 4 kappa square to the power minus 1 over a modulation of sin which is sin square gamma Z.

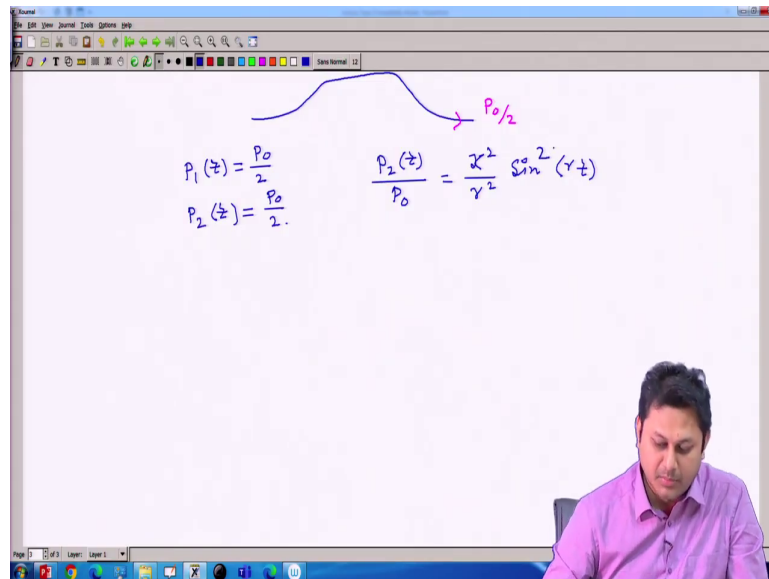
So, sin square gamma Z should have maximum value 1, but this amplitude suggest that it is less than 1 so, I can have amplitude here and the value of this amplitude is simply 1 plus delta beta tilde divided by 4 kappa square whole to the power of minus 1 whole to the power of minus 1 so, this is the amplitude we have.

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Well, now quickly jump to the working principle which I already discuss so, 3 d B power splitter in 3 d B power splitter what happen? So this is the structure we have we launch a power here P_0 and in the output I have a power here P_0 divided by 2 and P_0 divided by 2. So, the power will going to split equally to 2 branches.

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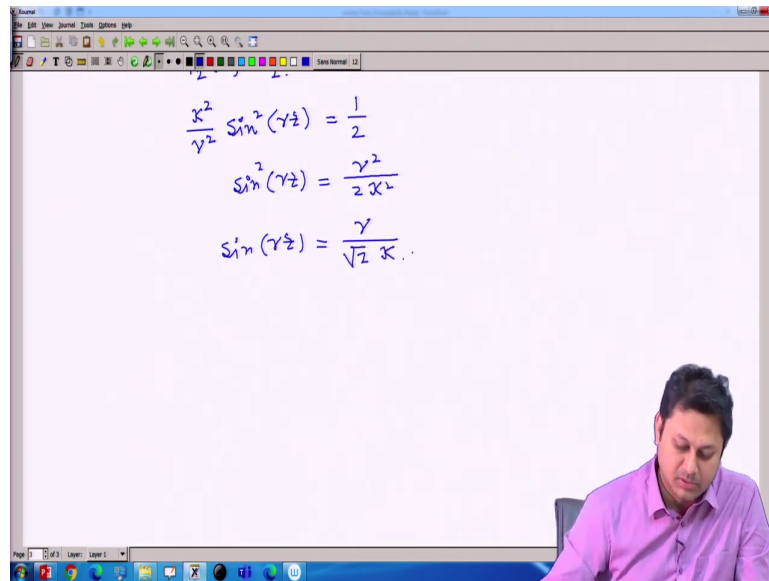


The whiteboard shows a diagram of a rectangular pulse with a peak value of $P_0/2$. Below the diagram, the following equations are written:

$$P_1(z) = \frac{P_0}{2}$$
$$P_2(z) = \frac{P_0}{2}$$
$$\frac{P_2(z)}{P_0} = \frac{\kappa^2}{\gamma^2} \sin^2(\gamma z)$$

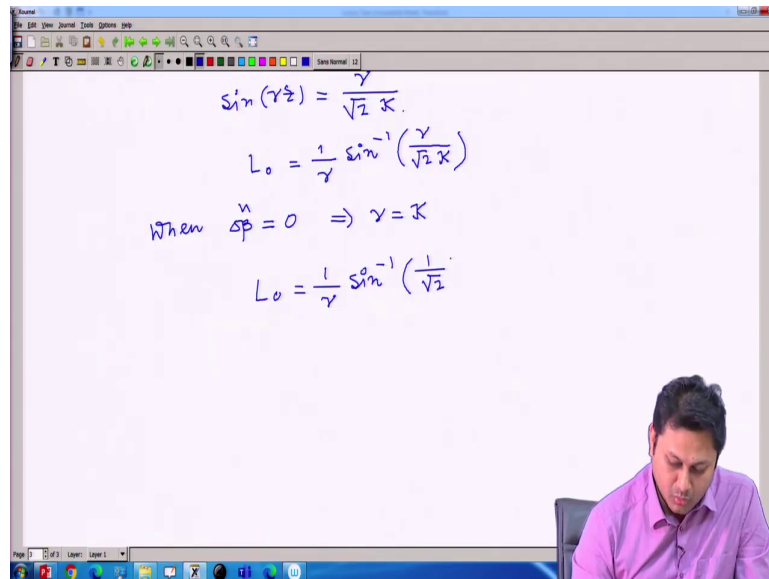
So, I have P_1 at Z is P_0 divided by 2 and P_2 at Z also this value. Now, P_2 which is a function of Z divided by P_0 is κ^2 divided by γ^2 according to our calculation we have $\sin^2 \gamma Z$. So, we have $\sin^2 \gamma Z$ ok I need to write this square properly $\sin^2 \gamma Z$.

