Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber Optics Components
Lecture - 37
Coupled Mode Theory

Hello student the course of Physics of Linear and Non-linear Optical Waveguide. Today we have lecture number 37 and in the last class, we started a very important concept Coupled Mode Theory, very important theory. So, we will going to continue the calculation that we started in the last class.

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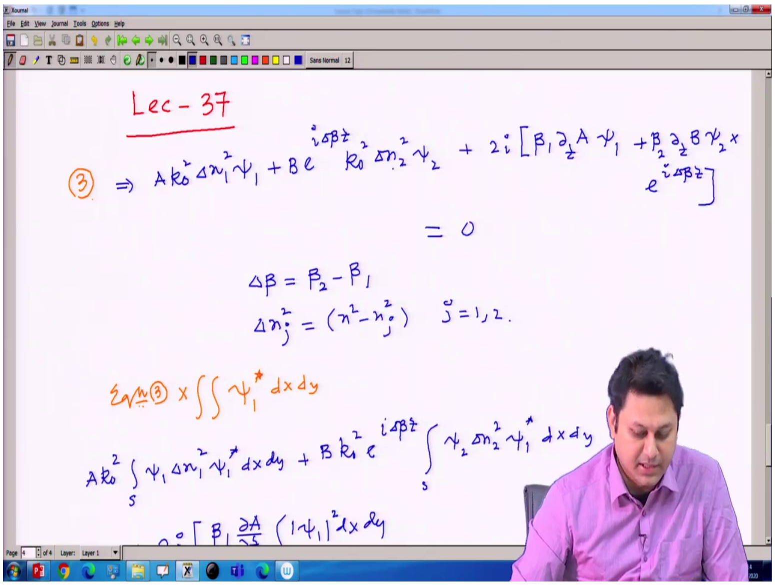

So, if you remember in the last class, we had an equation in our hand. So, let me write down this equation once again that,, Ak 0 square delta n square psi 1 plus Be to the power of i delta Bz k 0 square delta n 2 square psi 2 plus 2i beta 1 del z A psi 1 plus beta 2 del z B psi 2 into e to the power of i delta beta z bracket close is equal to 0 that was the equation.

Where delta beta was beta 2 minus beta 1 and delta n j square is n square minus n j square, j is 1 and 2. Well, now let me write this equation as say equation. So, in the last class I had a node here.

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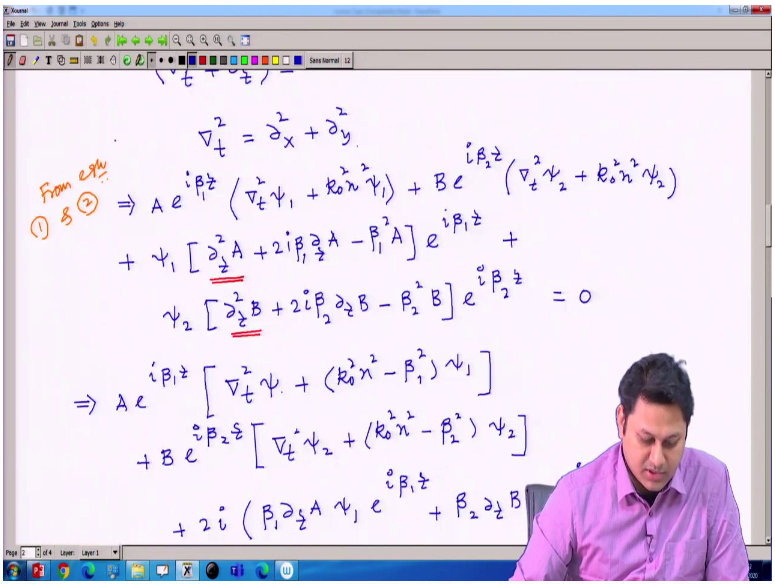

So, in the last class, I wrote this equation 1 and 2 equation was named. So, let me write this equation as equation 3. So, now, what I do; equation 3 I will going to multiply a term like psi

1 star dx dy ; I am going to multiply this psi 1 star dx dy to the entire equation, so sorry equation 3 into this term, ok.

Let me erase this. So, equation 3 whatever the equation I have and then multiply with this term. If I do, then I have Ak 0 square integration psi 1 delta n 1 square psi 1 star dx dy plus Bk 0 square e to the power of i delta beta z integration; this is over some surface psi 2 delta n square psi 1 star dx dy .

