Physics of Linear and Non Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 33
Optical Fiber Mode Morphology

Hello, student to the course of Physics of Linear and Non-linear Optical Waveguides. Today, we have lecture number 33 and in this lecture, we will continuing with Optical modes and try to understand the Mode Morphology. So, before going to the topic, let me once again remind what we have done in the last class.

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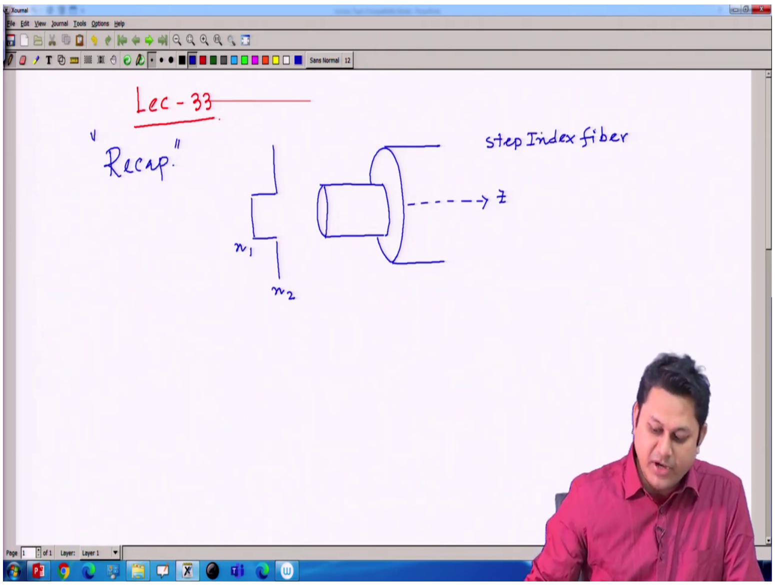

So, it is recap, a quick recap. So, this is the structure of an optical fiber, this is the core part and this is the cladding part, this is the geometry, this is z, co-refractive index is n 1 and cladding is n 2 for step index fiber. So, this is a step index fiber. So, we try to find out the modes in step index fiber which is a simpler one.

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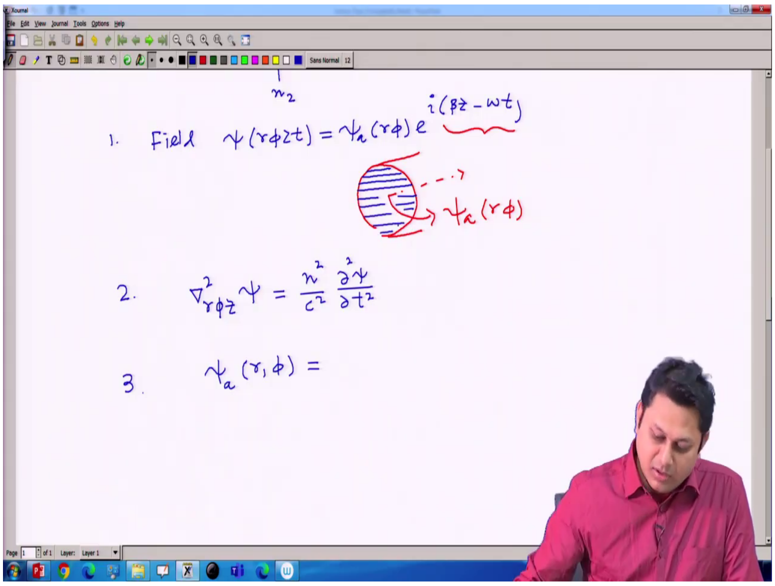

So, 1st if you remember we define the field as a function of r phi z and t as this that was our field where r phi is a distribution in this region; that means, in the cross-sectional region. So, if this is the cross section, so, at this region so, this is the fiber core. So, the field is defined a is defined in this region.

And another part which is propagating through the fiber along z direction and that is taken care of this part. In the 2nd point, we going to use this x this equation which is essentially the wave equation with the field given field. Next during the wave equation, when we try to evaluate this differential equation.

