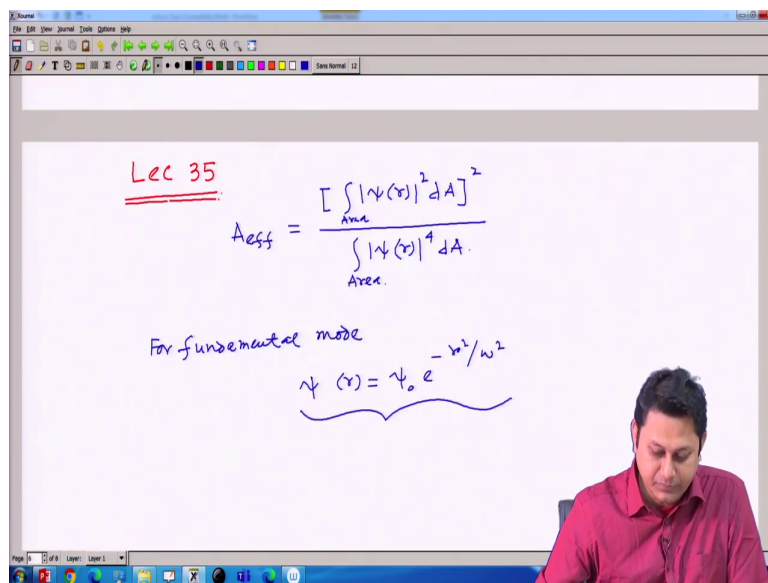


Physics of Linear and Non Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 35
Effective Area of Mode, Fiber Optics Components

Hello student to the course of Physics of linear and non-linear optical wave guides. So, today we have lecture number 35 and in this lecture we will going to calculate the Effective Area of the Modes, and we start a new topic which is related to Optical Fiber Components ok.

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Lec 35

$$A_{eff} = \frac{\left[\int_{Area} |\psi(r)|^2 dA \right]^2}{\int_{Area} |\psi(r)|^4 dA}$$

For fundamental mode

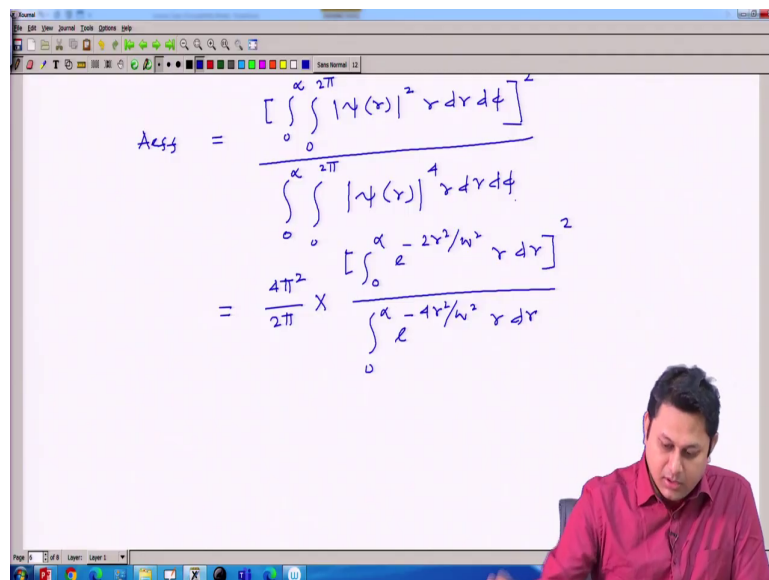
$$\psi(r) = \psi_0 e^{-r^2/w^2}$$

So in the last class, if you remember we defined the effective area of a mode as this; by definition this is over area this is over area. So, by definition this is the effective area now, we

will going to calculate the effective area for a fundamental mode. So, for fundamental mode we have $\psi_0(r)$ as sorry better to write is as $\psi(r)$ is $\psi_0 e^{-r^2/w^2}$ to the power of minus r square ω square.

