Physics of Linear and Non Linear Optical Waveguides
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Module - 04
Fiber Optics Components
Lecture - 36
Directional Coupler

Hello student to the course of Physics of linear and non-linear optical waveguides. So, today we have lecture number 36. And in today's lecture, we will try to understand the very important theory called couple mode theory, which is related to coupler.

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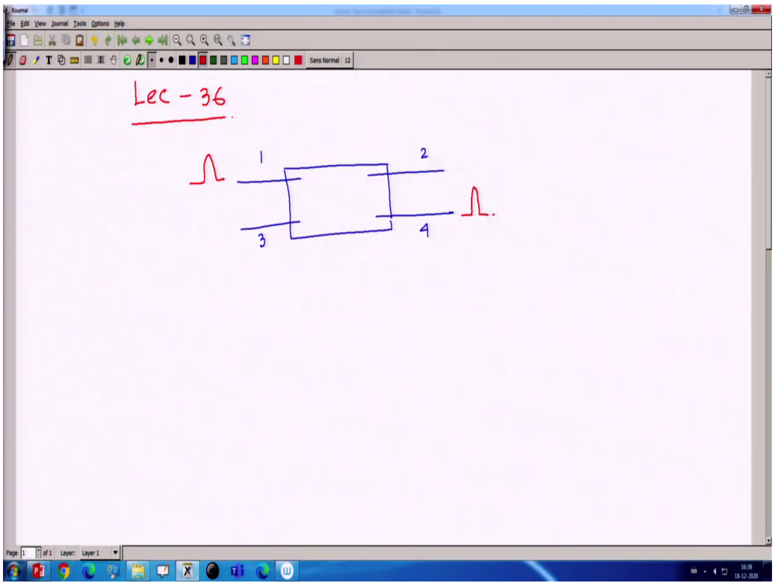

So, in the last class, we already started the concept of Directional Coupler, where we mentioned that, if I have a structure like that having 4 ports; then if I launch a light here in this port, there is a possibility that I can have the light here which is called switching.

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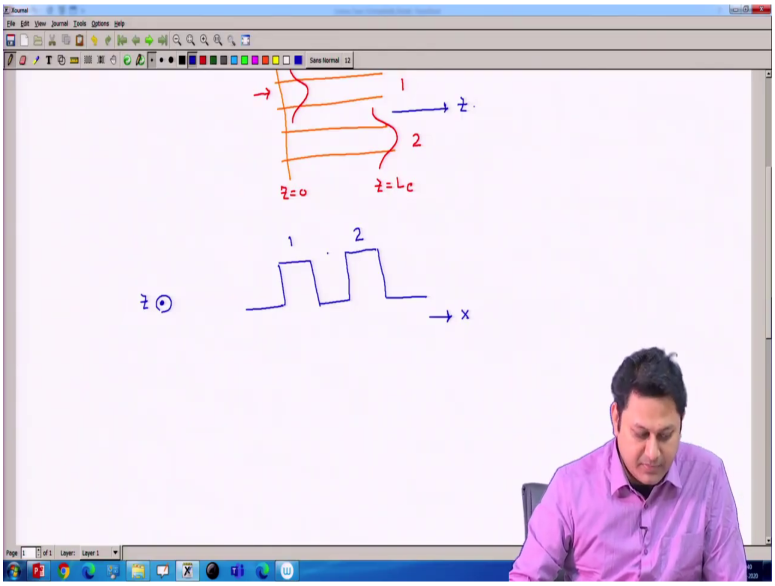

So, that means, there is a transfer of energy from one waveguide to another waveguide, which is placed very close to each other. So, let me draw a rough schematic diagram. Suppose these two waveguides are placed very close to each other and I have a wave here initial wave. And at certain distance what happened, this wave will come here at this waveguide.

So, this is waveguide 1 and this is waveguide 2. So, I launch the light here initially say at z equal to 0 point. And some z equal to say some length L c; I can find that this is a critical or coupling length and what happened that entire energy can transfer to these new waveguides, along this direction we have the propagation. So, this is my z direction.

So, I can now write this waveguide in this draw the way the refractive index profile of these two waveguide in this way, two waveguides placed side by side. So, this is the structure of the waveguide; this is waveguide 1 and this is waveguide 2 and they are placed very close to each other and z is perpendicular to the plane.

So, along this direction we have z and along this direction, suppose we have x. So, refractive index is varying along this direction, two waveguides they are placed side by side. So, what happened that, we can have some general modes of these waveguides; these this waveguide can be considered to a single waveguide.