We need to find out the solution and in order to find the solution for this distribution which is over this region as I mentioned in this region in the spatial distribution if I want to find out which is essentially called the mode. So, in this region I divided this function as this by using the separation of variable method.

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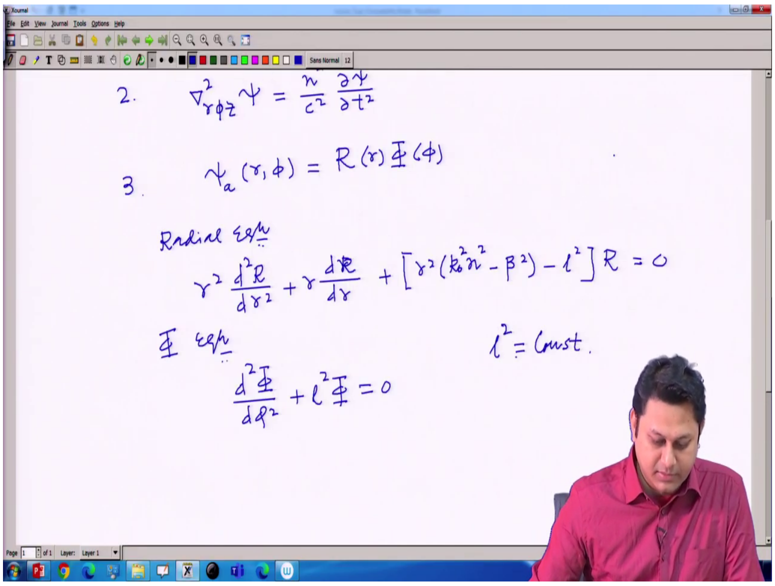

This is a standard method to solve the differential equation and I separated this function in this way. After separating this function we had 2 equation one is the radial equation is the radial differential equation having the form this this is r dR and one was the phi equation or the azimuthal equation which was simply this, where l square was some constant.

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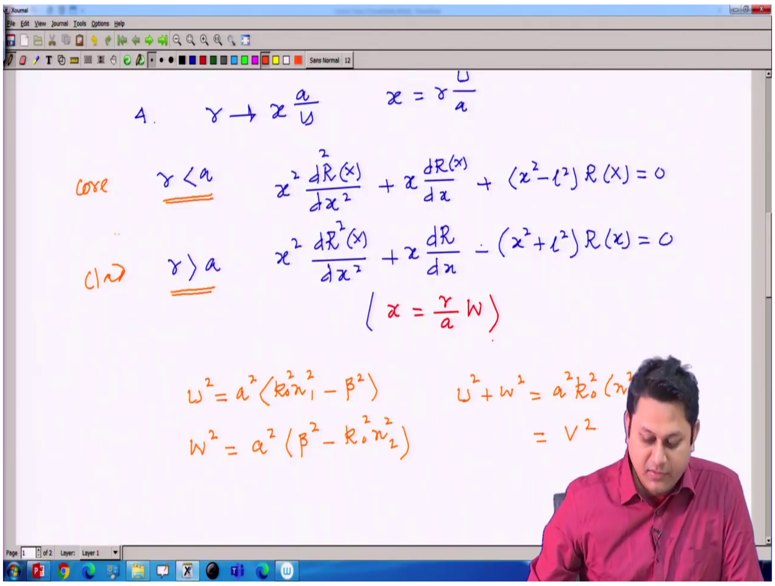

Next after that we make a rescaling of this parameter like this such that my x become r U by a and after doing this rescaling for r less than a we had a well known differential equation of the form like this and for r greater than a we had almost a similar kind of equation, here we had a minus sign. So, only difference is coming in this portion.

Here, mind it, my rescaling factor x was r by a W, U and W are 2 parameters important parameters let me define it here U square was a square k 0 square n 1 square minus beta square and W square was a square beta square k 0 square into square such that U square plus W square if I add these two term, then it should be simply a square k 0 square n 1 square minus n 2 square which is our V parameter I should write it V square that was the structure we had.

After that after that we had the total solution. So, this is for in the region when r less than a and we had a basal equation and for r greater, than a this is the this is in the region of core and this is in the region of clad. So, in the core region we have a basal solution of the first kind and in the clad region we had a basal solution modified basal solution.