So, you can see by integrating this, I am basically making some sort of coupling between the psi 1 and psi 2. Here I can have the first, this is the self-coupling kind of term, this is the cross coupling kind of term. So, I am making some kind of coupling here.

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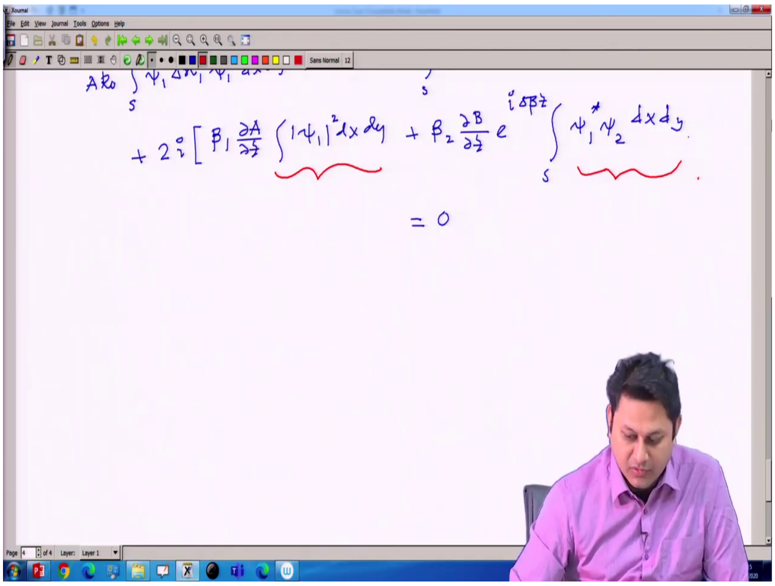

Plus 2i, then beta 1 del A del z; then integration of mod of psi 1 square dx dy. Here I have this term n 1 square, here I have this term delta n 2 square, so this term is now taken care. Now, here when I integrate this integration; here I have psi 1 and psi 2. So, one integration should be mod of psi 1 square and another integration will be beta 2 del B del z; with the beta term I always have the phase, this phase and then integrate as psi 1 star psi 2 dx dy this I have,, which is equal to 0..

Now, as I mentioned I have some sign some kind of, some sort of coupling term here and this is some kind of overlapping term, sorry overlapping term; this is a self overlapping and this is the cross overlapping.

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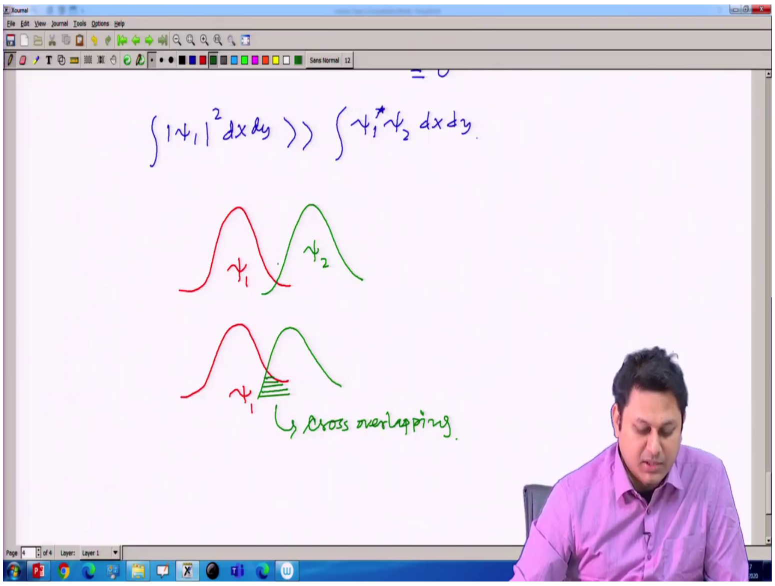

So, normally what happened that the self overlapping term, which is overlapping between the same psi with itself, psi 1 mod square dx dy is much much greater than the cross overlapping term psi 1 star psi 2 dx dy .

One can qualitatively understand that fact that, if I have a field like this; say this is my psi 1 and if I have a field like this say, this is my psi 2, this is my psi 1. So, the self-overlapping means, I am overlapping psi 1 over psi 1. So, the overlapping factor is very high, but when I have a cross overlapping; so only these term which is overlapping, so only this portion which is overlapping, suppose I draw is field like this.

