Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber Optics Components
Lecture - 38
Coupled Mode Theory (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number-38. And we will continue, we will going to continue the calculation related to the Coupled Mode Theory.

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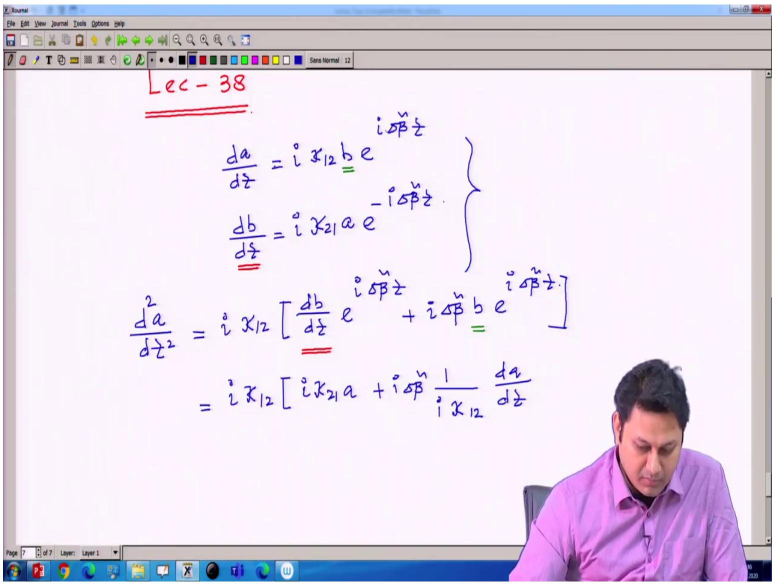

So, in the last class, we have we figure out the differential equation of two amplitudes of these two modes which going to evolve over the system in this way. This was one equation, and another equation was this. So, today we will going to solve this equation, decouple this equation because this is a coupled differential equation and we will going to solve this and after putting some boundary condition try to find out what is the explicit form of a and b.

Well, let us make a derivative. So, in order to decouple that, let us make a derivative of the first equation with respect to z. So, that I can have this I simply make a derivative the first equation. And I am having this. There is a standard way to decouple that this kind of equation. Now, after making the derivative, I have this term in my hand. So, I can replace this term here.

And also I have a b here which I can replace from the first equation itself because b is here. So, I have this term here, I can replace from this. And I can also have let me define in a different I also have a b which is sitting here. So, I will going to replace this every. So, entire equation will come into the form of a. So, if I do, if I do, then it should be i kappa 12, then this term will be i kappa 21.

And you can see it is a multiplied by e to the power of minus of i delta beta delta beta tilde z. So, this term and this term will go to cancel up.

And another term will be simply 1 plus i delta beta tilde b, I will going to replace as 1 divided by i kappa 12 and then da dz because I just replace my b from here. So, it should be da dz divided by 1 by kappa 12. And then it should be one delta beta with a negative sign. So, this again will going to cancel out with this one. So, I have this thing.

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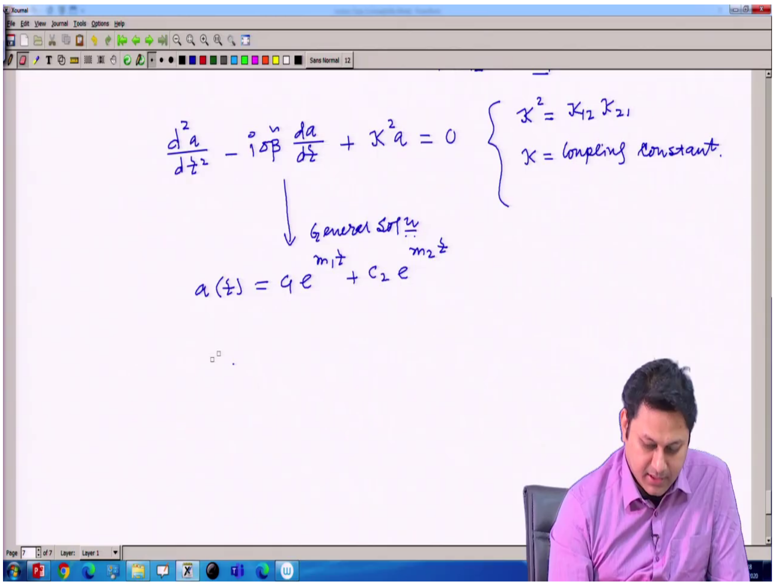

So, eventually I have d 2 a dz square minus i delta beta tilde da dz plus I say kappa square a equal to 0, here this kappa square, I write it as kappa 12 multiplied by kappa 21. And this kappa is called as kappa as a name which is called the coupling constant. So, I can have a general solution for that. And for this kind of differential equation, we know how to calculate the general solution.