Now, this is the field distribution approximate field distribution that is given to you so, this is for fundamental mode. Now, I am going to use these field distribution to find out the effective area, which is defined already defined here.

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$$A_{eff} = \frac{\left[\int_0^\infty \int_0^{2\pi} |\psi(r)|^2 r dr d\phi \right]^2}{\int_0^\infty \int_0^{2\pi} |\psi(r)|^4 r dr d\phi}$$

$$= \frac{4\pi^2}{2\pi} \times \frac{\left[\int_0^\infty e^{-2r^2/w^2} r dr \right]^2}{\int_0^\infty e^{-4r^2/w^2} r dr}$$

So, my $A_{effective}$ should be equal to 0 to infinity integration 0 to 2π then, mode of $\psi(r)$ whole square $r dr d\phi$ this is the element area element for in polar coordinate. Because I need to use the polar coordinate here, since this since the geometry is a cylindrical geometry divided by integration 0 to infinity integration 0 to 2π then, $\psi(r)$ of r whole to the power 4 and the area element.

This field the fundamental mode this fundamental mode is does not it does not depend on the ϕ . So, that is why this in this integration this ϕ there is it will just going to integrate over 0 to 2π over this ϕ .

So, there is no ϕ dependency inside this integration. So, that is why it should be simply 2π here, and in the denominator also it should be simply 2π so, I can directly write it as 4π square because it is a square associated with the numerator so, it should be 4π divided by 2π .

And the rest of the integration is integration 0 to infinity then, e to the power of minus of $2r$ square divided by w square and then; $r dr$ and the has to be squared and we have here 0 to infinity e to the power of minus of $4r$ square divided by w square $r dr$. Mainly this is to the power 4 that is why this 4 will come here.

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$$\int_0^{\infty} e^{-2r^2/w^2} r dr$$

$$2r^2/w^2 = x$$

$$\frac{4}{w^2} r dr = dx$$

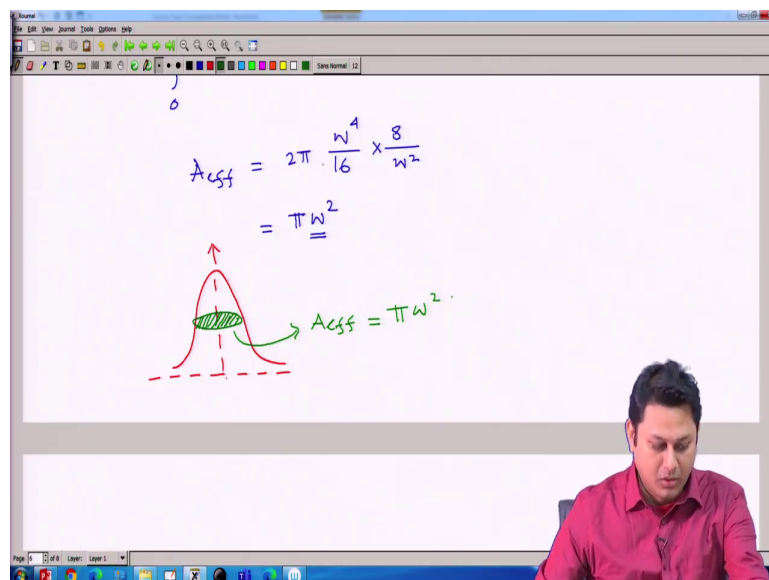
$$\frac{w^2}{4} \int_0^{\infty} e^{-x} dx = \frac{w^2}{4} \left[\frac{e^{-x}}{-1} \right]_0^{\infty} = \frac{w^2}{4}$$

$$\int_0^{\infty} e^{-4r^2/w^2} r dr = \frac{w^2}{8}$$

So, if I now integrate this portion quickly let us do that e to the power of minus 2 r square divided by these things r d r. So, I can choose 2 r square as a new variable x so, that I can have 4 divided by omega square sorry w square then r d r equal to d x. Now, we will going to put this d r here so, simply the equation will be w square divided by 4 integration 0 to infinity e to the power of minus x because, this portion I take x and then d x.