So, this is the portion which is overlapping, this is a cross overlapping. So, this integral will be much much smaller compute ah compared to the self coupling terms. So, I can neglect this term safely.

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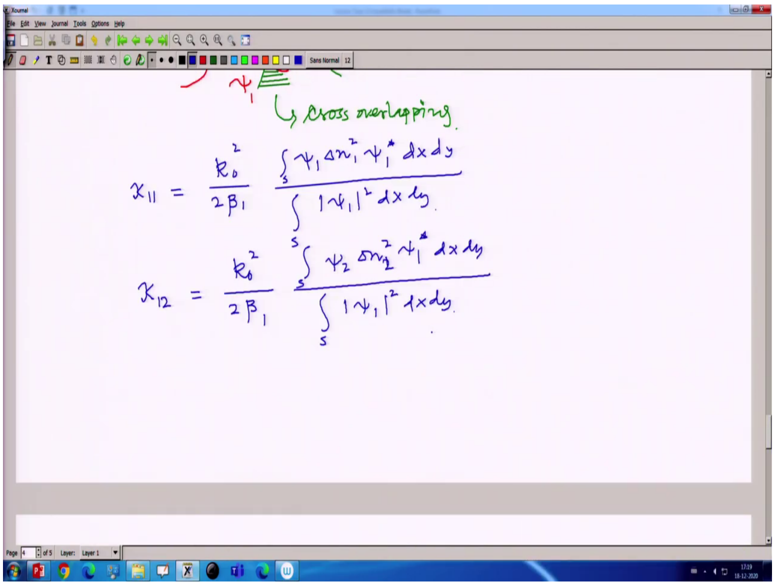

So, if I do and if I define something more, which is say kappa 11; I define this as k 0 square divided by 2 beta 1, then integral psi 1 delta n square, because this term I already have in my equation dx dy divided by this self overlapping term,, mod of psi 1 square dx dy .

So, in the equation I have this term. So, I will going to neglect this term and then I divide the entire equation with this term, this integral; here and here this term will come and also I divide everything with 2 beta 1. So, I can have a term like this.

Also I can have another term say, kappa 12 as this,, which is psi 2 delta n 1s delta n 2 square psi 1 star dx dy divided by the usual self coupling term over some surface. So, dx dy is the, is a cross surface area of the of this, cross surface area of this waveguide, whatever the waveguide structure. So, we have a waveguide structure like this.

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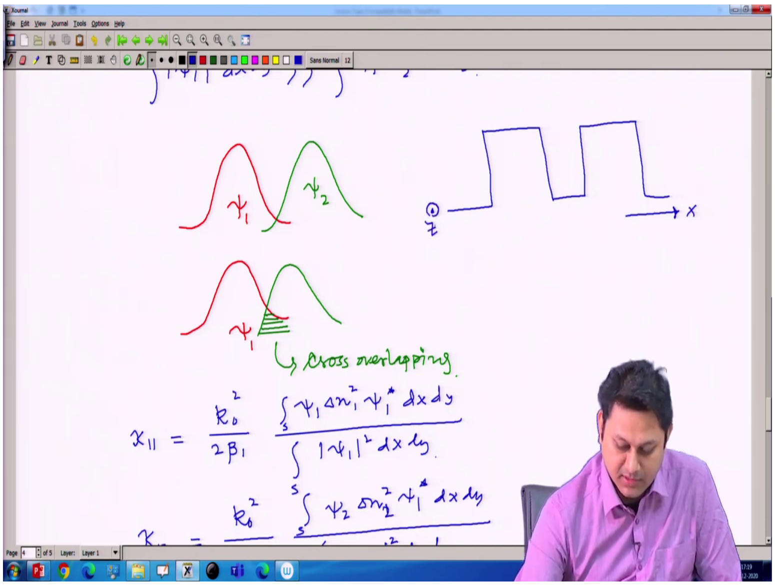

So, I am talking about. So, this area whatever the area we have is basically my x y and this direction it is my z.

Well after having this kappa 1 and kappa 2; my differential equation become much more, whatever the equation we have here is much more simpler.