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$$\frac{\kappa^2}{\gamma^2} \sin^2(\gamma Z) = \frac{1}{2}$$
$$\sin^2(\gamma Z) = \frac{\gamma^2}{2 \kappa^2}$$
$$\sin(\gamma Z) = \frac{\gamma}{\sqrt{2} \kappa}$$

So that means, according to our condition sin square gamma Z with kappa square divided by gamma square has to be equal to half, this is the condition for equal power coupling it should be half. If it is half then, sin gamma Z will be gamma square divided by 2 kappa square sin square, and sin gamma Z will be gamma divided by root over of 2 multiplied by kappa.

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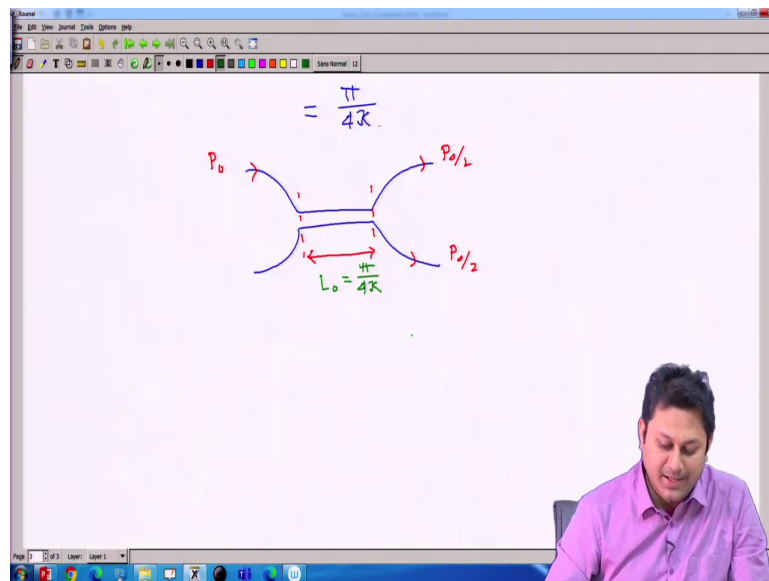

$$\sin(\gamma z) = \frac{\gamma}{\sqrt{2} \kappa}$$
$$L_0 = \frac{1}{\gamma} \sin^{-1}\left(\frac{\gamma}{\sqrt{2} \kappa}\right)$$

When $\delta\beta = 0 \Rightarrow \gamma = \kappa$

$$L_0 = \frac{1}{\gamma} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

So, eventually I can have a length say L_0 for which this condition is satisfied so, if I want to find out what is the condition it should be 1 divided by γ $\sin^{-1}$ of γ divided by root over of 2 then κ . So, note that when, $\delta\beta$ is equal to 0 that means, γ is equal to κ then this L_0 value becomes 1 divided by γ $\sin^{-1}$ of 1 by root 2 now, $\sin^{-1}$ of 1 by root 2 we know what is the value.

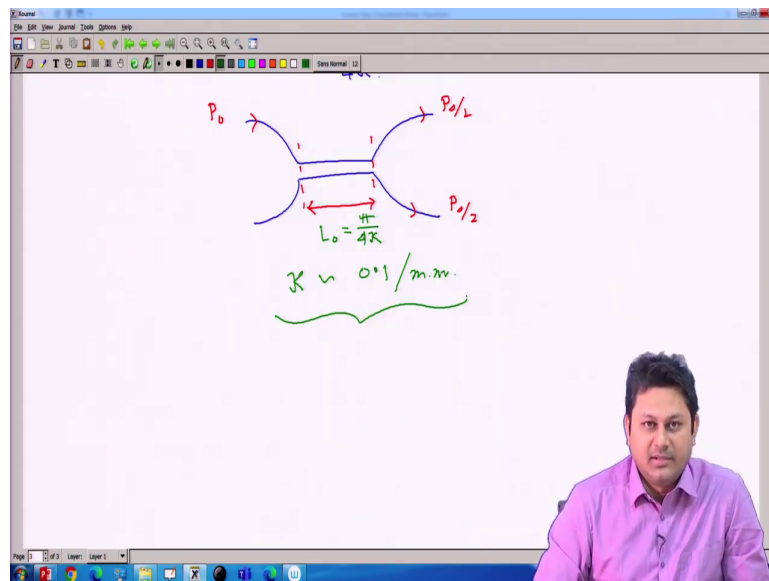
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So, it should be simply and also gamma is equal to kappa so, I can write in terms of the coupling coefficient. So, this value should be simply pi divided by 4 kappa. So, if I know the value of the kappa then I can design the waveguide the coupler in such a way so, in this case the system are something like this so let me draw once again.

So, I launch P_0 here and at the output I have $P_0/2$ $P_0/2$ so, this length now I can find out and according to the coupler mode theory I calculate and I find that if the coupling coefficient is known to me I can have this length as L_0 equal to $\frac{\pi}{4\kappa}$. So, if the kappa is known then I can calculate.

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So, typically kappa is of the order of say around 0.1 say per millimeter so, typically this is the order of the kappa so, if I use this value. So, I can have an idea that what should be the length of the coupler which basically divide the energy into two equal parts to two equal branches; this is a very useful component in communication optical communication or other uses.

So, this is the principle or this is the physics behind it through which it basically works, using the couple mode theory we calculate how the power is distributed, what is the evolution of the power distribution, and from that we calculate the length of the coupler and this length is related to the coupling length inversely related to the coupling length the coupling constant.

So, if the coupling constant is known I can calculate, the length at which these phenomena happened. So, with this note I will like to conclude today's class. In the next class we will also learn another component which is called WDM.

So, thank you for your attention and see you in the next class.