So, I can assume a general solution of the form say c 1 e to the power of m 1 z plus c 2 e to the power of m 2 z. And then I put this solution here to execute the value of m.

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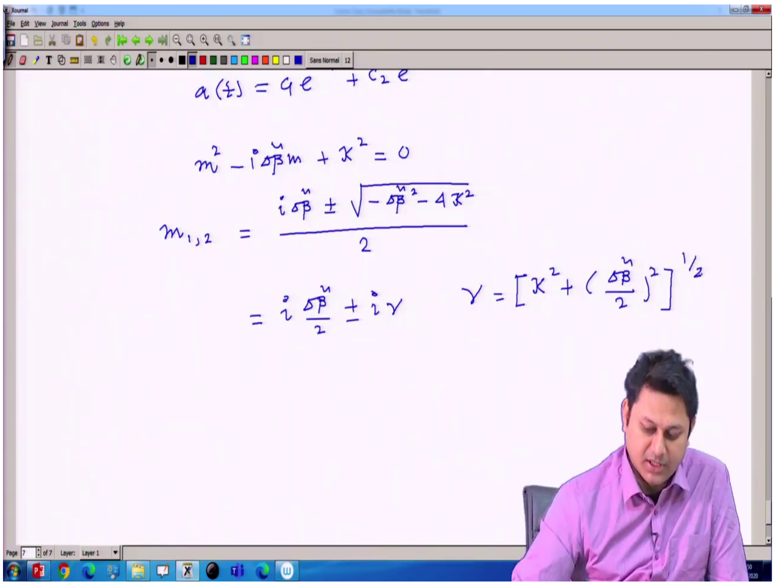

And m 1, 2 can be if I put this there in the equation ok, let me do this step one by one. So, I put this here. And after putting that I can have m square plus i it should be minus i delta beta tilde m plus kappa square equal to 0. So, I can have my m 1, 2 as minus b.

So, the solution of this quadratic equation is this minus of delta beta square minus 4 of kappa square straight forward calculation 2 which I can write at i delta beta divided by 2 then plus minus of i one other term gamma, because this is a negative sign so it should be come out as. Where gamma I can write as kappa square plus delta beta tilde divided by 2 square of that and whole to the power half. So, this is my gamma.

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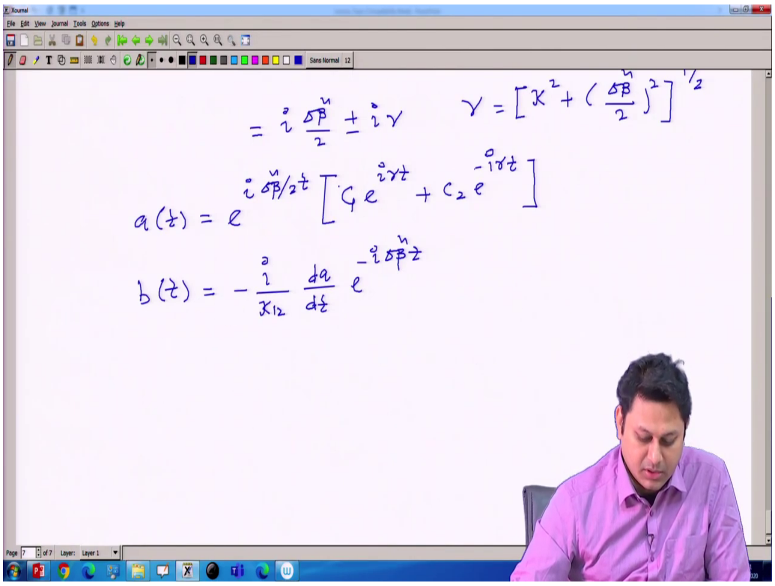

So, my a z will be simply e to the power of i delta beta tilde divided by 2 z and then c 1 e to the power of i gamma z plus c 2 e to the power of minus i gamma z. So, I have the solution for a z in this form. Mind it, c 1 and c 2 are the solutions and we need to evaluate this solution putting the boundary condition.

So, b, I can also figure out. So, b z we know in terms of a and which is minus of i kappa 12, then da dz e to the power of minus i delta beta tilde z. So, this is the b in terms of a, and that I can find from this equation this first equation. So, now, I know what is the value of a z because I already do solve the differential equation.