So, this is simply w square divided by 4 and if I execute this integration e to the power minus x it will be e to the power of minus x minus 1 0 to infinity it should be simply divided by 4. In the similar way integration 0 to infinity e to the power of minus 4 r square r d r one can execute that and the result will be simply I am not doing this integration because, this exactly the same procedure and one can find this value with quite easily.

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So, eventually my A effective will be we already have 4 pi square divided by 2 pi so, it is essentially 2 pi. Then, it should be omega to the power w to the power 4 divided by 16 because, I need to have the square of this quantity w square by 4 because it is square is sitting here divided by this quantity.

So, it should be multiplied by 8 of w square. So, it is essentially pi of a very straight forward expression phi of w square so, you can see it is this omega this w is essentially a spot size. So, if I know the spot size then I can readily find out the effective area. So, if this is the field distribution.

So let me draw in a different way, if the distribution is a Gaussian distribution the field distribution is a Gaussian distribution like this if it is approximated as a Gaussian distribution then the spot size if I know then effective area I can calculate very quickly. So, this is the

effective area of this corresponding field we just draw this the last class so, this is basically the $A_{\text{effective}}$ which is simply π multiplied by the spot size square.

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empirical relationship between w & a

$$\frac{w}{a} \approx \left(0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} \right)$$

w = Spot Size
 a = Core Radius of the fiber.

$0.8 < V < 2.5$

$$V = a k_0 (n_1^2 - n_2^2)^{1/2}$$

$$= a \cdot \frac{2\pi}{\lambda_0} (n_1^2 - n_2^2)^{1/2}$$

LP₀₁

Well one can have one important empirical formula regarding the spot size and the fiber diameter. So, that I am just going to mention here, which is empirical relationship between spot size and a where a is a fiber diameter. So, this relationship is something like this this is purely empirical relationship and valid only for certain range of V . So, I will go going to give you the range of V as well.

So, V to the power 3 by 2 plus 2.879 divided by V to the power 6 and these valid this is valid for the range of $V < 2.5$. So, in these range of V so, what this empirical formula says that if I have a V value in the range 0.8 to in between point a to 2.5 that means, essentially we are working in the region where the fiber is allowing only one mode or in single mode region.

Then for the single mode region only one mode will go to excite and that is the fundamental mode; and we know that the distribution of the fundamental mode is Gaussian one. So, this is the distribution of the fundamental mode approximated as a Gaussian one.

So, this is the fundamental mode or rather right $l p 0 1$ mode $LP 0 1$ mode and for this $LP 0 1$ mode depending on the value of v parameter I can change the value of v parameter by changing the wavelength keeping a fixed I I can also change the value of v parameter by changing the wavelength or by changing the value of

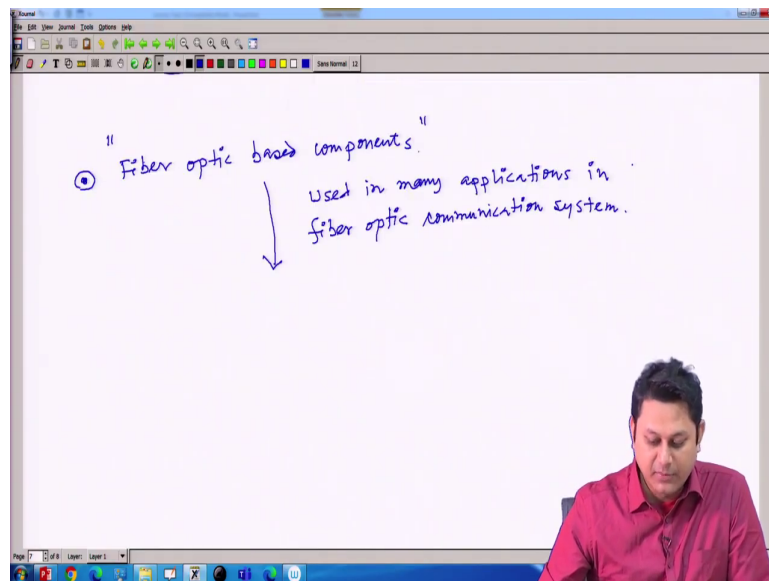
so, let me write down what was initially the v parameter it is a $k_0 n_1^2$ square minus n_2^2 square whole to the power half. So, it is a 2π divided by λ_0 it is operating wavelength and n_1^2 square minus n_2^2 square whole to the power half. So, I can change the value of n_1 and n_2 and then I can change the value of v I can change also the λ_0 and I can change the value of v .