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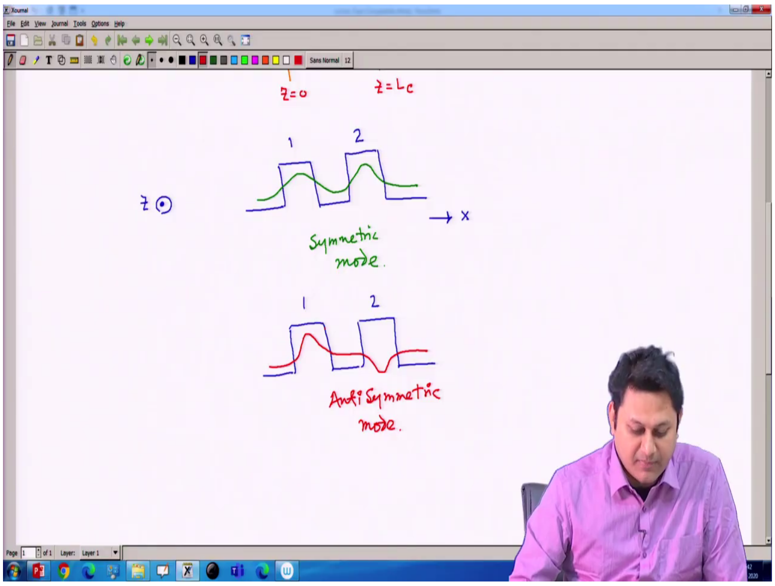

And if I consider this a single waveguide, I can have a general mode same this this form, this is called a symmetric mode; also for this waveguide, I can have a mode like this, which we call the anti-symmetric mode.

Now, using the first principle, we will try to understand the propagation of these kind of modes in the waveguide and how these energy transfer is possible in the waveguide.

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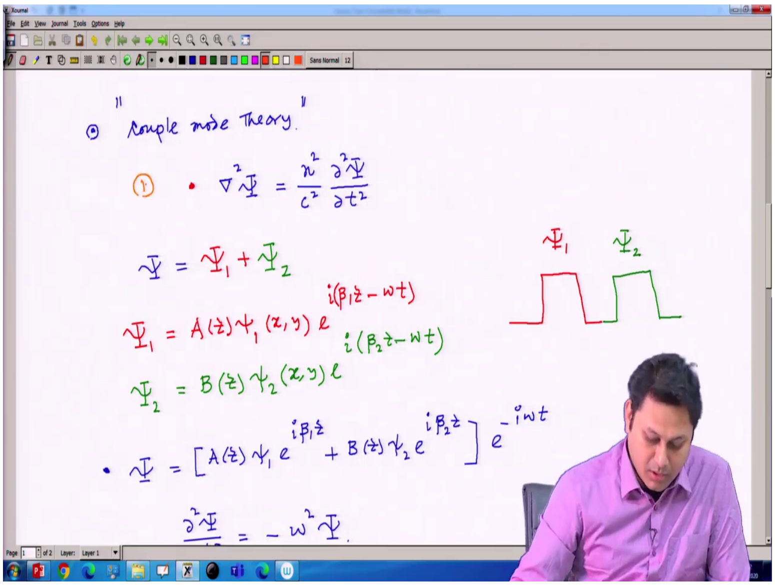

So, let us start with the very first thing and this is a very important theory; we call this is a coupled mode theory. Using the coupled mode theory, we try to understand how this mode will going to evolve inside the waveguide; not only that how the power will going to transfer from one mode to another mode and all the necessary information are going to extract.

So, in this particular course we will calculate this couple mode theory rigorously; all the rigorous mathematics will be done here, so it will be little bit lengthy. So, let us start with the first equation that we have and always we use and that is the wave equation; because when we deal with the wave, then we need to write this wave equation at first. So, that we know; this is the Maxwell's wave equation I am writing, where psi is a general mode in this waveguide.