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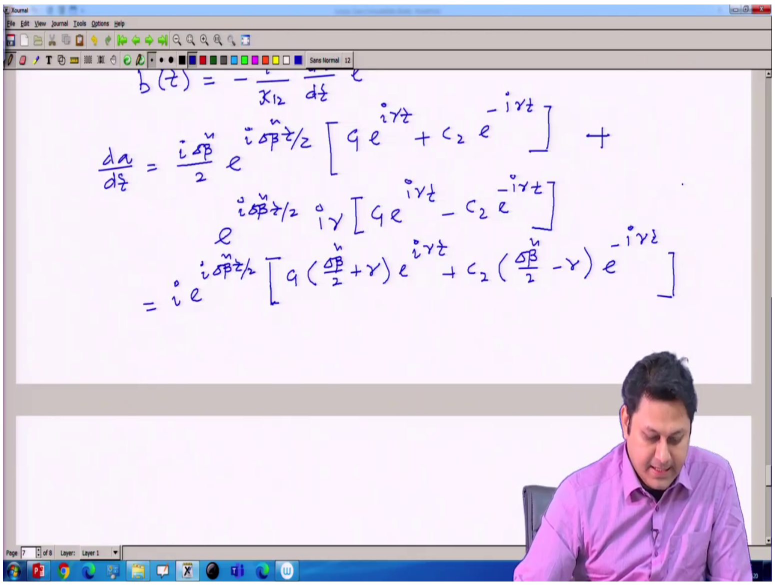

So, I can extract the b out of that. So, in order to do, I need to calculate da dz. So, da dz will be i delta beta tilde divided by 2 e to the power of i delta beta tilde z by 2 the derivative of the first term multiplied by the second term e to the power of i whatever we have gamma z plus c 2 e to the power of minus i gamma z plus derivative of the second term e to the power of i delta beta tilde z divided by 2 and then i gamma c 1 e to the power of i then minus c 2 e to the power of i gamma z.

So, I have this expression for del a del z which I can write as i e to the power of i delta beta tilde z divided by 2. And then if I take say c 1 common, then it should be delta beta tilde by 2 plus gamma i take c 1 common from this two, and then e to the power of i gamma z will be here and plus c 2 delta beta tilde by 2 minus gamma and i should have e to the power of minus i gamma 2. So, here I have a minus sign sorry this one. So, da dz I calculate.

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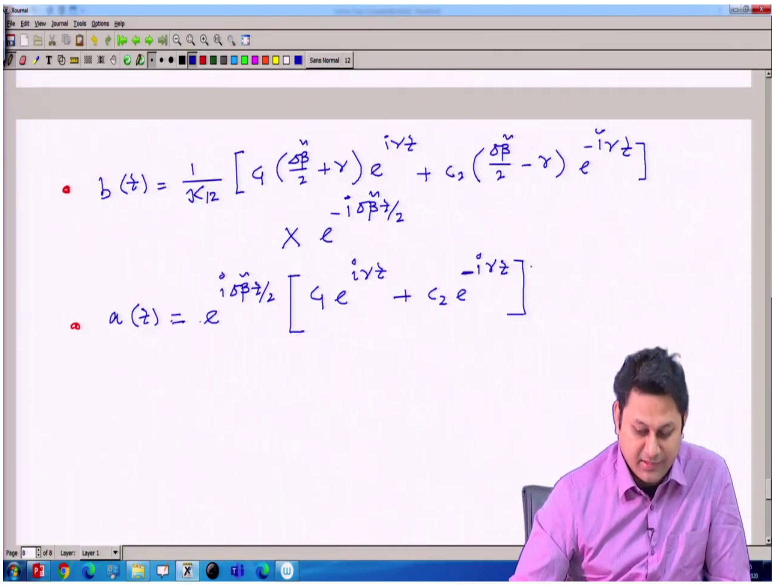

So, I can calculate my b. And my b will be simply 1 divided by kappa 12 then c 1 delta beta tilde divided by 2 plus gamma. This calculation is little bit lengthy, but straight forward. I suggest all the students, so please go through the calculation at least once, and then you can realize that how one can extract all the information out of starting from this Maxwell's equations – Maxwell's wave equations z and then multiplied by the phase term z by 2.

So, I just extract da dz from the solution a that I figure out here. And then I put this value here. And when I put this value, I have the value of b here. So, I find finally, I find my b and let me write it once again what a value I calculated. So, my a z was e to the power of i delta beta tilde z by 2 multiplied by c 1 e to the power of i gamma z plus c 2 e to the power of minus i gamma z that was the value of a i figure out.

So, I, now I know what is my a, what is my b. And c 1, c 2 are the constant that will going to evaluate with the boundary condition, so that is important.