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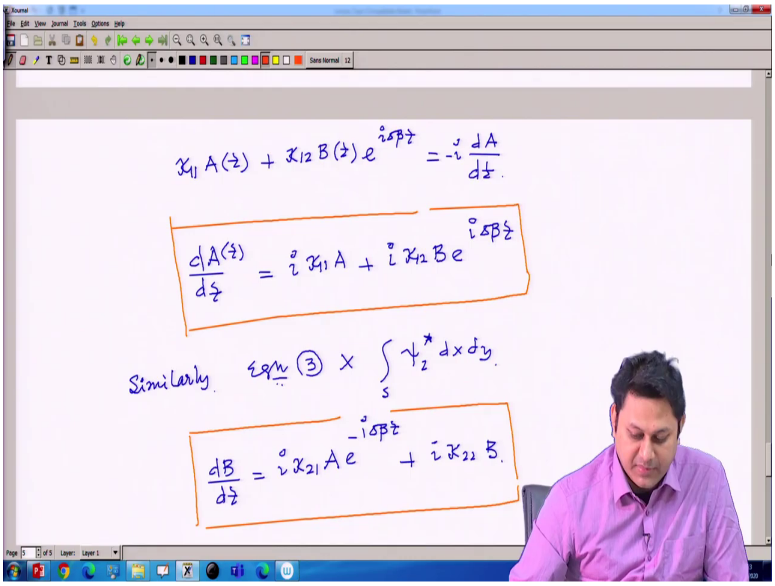

And the simpler form I can have is this kappa 11, after defining that A which is a function of z, then kappa 12 beta z; then e to the power of i delta beta z is equal to minus of i dA dz.

So, I simplified it more and eventually I have a very interesting equation, which I wanted to find and this equation is the evolution of the amplitude over z is equal to i kappa 11 A plus i kappa 12 Be to the power of i delta Bz . So, I have a differential equation of the evolution of the amplitude Az.

And this equation I figure out by integrating by putting a very important step here; I like to recall this step that, I have an equation 3 and this equation 3, then I multiply this equation 3 with psi 1 star dx dy and I am getting this equation, whatever the equation is now written here.

Now, you can do the similar way. Similarly,, I have equation 3 and I am going to multiply this equation with this term psi 2 star dx dy that I can do; this is my choice. And if I do that and do all the procedure, follow all the procedure that has been done so far for A; I can have, I can end up with these equation,, a similar kind of equation. And I am writing this equation here this; this is a differential equation for the another amplitude B, how the B will going to change it is given here.

Note it in the first case I have delta b, but in this case I have minus of delta B; because my delta B, delta beta rather is defined as beta 2 minus beta 1

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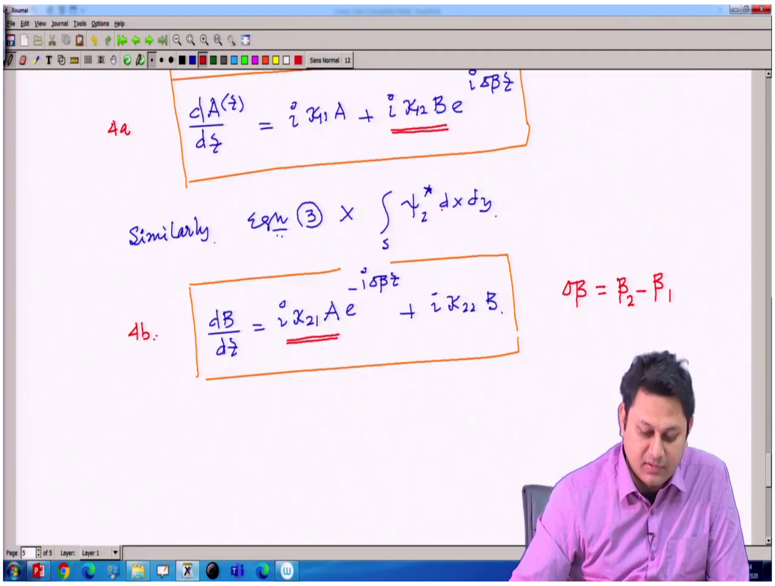

So, in the first case I have a term beta 2 minus beta 1 and in the next case if I if you do the calculation rigorously, you will find; you will have beta 1 minus beta 2. So, in order to make this beta 2 minus beta 1, I need to put a negative sign. So, that is why this negative sign will be here.