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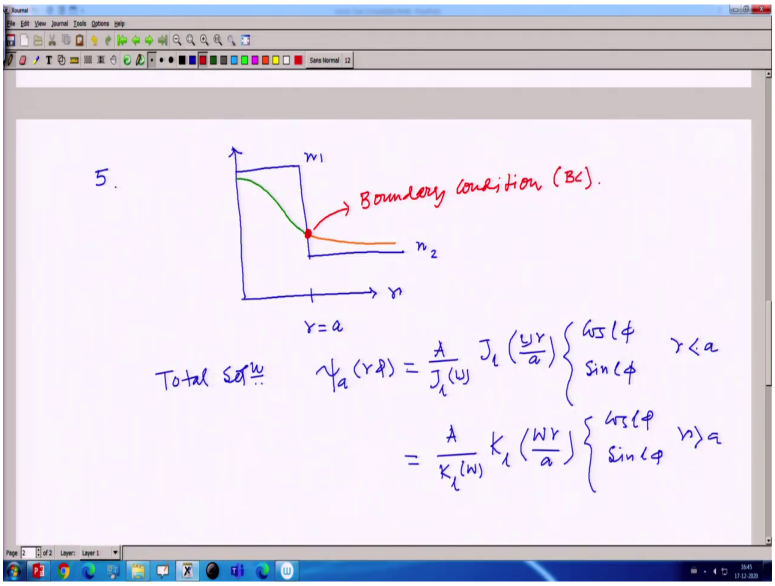

And, if I plot this solution then as 0.5 we did it. So, this is the fiber structure. We had a refractive index n 1 here and refractive index n 2 here. This point is r equal to a and along this direction is my r and the field is something like this. This is for equation 1 and another part this is for equation 2. Equation 1 means here this one first x square d 2 R dx square this equation the solution is given here, the green one and the orange one is a solution of this.

So, these 2 solutions should have a matching point here, this is the boundary condition. So, in general the solution was written the total solutions were psi a r phi was A J l U. And, then J l Ur by a then cos l phi sin l phi that was the solution we had in the co-region.

So, when r is greater than sorry r is less than a, r is less than a and another solution was this when r is greater than a. These are the 2 solutions 2 set of solutions we had for core and cladding part. Next we put the boundary condition as I mentioned here in this point we had a boundary condition.

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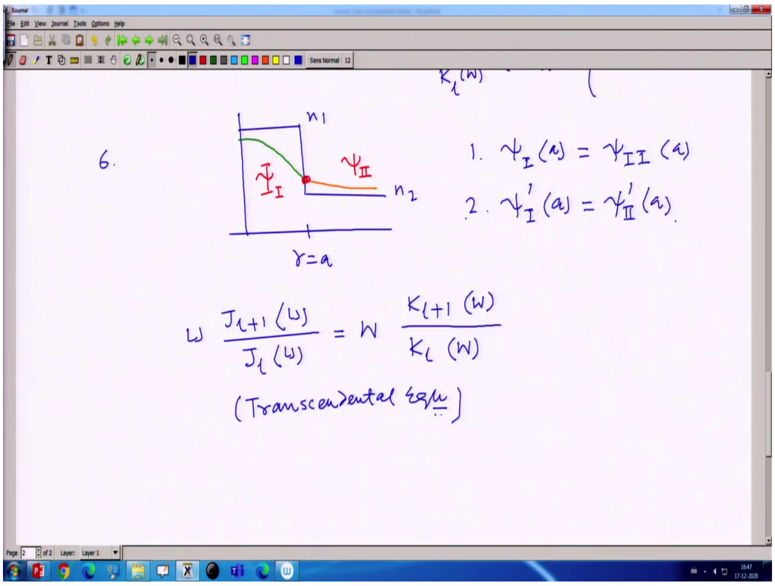

So, this is the point where we have a boundary condition. So, the boundary condition tells us so, we had a boundary condition here. So, the boundary condition 0.6 tells us so, quickly let me draw this n 1, n 2. Here we want to find the boundary condition. The solution here say psi 1 and the solution I just write psi 2 and the boundary condition was two boundary conditions one we already take account with these solutions that a which is the value of the two function at this point r equal to a small a.