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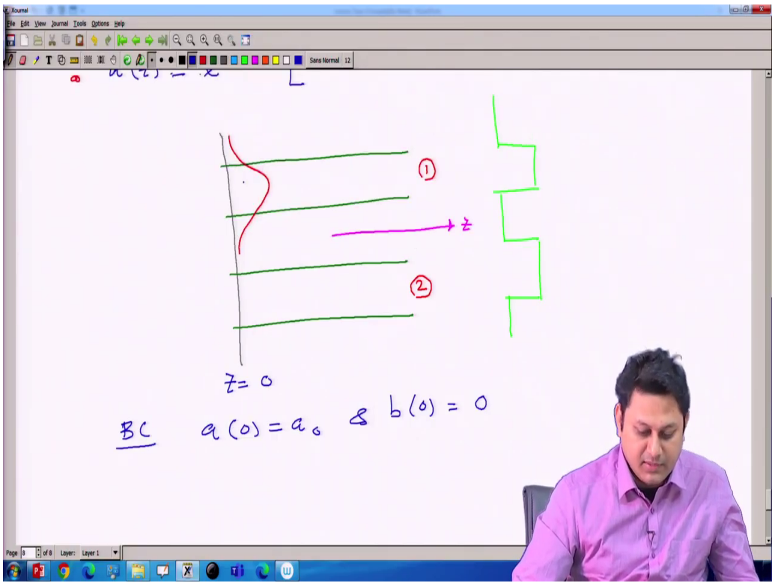

So, boundary condition I already mentioned earlier that this is the structure of the waveguide say, and this is waveguide 1. So, at waveguide 1, I launch a light here. So, this is waveguide 1, and this is waveguide 2. And this along this direction, I am having my z. The refractive index profile if I draw, it should be something like this. So, this is the refractive index profile that two waveguides place side by side.

So, the boundary conditions suggest that simply this is at z equal to 0. So, the boundary condition simply suggests that a at 0 is some value a 0, and b at z equal to 0 is simply 0 because there is no b at all at waveguide 2. So, with these two boundary conditions, we can evaluate the constants.

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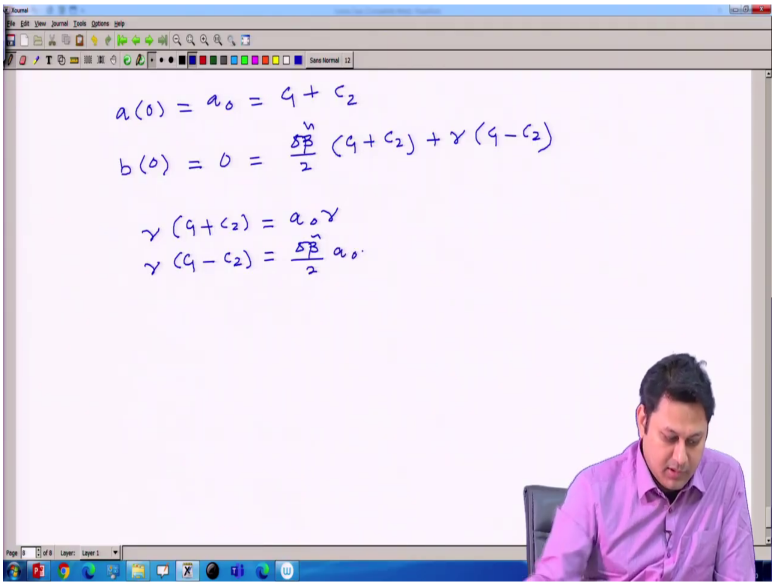

So, let us try to find out the constant. So, a 0 which is a 0 should be equal to c 1 plus c 2. Because a if you look carefully if I put z equal to 0, z equal to 0, it should be c 1 plus c 2, and here z equal to 0 1. So, simply c 1 plus c 2 is a 0. And for b 0 which is equal to 0 is equal to delta beta tilde divided by 2 c 1 plus c 2 plus gamma c 1 minus c 2.

Because here if you look carefully if I put this equal to 0, and this equal to 0, then this is 1, and this is 1. So, it should be something like delta beta, and this 1 by kappa 0 is in the denominator. So, this term will going to be 0, so it should be c 1 and c 2 c 1 plus c 2 delta beta by 2, and the rest term plus gamma c 1 minus c 2 these things will be 0. So, I can have gamma c 1 plus c 2 as a 0 gamma and gamma c 1 minus c 2 as delta beta tilde divided by 2 a 0.

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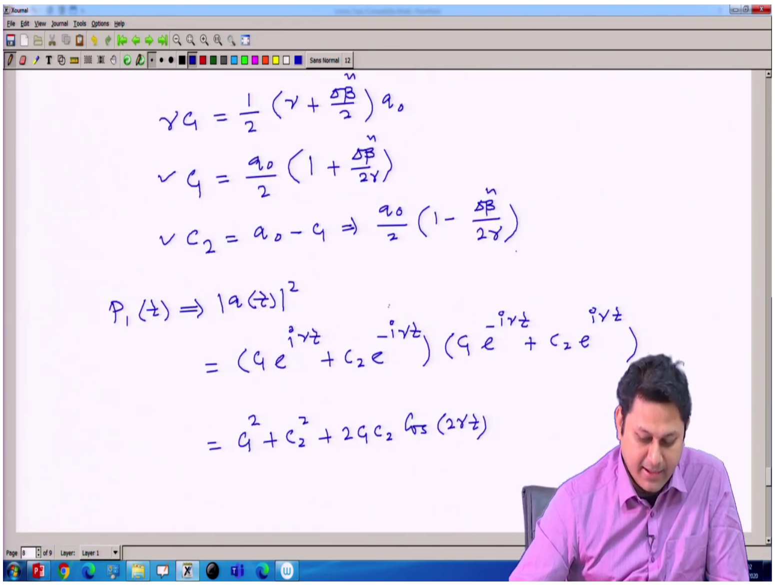