So, now I have two very important equation and this equation I write it as equation 4a and equation 4b, these are the coupled equation mind it, sorry this will be 4b. So, the variation of the a will depend on b and the variation of b will depend on a through this coupling term, through this coupling term.

So, first case I have a coupling term here and in second case I have a coupling term here; but still I am having one term like this. So, this is some term, which I can also identify.

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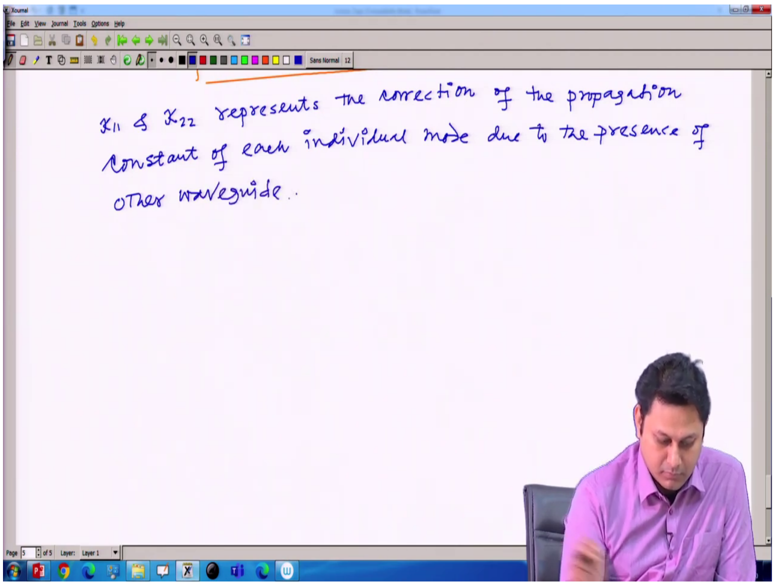

So, kappa 1 and kappa 2 is essentially. So, let me write down, kappa 11 and kappa 22 represents the correction of the propagation constant of each individual mode due to the presence of other waveguide. So, what was that?

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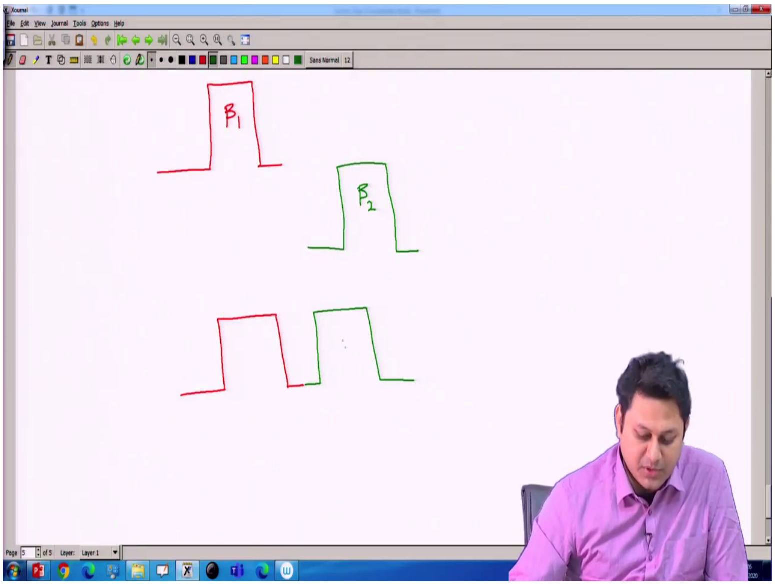

So, initially suppose I have one waveguide and I have a propagation constant beta here. Then I have another waveguide, individual another waveguide and this is beta 2. Now, what happened that, I now I have both the waveguides,, sorry let me draw in a logical way.