So, if I change the value of v with these changing by n_1 n_2 or by changing λ_0 then, that value if I put here I can have a ratio of w divided by a if a is given I can find out the value of the spot size.

So, readily we can know that how the mode is distributed what is the characteristics of the mode, the mode will be definitely a Gaussian type, but with these information with this empirical formula I can find out the value of the spot size what should be the spot size of the mode.

So, that is the use the useful formula so, here w is a spot size. So, let me write it so w is basically the spot size and a is the core radius of the fiber ok. So, with this I like to conclude the topic here, which is related to modes almost everything we covered here qualitatively we understand the mode distribution as well.

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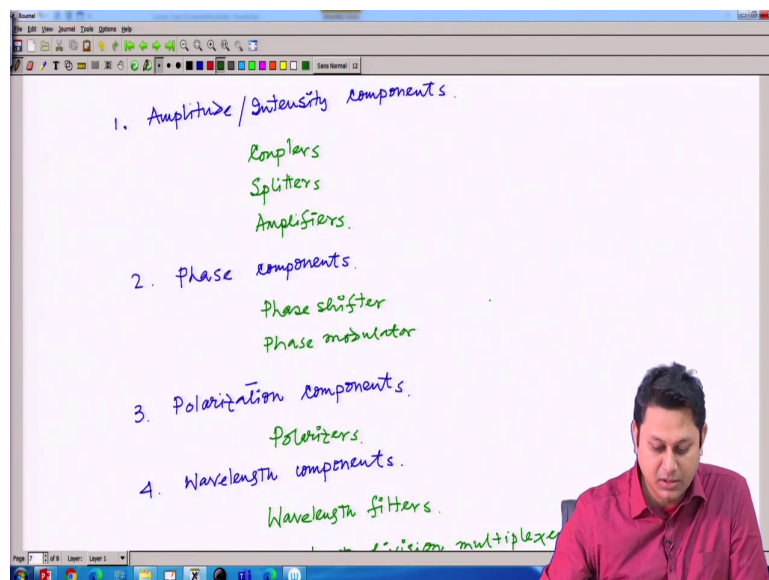


And now, we will going to jump a new topic and that is related to the Fiber Optic Components Fiber Optic based Components. The fiber optics based components are very useful because, we can have different kind of optical components. So, what are those components? Let me list it down, but before that.

So, this is used in many applications so, these are the optical components optical fiber is not only used to just transmit light from one point to another point or information from one point to another point; it can also used as different optical components through which we can do many things used in many applications in specially in fiber optics based communication system.

So, there are many components fiber based component that is used in fiber optic based communication system.