So, the 1st boundary condition is the function in the co-region and the function in the cladding region has the same value at the boundary. So, that is one and 2nd boundary condition is the derivative of that stuff at a and these two matches. So, if I put this boundary condition.

Then we eventually have a transcendental equation of the form like this. In order to find this transcendental equation sorry, it should be W. In order to find the transcendental equation we use some standard relation we call this recursion relations and we get this. This is the transcendental equation.

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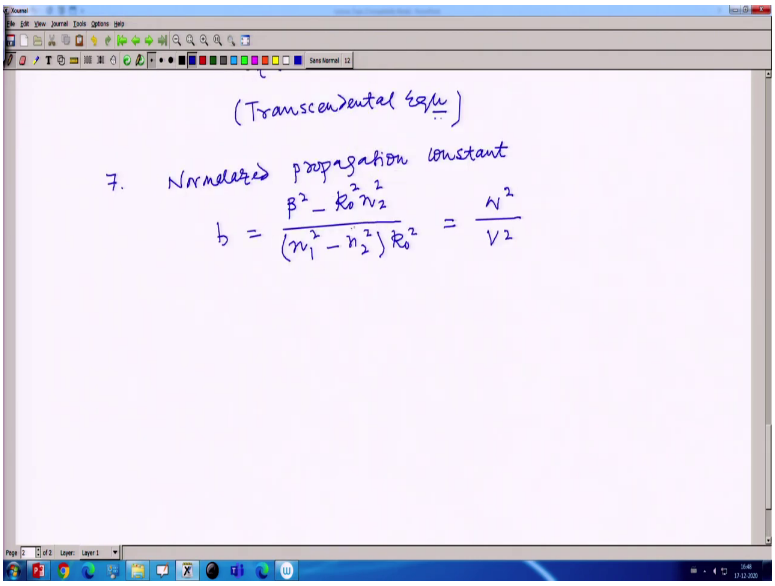

One can have a solution graphically for such equations. Then after that we modified this transcendental argument of this transcendental equation by introducing the normalize propagation constant. We always use this normalized propagation constant as due to the convenience because we know. So, this value was beta square minus k 0 square n 2 square, then n 1 square minus n 2 square k 0 square which is eventually w square by V square.

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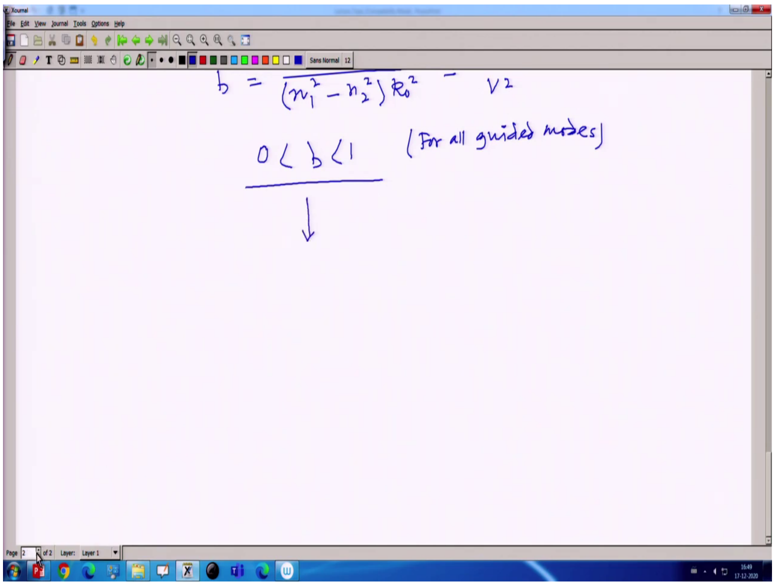

And, using this normalization, so, b has a restriction and it can be in between 0 to 1 that is the greatest advantage. We have for b this is a normalized value and it can vary for all guided modes for all guided modes it varies from 0 to 1. And, then using that we eventually figure out that V, the transcendental equation now one can write in this form um.