Now, instead of having one waveguide, I am having both waveguide together. So, now, this beta 1 and beta 2 will going to change, because of the presence of this waveguide and that change, this correction is given here

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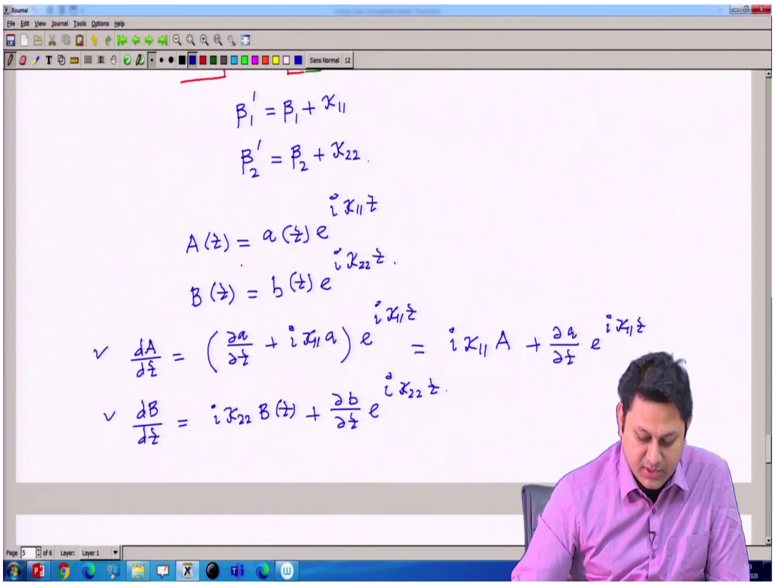

So, I if I have a corrected term say, beta 1 prime and beta 2 prime. So, beta 1 and beta 2 prime now be corrected. So, beta 1 prime should be beta 1 plus some correction and this correction say K 11 kappa 11 and beta 2 prime should be beta 2 plus some correction say kappa 2 2 that, correction here we can we are talking about.

So, now after having this concept, I can write my A z; since I know this is a correction term, I can write my A z as a new notation with e to the power of, because I know this is a correction. So, I can write these as a proposition constant like this and B z, the amplitude term is also having a contribution of the correction as this.

So, now A and B which was initially amplitude is associated with the phase term and this phase is basically the correction in the propagation constant that is all. So, here I can have dA dz is equal to del z say plus i kappa 11 a e to the power of i kappa 11 z. And so, and dB dz will be simply the similar way, I can write i kappa 22 Bz ; because this is if I write, so it should be i kappa 11 A 2 A e to the power i K z. So, i kappa 11 z, I can write it as A.

And then the rest of the term plus say d A d z e to the power of i kappa 11 z like that and B I can write also in this way. So, I can, I can put this and eventually I can put this here in this equation, whatever the equation I have and if I put this equation these two in these two equation 4a and 4b.

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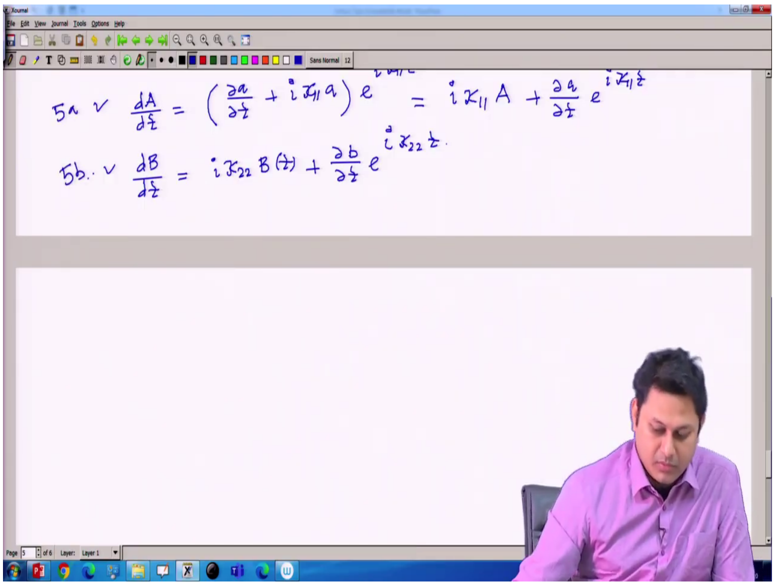