And if I do that calculation I can find that gamma c 1 is equal to half of if I just add half of gamma plus delta beta tilde divided by 2, then a 0. So that c 1 is equal to a 0 divided by 2, then gamma plus delta beta tilde divided by 2 c 1 ok, this gamma will going to cancel out. So, it should be 1. It should be simply 1 plus gamma should be here, and c 2 is a 0 minus c 1 which is simply a 0 divided by 2 1 minus delta beta tilde divided by 2 of gamma.

So, I have c 1 and c 2 in terms of a 0 which is my boundary condition, the initial amplitude and delta beta 0 and gamma. Now, the important thing is the power because that I eventually going to measure.

So, the power is related to the mod of a z square, the amplitude square is related directly related to power. Ignoring the proportionality constant, it will be simply, if I write it will be simply c 1 e to the power i gamma z plus c 2 e to the power of minus i gamma z, and then c 1 e to the power of minus i complex conjugate of these things plus c 2 e to the power of i gamma z.

Note that c 1 and c 2 both are real, so that is why do I should not put any kind of star over here. Well, these things can be written as c 1 square plus c 2 square plus 2 c 1, c 2 cos of 2 gamma z. If I multiply, you will going to get this one.

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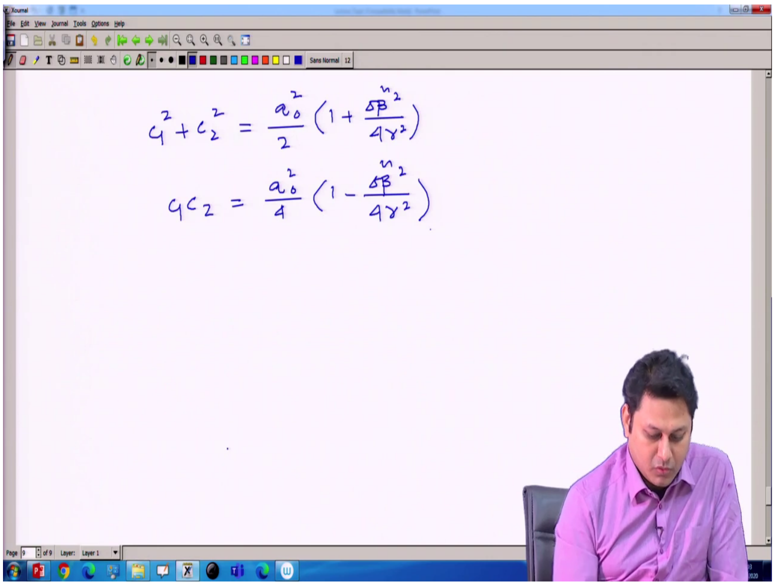

Well, c 1 square plus c 2 square, I can also extract the value. And this value will be simply this one 4 gamma square. I just have my c 1 square; I have my c 2 square. If I make a square and add, then I will going to get this one. And c 1 and c 2 also I can calculate, it should be a 0 square to the power a 0 square divided by 4, and then a plus b into a minus b. So, a minus delta beta by 4 gamma square.

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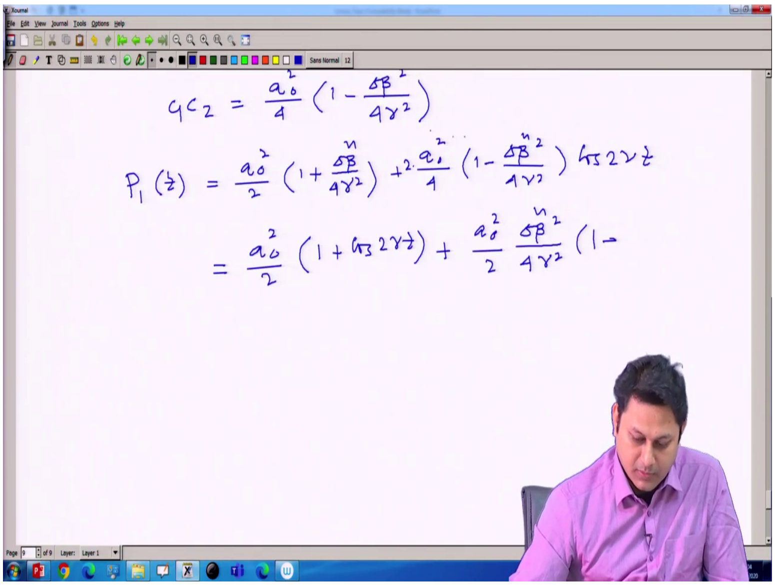