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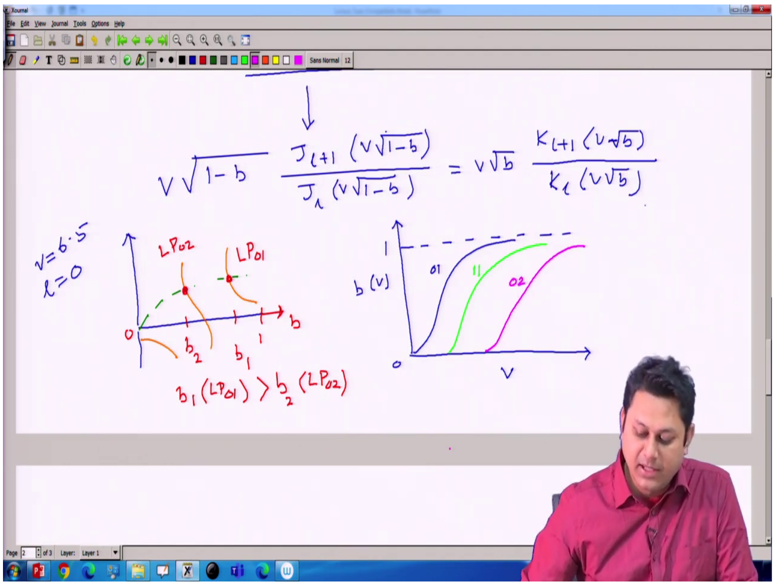

This form, where if I want to find out the solutions for transcendental equations the solutions for example, V is equal to 6.5 and l equal to 0, for these two values given values we plot the transcendental equations plot the left hand and right hand side. So, the value of b is 0 here and 1 here.

So, along this direction I measured the value of b and we had certain cutting points and it was something like this and these are the points of the solutions. This is for example, this is for LP 01 mode.

This is for LP 02 modes and I we find certain value of b, say this is b 1 and this is b 2; obviously, b 1 is for LP 01 mode which is greater than the value of b 2 which is for LP 02 mode. And, from this b 1 and b 2 values I can eventually find out this beta which is our propagation real propagation constant this beta we can figure out.

Also we can plot we can map all the modes by plotting V as a function of b as a function of V. So, b can be in between 0 to 1 this is b as a function of V and along this direction I just plot V and we find that the curves are like this. This is one curve, this is another curve, this is another curve and so on.

The first set of curve is for say LP 01, the second set of curve say LP 11 and each set of curves is basically defined is defined for a specific l and m values in general it is LP lm.

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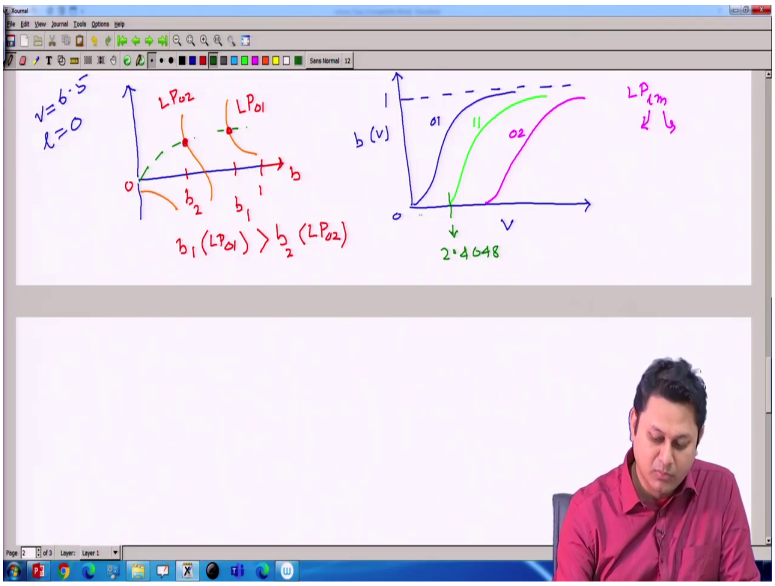

l and m are these two values I am putting here.