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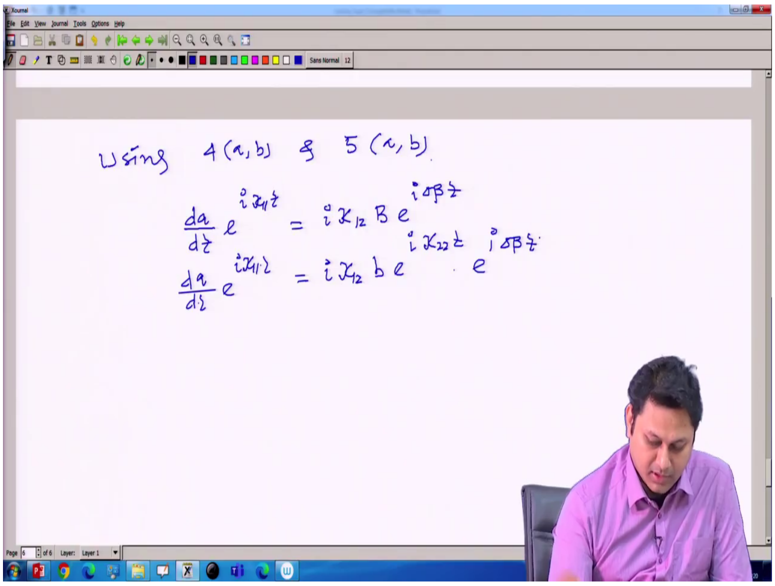

So, so let us write it as 5 a and 5 b. So, using 4 a, b and 5 a, b; we can have something like da dz e to the power of i kappa 11 z is equal to i kappa 12.

Then B, I can write this B as say,, ok. Let me write the total term, then I will. So, it should be Be to the power of i delta beta z, it was original in the equation; this thing is, so da dz e to the power i kappa 11 z is i kappa 12, B I can write as small be to the power of i kappa 22 z into e to the power of i delta beta z.

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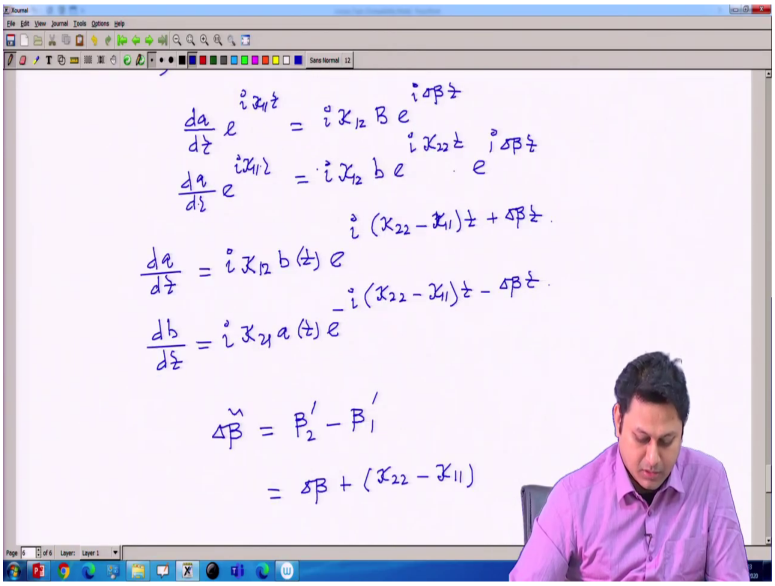

So, eventually I can have d a d z is equal to i of kappa 1 2 Bz e to the power of i kappa 2 2 minus kappa,, sorry it is kappa naught K; so kappa 2 2 minus kappa 1 1 z plus delta beta z. In the similar way, exactly in the similar way one can have d b d z is equal to i kappa 2 1 a z e to the power of minus of i kappa 2 2 minus kappa 1 1 z minus delta beta z.

Now, we can define, because we know that kappa 1 1 and kappa 2 2 as the modification of the propagation constant delta, propagation constant beta. So, I can have beta tilde in general as say beta 2 prime minus beta 1 prime, which is delta beta plus kappa 2 2 minus kappa 1 1; beta 2 prime and beta 1 prime I have already defined here.