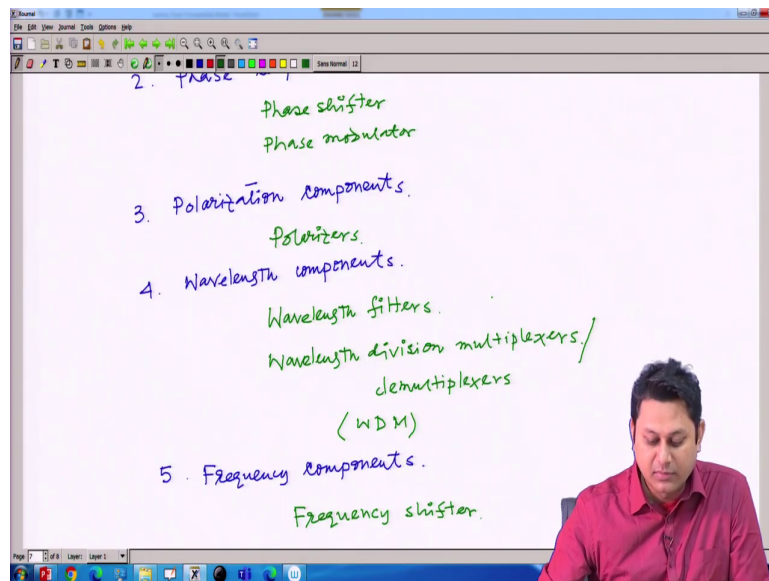
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For example, like amplitude or intensity components, what are this amplitude like coupler very important we will going to study this coupler in this course in detail then splitter, then amplifiers etcetera. Also we have a different kind of phase components like, phase shifter if I want to shift the phase then phase modulator if I want to modulate the phase.

Then, the polarization component if I want to play with the polarization of the light so, polarization components under polarization components we have mainly the polarizer's which

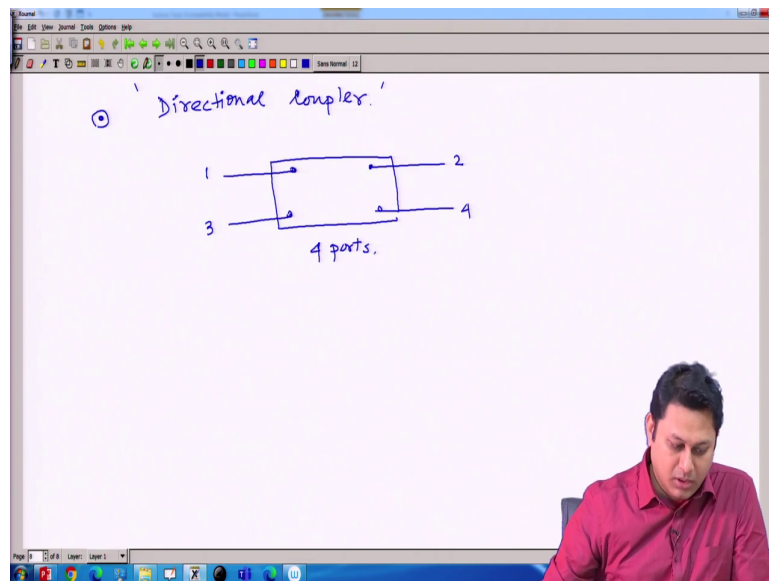
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Then, I can have say wavelength based component like wavelength filter wavelength filters or wavelength division multiplexing wavelength division multiplexing is a system where you can launch a different wavelengths and then the output this wavelength can be separated out. Wavelength division multiplexers or de multiplexers in short it is called WDM.

Also we can have like some frequency components like frequency shifter. So, the least is large so, I am just giving you few of the names of the components that one can think of using the fibers. Now, we will going to choose one very important component out of that and try to understand the physics behind that, which is called the directional coupler.

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So, next try to understand the physics of the working principle of say directional coupler. So, definitely the name suggests that some coupling is associated with that and this coupling is essentially the light coupling so, I can couple the light from one waveguide to another waveguide and as a result we can have we can use this kind of this kind of system to different applications; and we will going to discuss this application in brief here.

So normally what happened here, we have 4 ports like this, suppose I have a black box and I am having 4 ports 1 say 2, say 3, 4. So, I am having 4 different ports here and I can use this 4 ports for different applications. So, 4 ports one kind of application is I launch a light here in one and I want this light to come in this 4 port so, this is called the switching.