Here at this point I had a cutting point and for step index fiber this value is around 2 point 2.4048; that means, that if the value of the V is less than 2.40 4048 then we have only one mode and this is this one and then it is called the single mode factor. So, this we also discuss.

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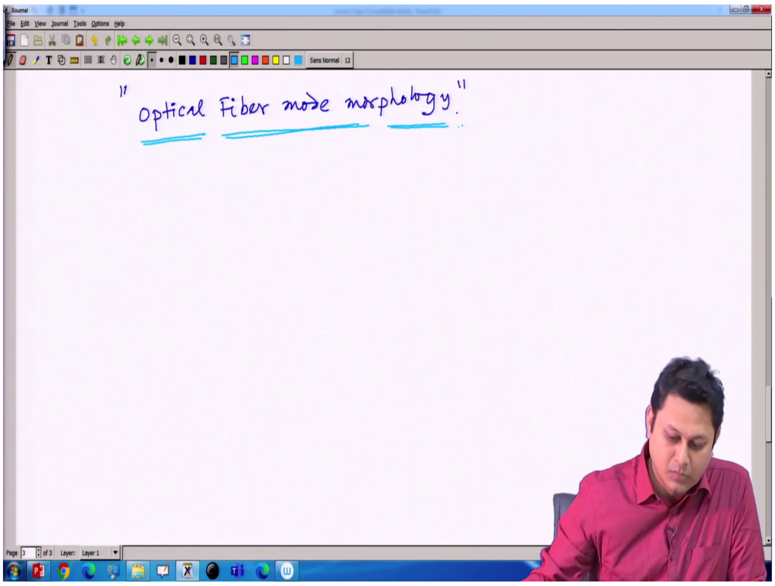

So, today we will now today we will going to find out the optical the optical fiber mode morphology. How the mode will going to distribute, how it look like we should try to find out some theoretical understanding behind all this mode distribution ok. So, optical fiber mode morphology.

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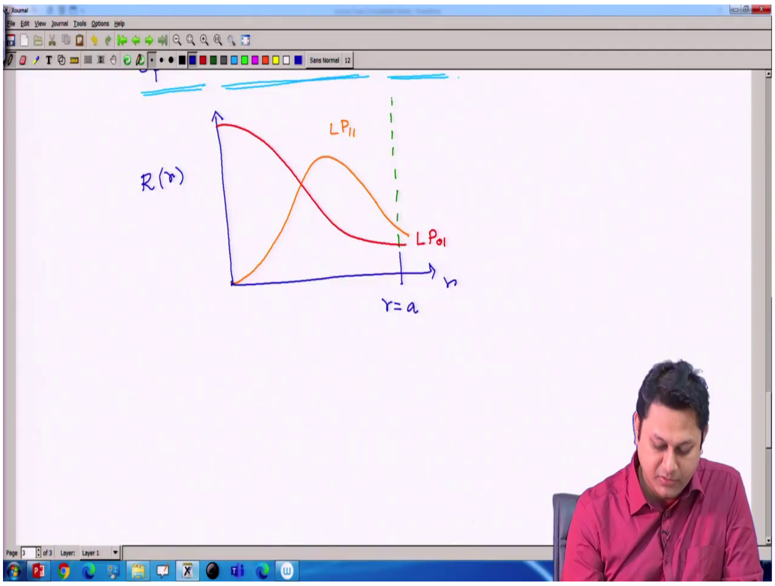

Well, if I try to find out what is happening in the core region, so, I should restrict. So, along this direction suppose I have r this is r equal to a core cladding boundary. So, let me put this boundary properly. So, this is the boundary and in this bound in this region if I try to find out the different modes.

So, the LP this is LP 01 is a basal function and the basal function look like this. For LP 11 the structure will be slightly different and it should be something like this this is for LP 11 in this way if I increase the value of the L, then I will going to find out these kind of structures inside the core.