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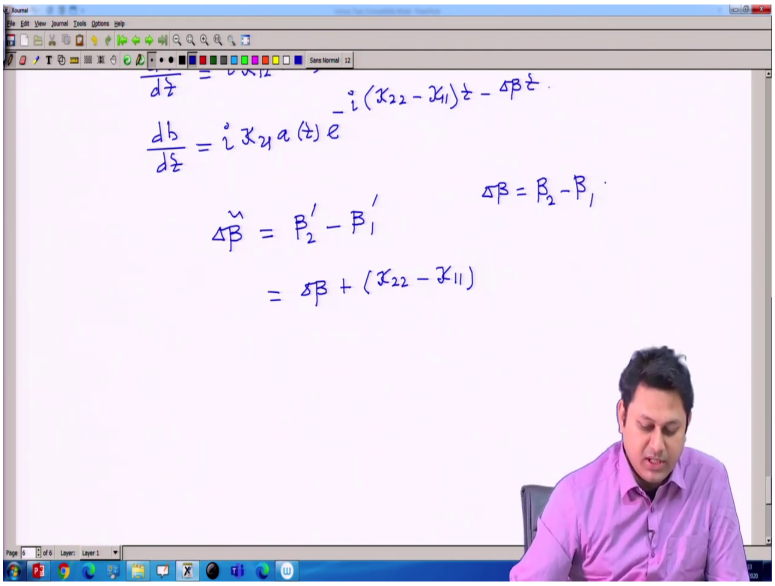

And if you put you can find that, beta 2 minus beta 2 prime minus beta 1 prime is delta beta; because delta beta initially defined as beta 2 minus beta 1.

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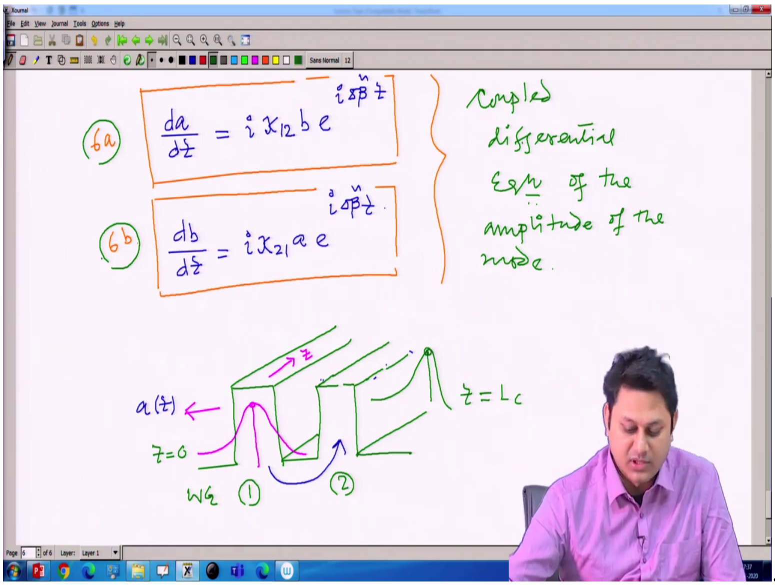

So, eventually I have a equation which is having the form like d a d z, which is a pure amplitude term is equal to, is evolving following this differential equation; then Be to the power of i delta beta tilde z. And this is one equation, so let me put in a bracket; say equation 6a. And another equation for b I have say 21 ae to the power of i delta beta tilde z,, say this is equation 6b.

So, these two are coupled differential equation and we almost there. So, these are the coupled differential equation of the amplitude of the modes. So, this is a couple differential equation of the amplitude of the mode; so that means this is my waveguide structure, I am drawing once again.

And suppose in the 3D it is like that. So, I have a wave here launched somewhere having some kind of mode structure and during the propagation this is along z direction, it will go to propagate. So, this is z direction.

So, this is the amplitude I am talking about, which is a function of z and this amplitude is going to change over the distance z. So, this is the distance z. So, what happened that it will going to change and there is a possibility that some of the energy can pass from this waveguide to this one and at certain distance I can have a field structure like this here.

My drawing is bad here. So, let me erase this. So,,, so I can have some field, some energy at some distance here. So, I can have some energy say here, during the propagation. Initially I do not have any thing here in waveguide. So, this is waveguide 1 and this is waveguide 2; WG 1 waveguide 1 and WG 2. And gradually it will going to change and I will have something here at some point say, at z equal to some distance say L c, this is z equal to 0.

So, how this amplitude will going to change over the distance that one can find out by solving these two differential equation that we derive here, these two very very important differential equation. So, in the next class, today I do not have the time to solve this couple differential equation.

So, in the next class, we will going to solve this coupled differential equation and then we have an idea that how the field is going from one waveguide to another waveguide. So, with this note, I would like to conclude my class. So, thank you for your attention and see you in the next class for the rest of the calculations.

Thank you.