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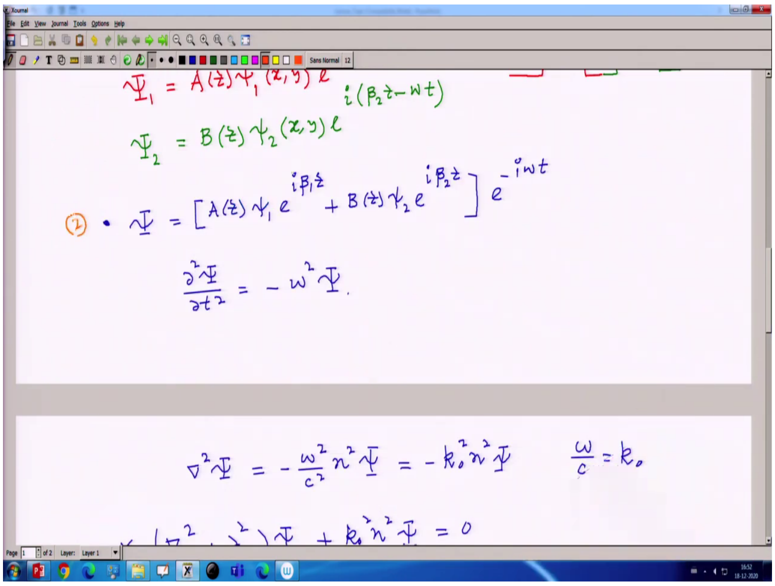

Now, I can defined psi, big psi which we called the general mode. The big psi can be defined as say psi 1 plus say psi 2, where my psi 1 is defined in this way. I have a amplitude which is varying with respect to z, that is the important term I am putting here; then I have psi 1 x, y, this is x, y dependent term and then the propagation term e to the power i beta 1 z minus omega t, ok.

Then psi 2 I can also define B z x, y e to the power i beta 2 z minus omega t. So, this way I define psi 1 and psi 2, assuming this modes are for, this fields are for these two waveguides like this. So, this is one waveguide and this is suppose another waveguide, I draw in this colour only. And the fields are defined here in such a way that, this is for psi 1 and this is for psi 2, so that the total mode is big psi which is the addition of these two.

So, superposition of psi 1 and psi 2 will give us the total mode of the system, whatever the system is drawn here. So, that means, my psi is A z amplitude; the important thing is the amplitude, this amplitude now become a function of z. Because when the mode is propagating these two; so what happened that, there is exchange of energy from this mode to this mode. So, that is why the amplitude will going to change over the z distance.

So, that is why A and B should be a function of z; then psi 1 e to the power of i beta 1 z plus B z psi 2 e to the power of i beta 2 z and then e to the power of minus i omega t that is my total field by definition.

Now, this total field will going to satisfy this this equation, this wave equation, Maxwell's wave equation. So, now, if I put this into this Maxwell's equation, I can have few terms.

So, the first thing I can do, I can extract this term quite easily; because the t dependence is only sitting here into the last term e to the power i omega t and if I make a double derivative with respect to time, then it should be minus of omega square and then big psi, that is my left right hand side term whatever is here.

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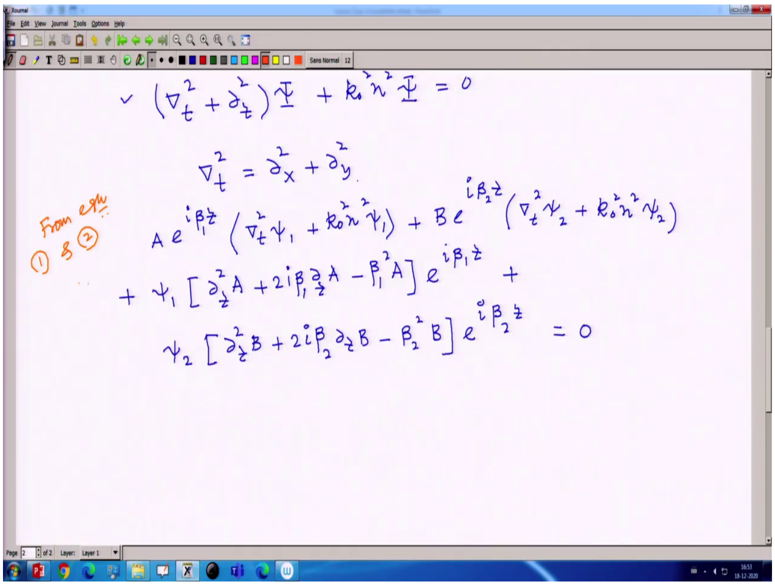

Now, also I can write del square psi is equal to minus of omega square c square n square big psi, which is minus of k 0 square n square big psi; because omega by c is nothing, but the propagation constant of the free space which is k 0, so that we find. Well, this can be written as, this is the notation we are being we have been using for long time that, I divide this del square operator into two part; one is the transverse component, and another is the longitudinal one, that is propagating that is the along the propagation distance z.