So, my p z which is P 1 z which is the power associated to waveguide 1 is simply a 0 square divided by 2 1 plus delta beta tilde divided by 4 gamma square then plus a square divided by 4 multiplied by because it is a 2 term associated with these 2 c 1, c 2. So, this delta beta tilde square divided by 4 gamma square and then cos of 2 gamma z.

So, this thing I can simplify as a square divided by 2 common 1 say plus cos of 2 gamma z plus another term is a square divided by 2 then delta beta tilde square divided by 4 of gamma square 1 minus because if I write it 1 minus cos of 2 gamma z.

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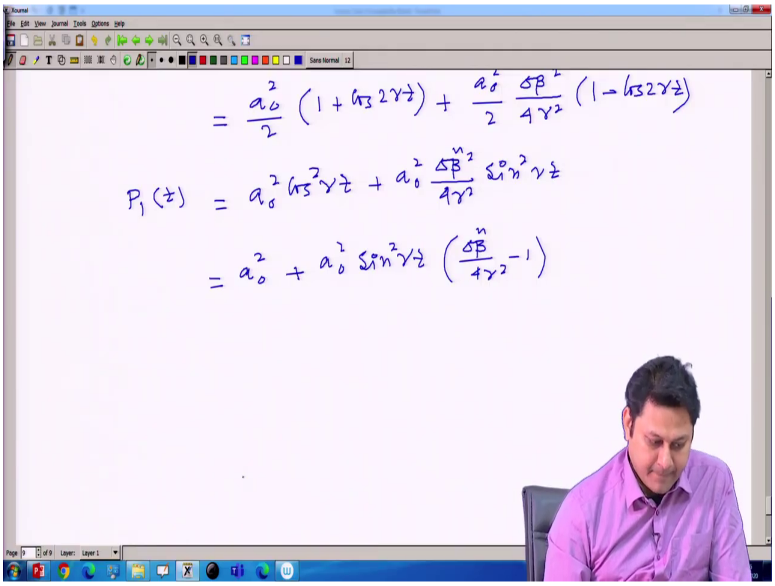

So, this I can write as say a 0 square then cos a square gamma z plus simple trigonometric identity 4 gamma square then it should be sin square gamma z. So, this is my P 1 z it will vary in this way. Well, I can also write this in the form of sign. So, it should be a 0 square plus cos square is 1 minus sin square. So, I will write cos square to 1 minus sin square.

So, then I have a 0 square delta beta sorry a 0 square, and then I have a sin square from this side and sin square from the first term. So, it should be sin square gamma z into delta beta tilde divided by 4 gamma square minus 1, it is one term.

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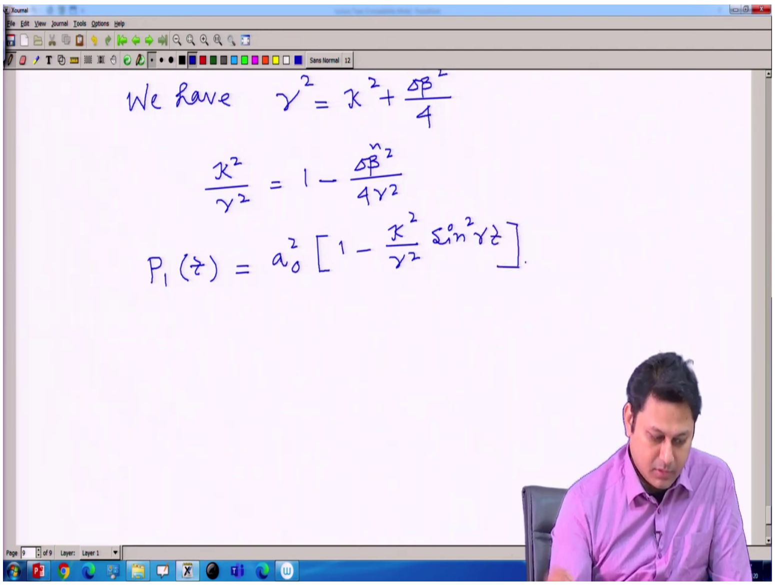

Now, I need to put the value of gamma. So, we have we defined gamma earlier. So, we have say gamma square equal to kappa square plus delta beta tilde square divided by 4.