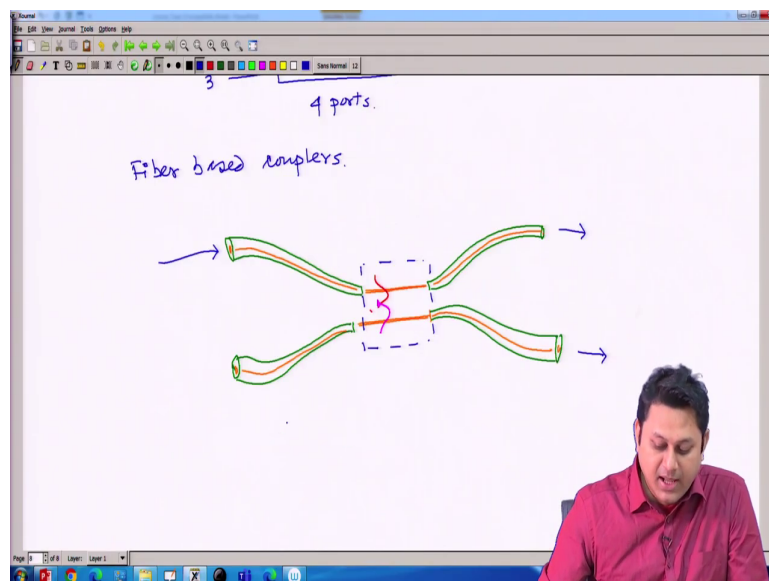
I will launch a light here with two different wavelength λ_1 and λ_2 ; and I want these 2 light to be separated in these two ports 2 to 4. So, I can have λ_1 here λ_2

here. So, this is also called this is basically called the wavelength division multiplication that I just mentioned the last page and so on.

I launch a light here with the power p_0 and I want this power to be splitted in these two branches 2 and 4 and it can be splitted according to my choice; one can split this light into 50-50 one can split this light to 30-40. Whatever the ratio they want they can split. So, many application one can think of with these kind of systems, which in general called directional coupler.

So, we will going to understand the basic physics and what is the mathematics or what is the physics associated with these kind of operations.

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Well, normally the most popular one is the fiber based couplers. In the fiber based couplers what happened that ok let me draw quickly that I have a fiber like this cut here and inside that I have a core so, this is basically my core of the fiber and I have this and then again from here.

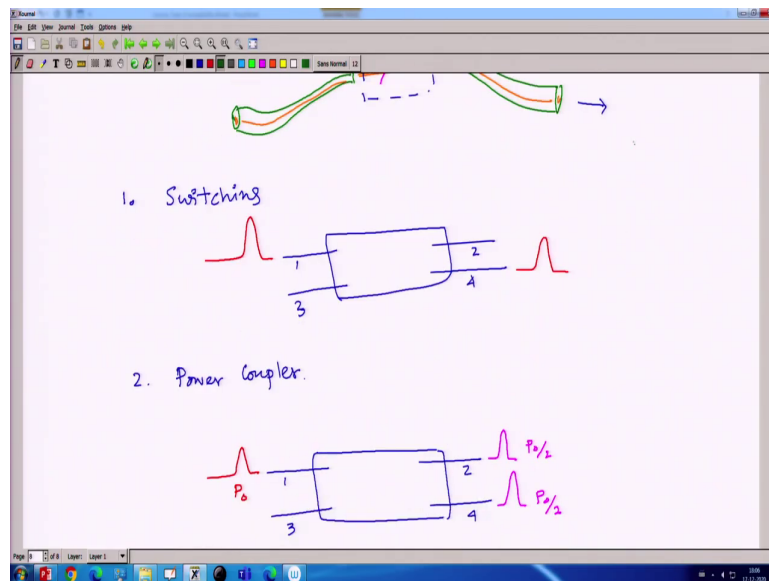
So, I am drawing this is roughly the structure of a fiber base so now, I have these two fibers and these two fibers is this is the core part and this core part is placed very close to each other; and the cladding is defined here with the green one. So, the cladding can be removed from the fiber and after removing the cladding I can put these two fiber very close to each other, so that the modes can talk to each other.

So, this fiber and this fiber are placed very close to each other. So, what happened that if a mode is distributed here like this and another mode which is distributed here, let us put in a different color say this one. So, there is overlapping between two and they will going to there is a crosstalk between these two so, they will going to talk to each other..