Then plus k 0 square n square big psi is equal to 0. Note it this transverse component of this quantity is this one, just is derivative. Well now I put this psi, whatever the psi I defined in the last page this one; I will going to put it here in this equation, in this equation. And if I do, I can have simply. So, A e to the power of i beta z; then the derivative with respect to x and y. So, I can write simply this one.

This is for psi 1. And for psi 2, I have B e to the power i ok; here we have beta 1, so beta 2 z, then the transverse part this one plus. Now, I have to execute this z derivative and if you look carefully, we have 2 z components here; one is sitting here aA z and another is here A function of z and B function of z and another is e to the power i beta 1 z.

So, we have to be careful enough to execute this z derivative which is a double derivative. And if you do carefully, then I can have a term like this plus 2 i beta derivative of x minus beta 1 square A e to the power of i beta 1 z that is one term plus another term is similarly exactly similar way I have this, this should be 1, this should be 2, it should be B.

Then minus of beta 2 square B e to the power of i e to the power I beta 2 z; this thing will be equal to 0. So, I put all the terms here and then after putting all this term. So, let me use. So, this is say equation 1. So, equation one and say this is equation 2 and this is from equation 1 and 2; we have this one. So, I just put this value of psi in equation 1 and I am get I am getting this one.

So, these things I can simplify a bit and if I simplify, ok.

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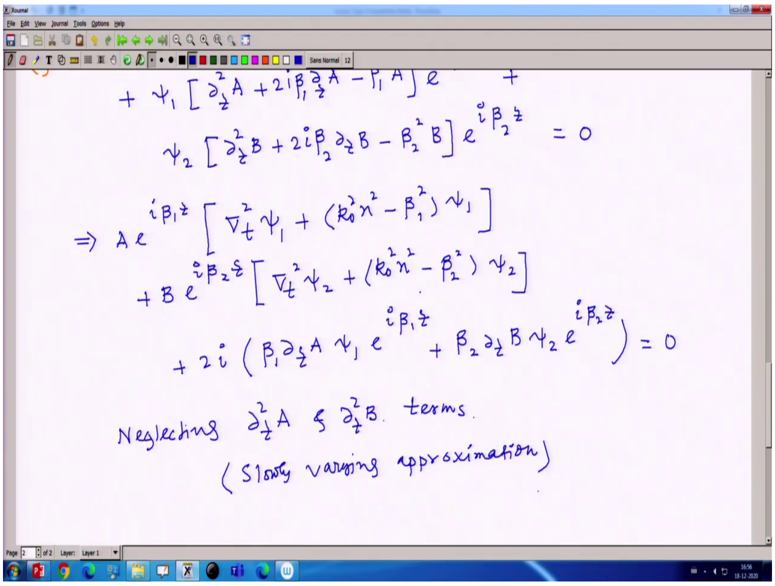

Let me write it again that, this thing is A e to the power of i beta 1 z; then I have a few terms like this psi 1 plus k 0 square n square minus beta 1 square psi 1, this is one term. Another term in the similar way I have B e to the power of i beta 2 z and then plus k 0 square n square minus beta 2 square psi 2 this one and plus 2 i beta 1 del z A psi 1 e to the power of i beta 1 z plus beta 2 del z beta.

Then psi 2, then e to the power of i from beta to z, bracket close is equal to 0. So, I am I am making one approximation; so let me first write down this approximation that, we neglect neglecting this second order derivative terms. So, this is a slowly varying approximation.

So, let us quickly understand what we have done here in this step. So, this is step 1 and this is step 2. So, from here I have step 2. So, in step 2 what happened that I just, I just take this beta 1 A term to here and beta 2 square beta 1 square A term and beta 2 square A term to in this equation, this part and this part. So, that I can have a term like this one and this one and rest of the things are straight forward; I have 2 i beta 2 first order derivative with respect to z B and 2 i beta 1 first order derivative with respect to z of A.

These two term I just put together and this is the term. And I neglect this term, this term and this term; because compared to the first order derivative, these two terms are small. So, we simply neglect this term and we called this is a slowly varying approximation. Now, let us go back to; now there is a I mean reason why we put this term here, this term and this term, we write it for a reason.