So, I can write say kappa square divided by gamma square is how much? It should be 1 minus delta beta square divided by 4 of say gamma square. I can have this. So, I can use this value here and eventually my power will be something like a 0 square into 1 minus kappa square by gamma square then sin square gamma z.

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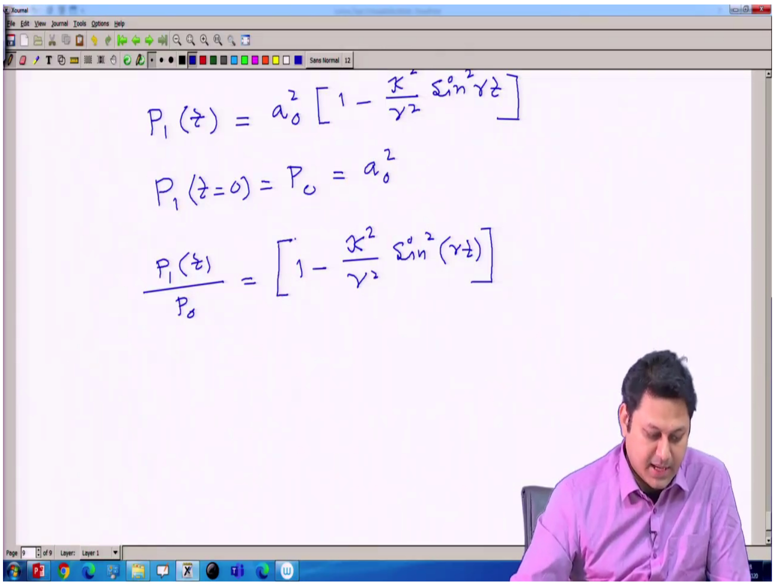

So, this is very interesting and also when z equal to 0 I have. So, P 1 at z equal to 0 which I can say my P 0 that is initial power it should be simply a 0 square. So, I can write this ratio as P 1 z divided by initial power P 0 which is at P 1 at z equal to 0. So, P 0 is P 1 at z equal to 0 simply it is 1 minus kappa square divided by gamma square sin square gamma z very important equation, I have very important equation I have.

Now, after doing all this lengthy calculation, I now conclude now I have a very meaningful equation that clearly suggests that if z increases, then this term will going to vary sinusoidally. So, this term will going to vary sinusoidally, that means, if I have a waveguide like this. So, let me draw this waveguide first.

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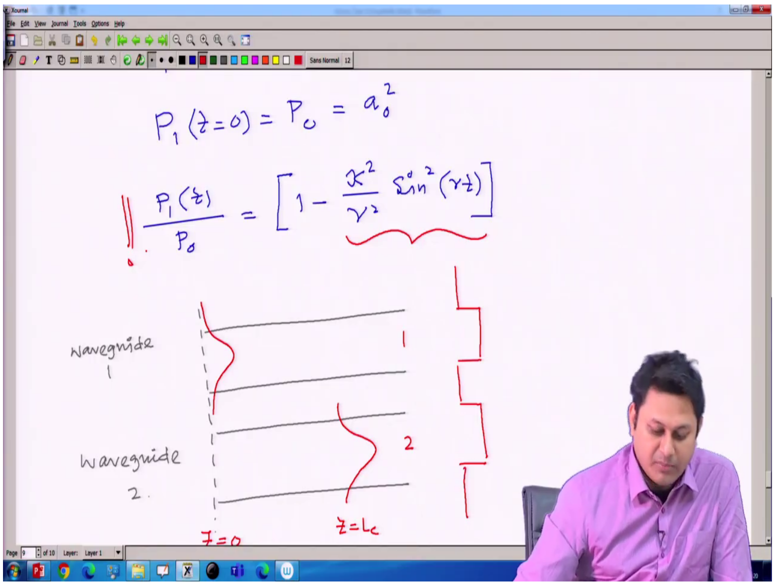

So, this is the waveguide I am having. So, this is waveguide 1, waveguide 1, and this is waveguide 2. I launch a field here at waveguide 1 at z equal to 0, this is my z equal to 0. This is 1 and this is 2. This is the waveguide structure. After certain distance, there is a possibility that this field will come here because my power will going to change sinusoidally. So, at certain length z equal to say L c the power will come here. So, this is basically the switching we are talking about.