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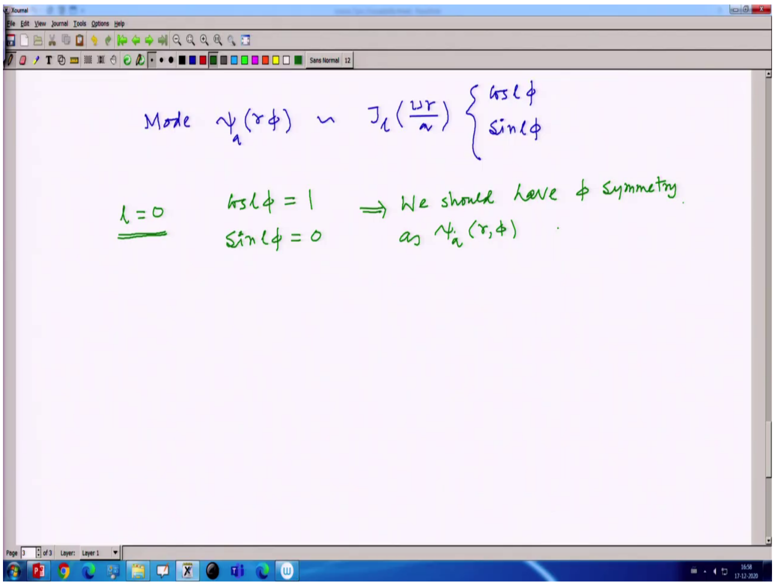

So, I am basically plotting the R as a function of r the radial part I am plotting. Also, we have a phi part, so, that will going to understand. So, in general the mode psi which is a function of r phi is of the order of say J l I just remove the constant which is here sitting here a divided by J l U. I am not putting right now because it is not that necessary to understand qualitatively the mode distribution. So, only the variable part I am putting here which is r associated with r and phi.

So, this is the structure this is the solutions of the mode. So, the mode should be in this way if I try to find out the distributions. Now, let us find out what happened for say l equal to 0 the simplest case l equal to 0.

When I we have l equal to 0, so, from this part we can see that cos of l phi will be since l equal to 0 so, will be 1 and sine of l phi will be 0. So, only cos of contribution of the cos of l phi will come when we put l equal to 0 and there is no phi dependency here at all.

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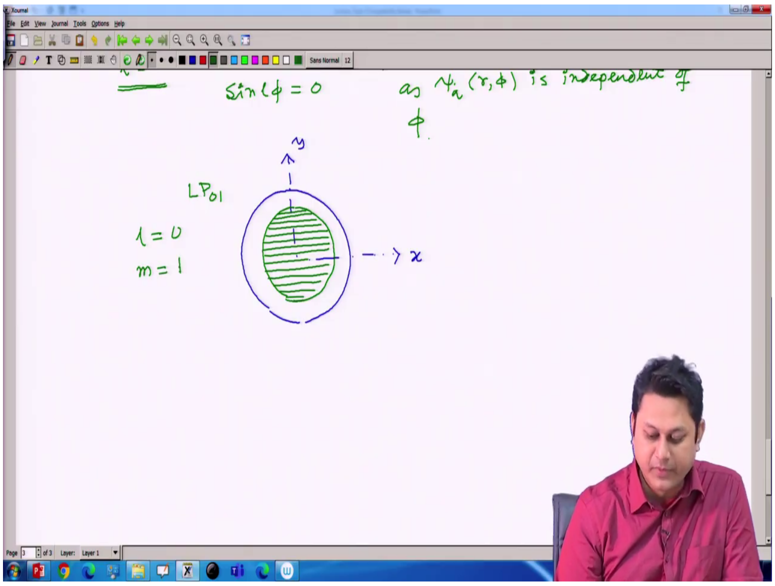

So, from that we can say for l equal to 0, we should have a phi symmetry as psi r sorry, it should be psi a according to our notation psi a function of r and phi is independent of phi. So, there is no phi dependency.

Now, under this condition if I put this is the core part of the fiber this is x, this is y and a field will going to distribute it inside the core like that this is the distribution of the field for l equal to 0 and m equal to 0 that is the l equal to 0 and m equal to one that is the first mode I am talking about. So, this is the first mode.