If they are talking to each other, so there is a possibility that there will be some exchange of information from this to that in terms of energy in terms of wavelength so, that we will going to understand. So, this is the principle. So, this region is region where we have the overlapping of mode and as a result if I launch a light here.

So, what happened that something will happen in these two ports? Either I can distribute the light I can or I can distribute the wavelength associated with the light in two different ports etcetera. So, I have four ports here also like I showed here and then I can distribute the lights in this way. So, that I can have the exchange of energy here or exchange of information here.

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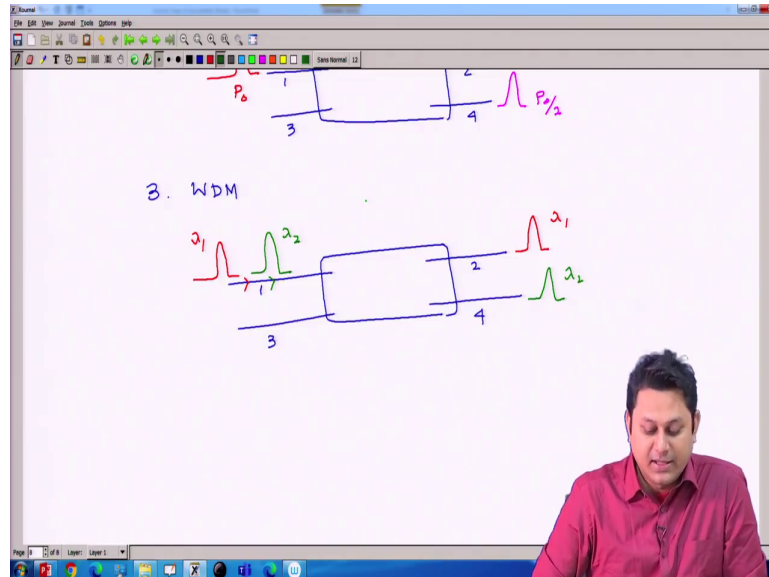
So, few applications let me quickly point out that how many what kind of applications one can think of so, first is say switching. So, this is the port this region dotted region and I have this one and I am launching a light here, this is 1 this is 2 say no this is 2 this is say 3 according to our notation and this is 4 and the light will come here.

So, I launch the light in port 1 and in the output the light is coming into port 4. So, this is called the switching so, I can switch the light. Next application one can think of is the power coupler. In the power coupler if I want to distribute the power from one port to another port that also we can do.

So, these are the 4 ports 1, 2, 3, 4; I launch a light here with the power say P_0 and in the output I can distribute this power according to my choice. So, in both the ports I now have lights and here I have light say $P_0/2$ by 2 here I can have the light. So, I can distribute the

power in a 50-50 distribution this is called the 3dB coupler, where I can distribute the light in a 50-50 portion in 50-50 ratio.

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Next one can think of WDM already I mentioned that very interesting application one can think of in WDM what happened this is the port again I have 4 ports here, I launched two light here with two different frequencies ok let me draw that in different color. So, I have lambda 1 and I have lambda 2, 2 light are passing through this system.

So, this is lambda 2 and this is lambda 1. So, I can make an arrangement with this coupler so, that I can distribute the light here in this way. So, in this port I can extract the lambda 1 and in this port I can extract the lambda 2. So, this is so you can see the structure is very very important here that it has to be very very close these two point has to be very very close so, that we can have the exchange of energy.

And when we have exchange of energy different kind of application one can think of like a switching like power coupler like WDM etcetera. In the next class, we will try to understand using a very very important theory called couple mode theory. So, we will try to understand using the couple mode theory how these things is happening the physics behind that.

So, today we do not have that much of time to discuss the couple mode theory. So, in the next class we will discuss the couple more theory to understand the physics behind these processes, how this things are happening in the real couplers. So, with that note I like to conclude my class here.

Thank you for your attention. So, see you in the next class.