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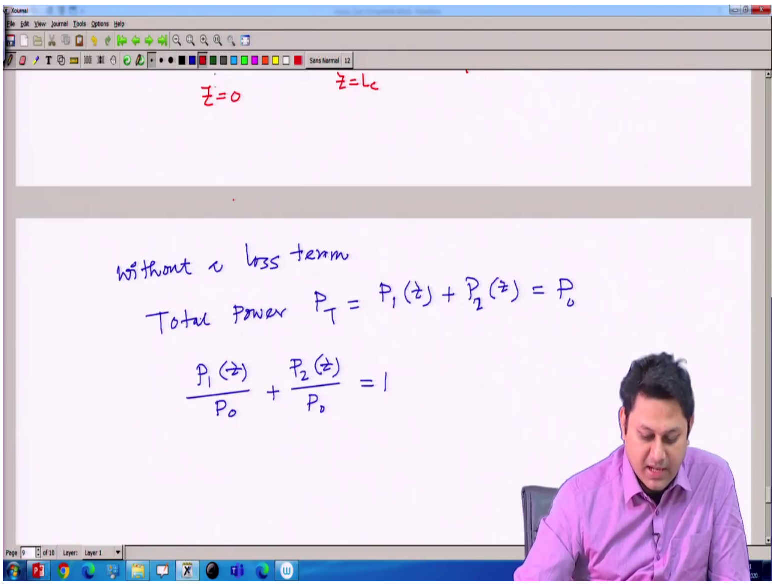

Well, I can write a general equation. If there is no loss, so without a loss term; without a loss term the total power remain conserved the total power I can write it is as P T is a total power it will be the sum of the power distributed in two waveguides. And since I launch a power in waveguide 1 which is P 0, so it should be equal to P 0.

Hence I have a equation P 1 z divided by P 0 plus P 2 z divided by P 0 will be equal to simply 1. P 1 z divided by P 0 I already evaluated and this value what I evaluated in is this one. So, this equation I already evaluated. So, from that, I can also calculate P 2 by P 0.

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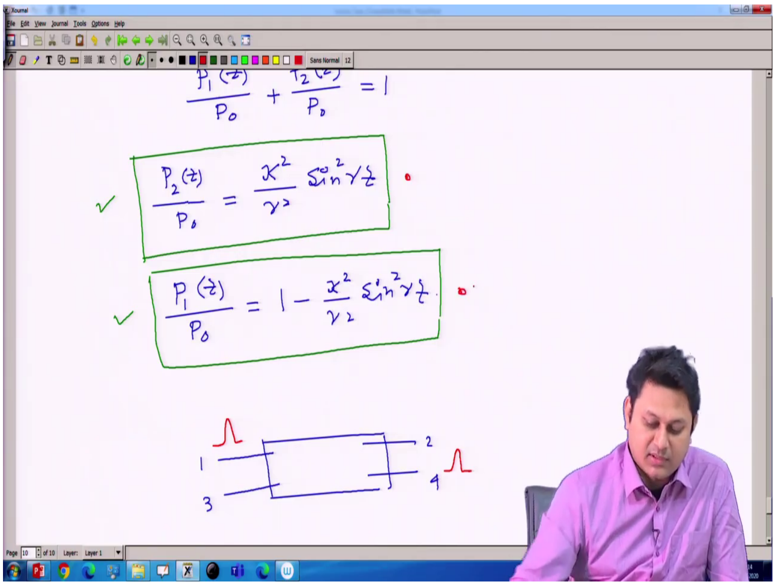

So, I can write P 2 divided by P 2 function of z divided by P 0 is simply kappa square divided by gamma square sin square gamma z. So, now, I have two equation in my hand. This equation suggests that how the power will going to exchange from one waveguide to another waveguide. And we find that this power will going to exchange in a sinusoidal fashion. So, let me write down the two equations side by side once again. So, this is one and another equation is this.

In most of the books, you will find these two equations without much derivation. But in this particular course, I want to calculate all the coupled mode entire coupled mode theory in detail, so that you understand that how this equation basically arrived how someone can arrive in this equation. And this is the way you can find these two equation.

So, in the next class, we will try to understand more about this equation. We try to solve this, we try to plot these two equation and then we will see that how the energy will going to transfer periodically from one waveguide to another waveguide. But anyway in today's class already you understand that since these power variation is related to a sin function, there will be a sinusoidal variation of the power from one waveguide to another waveguide.

So, in the switching problem, we are eventually doing the same things. I am just finally, drawing what the problem we started with that I had this port 4 ports. And this is one say 1, 2, 3 and 4. So, I launch a power here in port 1, and I find the power is coming to this one. So, this is basically switching problem. And this switching problem can really be understand with these two equations.

And in these two equations suggest that if we have two waveguide which is placed very close to each other and behave like a coupler; then the energy can couple from waveguide one waveguide to another waveguide.

And there is a periodic transfer from the energy from one waveguide to another waveguide. So, if you cut my coupler in such a way that this energy is completely transferred to here, then I can use this as a use I can use this as a switch optical switch.

So, with this note, I like to conclude. In the next class, we start with this equation and try to understand more about the principle of optical switch, and then principle of 3 dB coupler and WDM systems.

So, with this note, thank you for your attention. See you in the next class.