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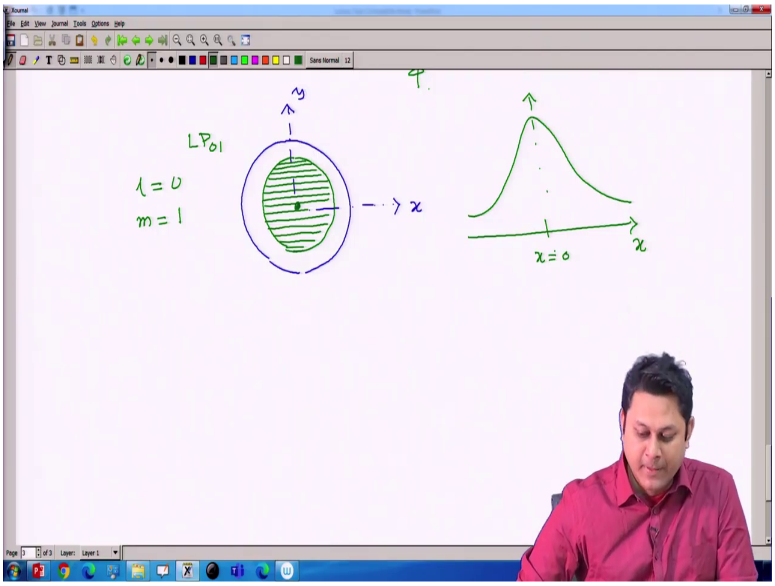

This mode is LP 01; that means, l equal to 0 and m equal to 1. If I want to find out the distribution over x-axis, this distribution will be something like this over x-axis. So, the intensity of the mode will be maximum here in the axis and then it goes like that. So, this is at x equal to 0 point. So, this is the distribution we have.

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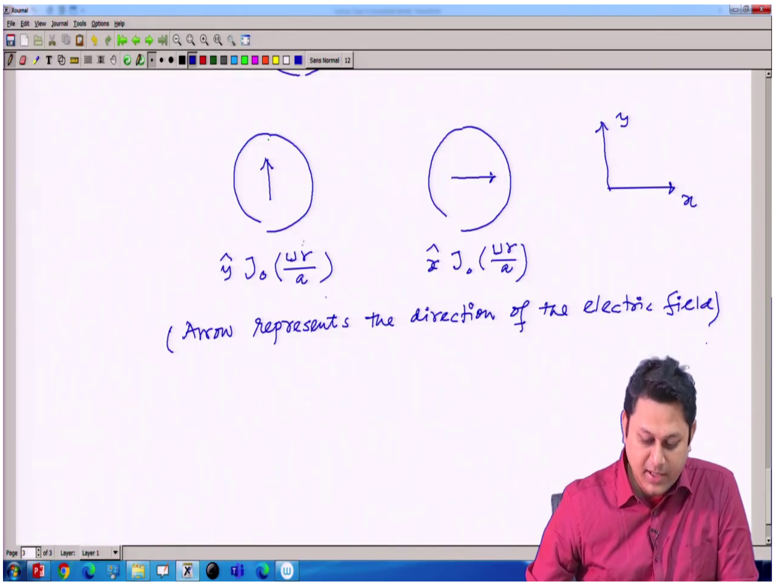

Now, also we can have the polarization or the electric field variation inside the mode and the electric field inside variation inside the mode if I try to find out along this direction and for x polarization mode, this will be around this direction when my x and y is already defined x and y.

So, for y polarization mode the electric field will vibrate in this direction and for x polarization. So, arrows basically arrow represents the direction of the electric field then. So, electric field inside the fiber is vibrating and the direction of the electric field one can find out in x-axis and y-axis it should be vibrate in this way for fundamental mode.

This is called the fundamental mode or LP 01 mode. So, today I like to conclude my class here because I do not have that much of time. So, in the next class we will continue with this mode morphology, try to understand more about the higher order modes which are rather interesting and try to find out how it depends on phi.

Please note that since l equal to 0, we do not have any phi dependency here, but as soon as I put the value of l equal to say 1 then the contribution of these phi through cos and sine will come and then we have a different situation and we should have a different kind of distribution. So, this kind of distribution we will going to discuss in the next class.

So, till then good bye, and thank you for your attention.