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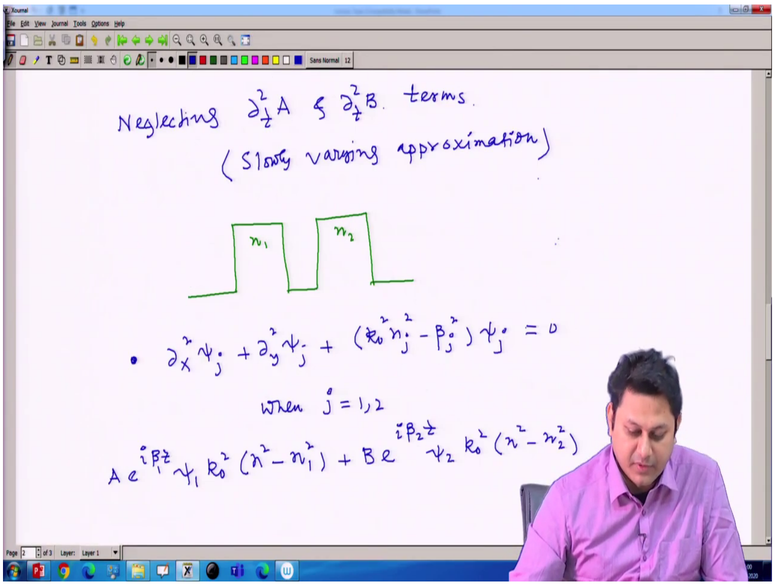

And let me draw once again the structure.

So, our structure was like this two waveguide placed side by side. This is refractive index say n 1, this is refractive index say n 2, two different refractive index. Then I can have a equation that should satisfy in general for these two waveguides is this one, when j is 1 and 2.

So, for psi 1 and psi 2 we can have these solutions of these waveguides; if these waveguides n 2 is not there, for n 1 waveguide I can have this equation and if n 1 is not there, I can have if I have only n 2, then I can have the Maxwell's equations or the mode equation for psi 2.

So, for psi 1 and psi 2 specially I have an additional equation in my hand and this additional equation reads like this. Now, I will going to use this equation here; because I already have this almost the identical form here, only thing you should note that this n. This n here is the refractive index of the entire system; whereas this n 1 and n 2 are the refractive index of the individual waveguides.

If I just now remove this waveguide in two; I have only one waveguide and then this equation going to satisfy by psi 1. If I do not have n 1, only have n 2; then again psi 2 will going to satisfy this equation. So, psi 1 and psi 2 individually will going to satisfy this general equation, I just write this general equation by putting a suffix j, so that I can understand that this equation will be satisfied by 1 and 2 or psi 1 and psi 2.

So, I will going to make use of this information here and try to understand that what happened after that. So, let me erase these lines, because this is just to highlight that. So, I want to use this here, ok.

So, now if I use this this equation there, then I can eventually have some equation like this beta z; then psi 1 k 0 square, n square already there. So, I incorporate another n 1 square here; plus beta it should be 1 e to the power of i beta 2 z and then psi 2 k 0 square n square minus n 2 square.

These two term additional term I have, because here if you look carefully; so this term that I have n, so this minus beta 1 square psi 1 square psi 1 I just replace here. So, these things is nothing, but the transverse components. So, when I have a transverse component, then. So, let me do one thing, because I need to write this.

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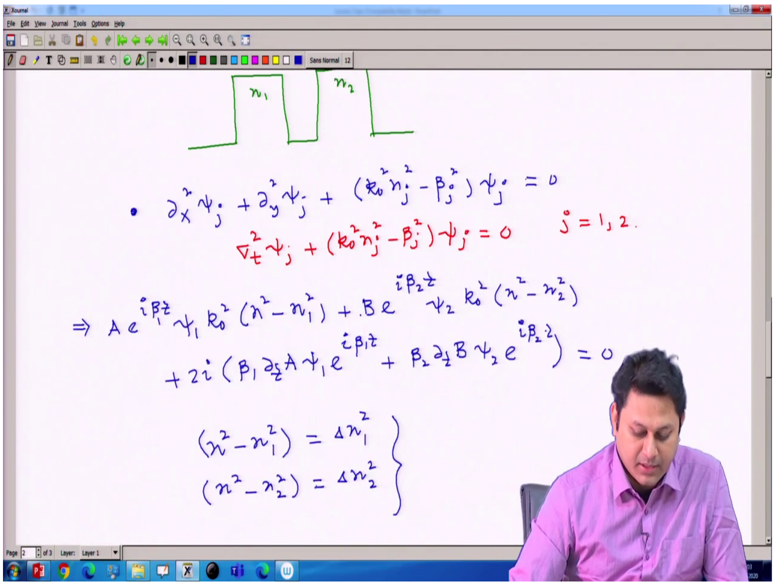

So, I will going to use this equation.

So, transverse component psi j plus k 0 square n j square minus beta j square psi j equal to 0; j is 1 and 2. I will going to use this equation here and eventually I have this equation. This plus, the rest term is as usual 2 i beta 1; then the first derivative with respect to z A psi 1 e to the power of i beta 1 z plus beta 2 first derivative with respect to z of B psi 2 e to the power of i beta 2 z, this is equal to 0.

This n 1 square this is the difference between the refractive index. So, I can write this difference as say delta n 1 square and n square minus n 2 square as delta n 2 square. So, I just write these two difference between these two refractive indexes. Mind it n is a refractive index of the entire system, if I consider these two as a single system and n 1 and n 2 are the individual refractive index of this individual waveguide.

So, n 1 is a refractive index when this waveguide is not here and n 2 is a refractive index when n 1 is not there, this refractive this waveguide is not there. But if I put these two waveguide together; then there is a general refractive index, there is overall refractive index of the system and that refractive index I defined as n. And psi 1 and psi 2 are the solutions are the field solution of individual waveguide.

Well after that, here I have one term called beta, and so let me write the next thing that A.

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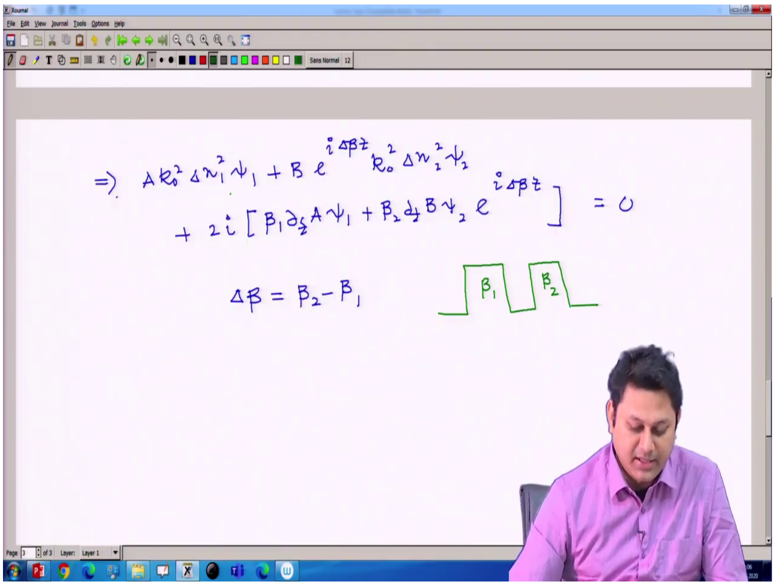

So, from this equation I can write A, I divide the entire equation with e to the power say e to the power of i beta z; if I divide the entire equation to e to the power i beta z, then I can have it Ak 0 square. So, let me see what was there, Ak 0 square, then delta n 1 square psi 1 plus B e to the power of i delta beta z; because I divide, so that is why I should have some kind of delta beta.

So, what delta beta I will I will show soon n 2 psi 2 plus 2 i beta 1 del z a psi 1 plus beta 2 del z beta psi 2 e to the power; again whenever I have B, I have this term, this is equal to 0. Note it, delta beta is equal to beta 2 minus beta 1.

So, what I essentially do that, this equation I have; I put n 1 n square minus n 1 square to delta n 1 square and then I divide the entire equation to e to the power i beta 1 z. If I divide, then this term will not be here; this term will be e to the power i beta 2 minus of i e to the power of e to the power of minus of i beta 1 z. So, that I write in this way delta beta z, where delta beta is beta 2 minus beta 1, which is the difference between the propagation constant of the two modes that is propagating here.

So, it is, this is the waveguide I have. So, this is the propagation constant here and this is the propagation constant here; in absence of these two. So, delta beta is a difference between this these two propagation constant of these two individual modes.

So, today I do not have much time to proceed more. So, I should start in the next class, I should start with this equation and then try to complete the calculation. As I mentioned in the starting of the class, this should be a lengthy calculation; but it is important to understand how for a couple mode theory, how we can extract the information of the amplitude and how the amplitude is changing with respect to z.

So, with that note, I like to conclude the class. So, see you in the next class with the additional calculation or the further calculation with this couple mode theory. So, see you and thank you.