Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 30
Modes in an optical fiber

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number 30. And, today we will going to study a very important topic, which is Modes in an Optical Fiber.

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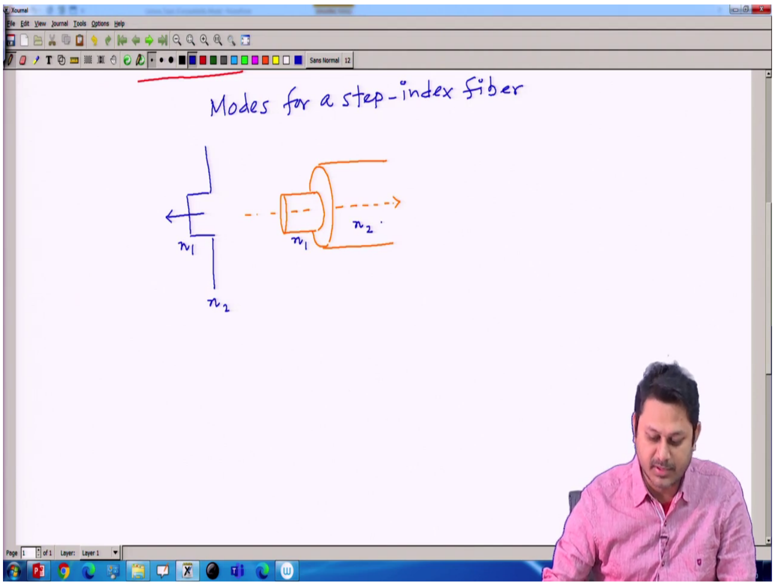

So, today we are going to learn modes for a step index fiber. So, far we are dealing with the modes for planar waveguides. So, today we will going to study the modes for step index fiber.

So, let me first draw, the structure of the optical fiber. We had a core part surrounded by a cladding. This is the structure geometrical structure of a fiber, where the refractive index profile is this first step index.

So, this is n 1, n 2 and the refractive index is n 1, this is n 2. So, we have a cylindrical symmetry of this geometry. And, we are going to use this cylindrical symmetry in the Maxwell's equation to find out the modes that is all.

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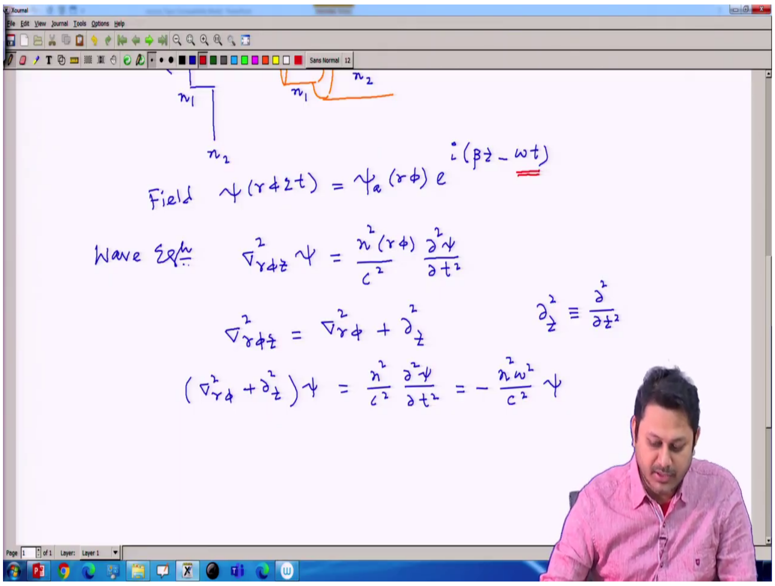

So, for this kind of system if I write the field, it should be psi r phi z t it should be a function of r phi z, because I am dealing with now a cylindrical symmetry. It should be divided into 2 part 1 is psi a, which is dealing with the r 5 part and a propagation part as usual with beta z minus omega t.

So, I have r phi dependency here and z and t dependency in the exponential term. So, this is a definition or this is a form of a optical field, that is going to propagate inside the fiber, which has a cylindrical symmetry. Now, we have the wave equation, which is of the form this. Now, here this operator the Laplacian operator, I should write in cylindrical coordinate that is something we need to take care.

So, if I write this operator it should be r phi plus this is a shorthand notation. So, if I write this; that means, I am eventually doing the partial derivative with respect to z like this second order partial derivative. Now, if I going to operate these things over here. So, I will have this quantity dels del square del t square psi, because I know what is my psi.

I can write it as minus of n square, omega square divided by c square and then psi. Because, when I make a double derivative of this given field, the term here sitting should come out the omega should come out as minus i omega twice I should have a negative sign. And, it simply n square, omega square divided by c square and then psi.

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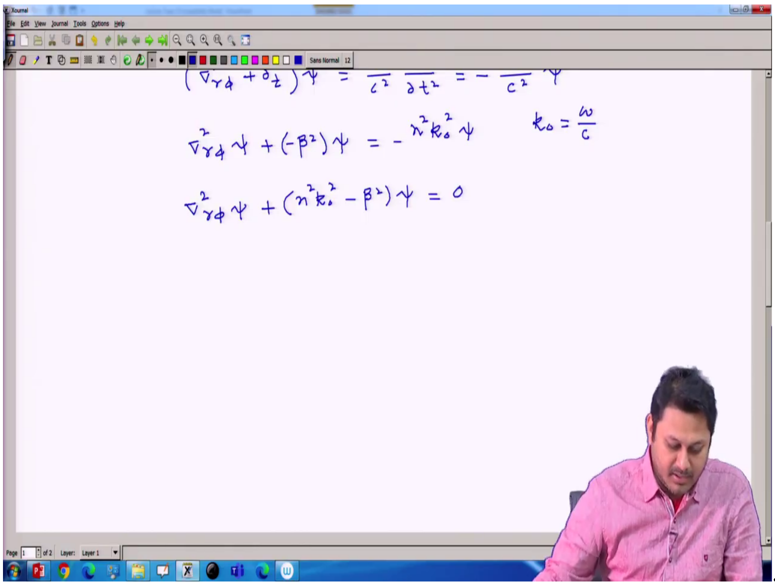

And, the first two part is simply; this portion I have a second order derivative with respect to z and I know this is my field.

So, if I do it should be minus of beta square psi is equal to minus of n square k 0 square psi, where my k 0 is omega divided by c, free space propagation, free space propagation constant or wave vector. So, eventually I have this, this is the equation I am right now having. Now, I will I can write this part in x in a expanded form.

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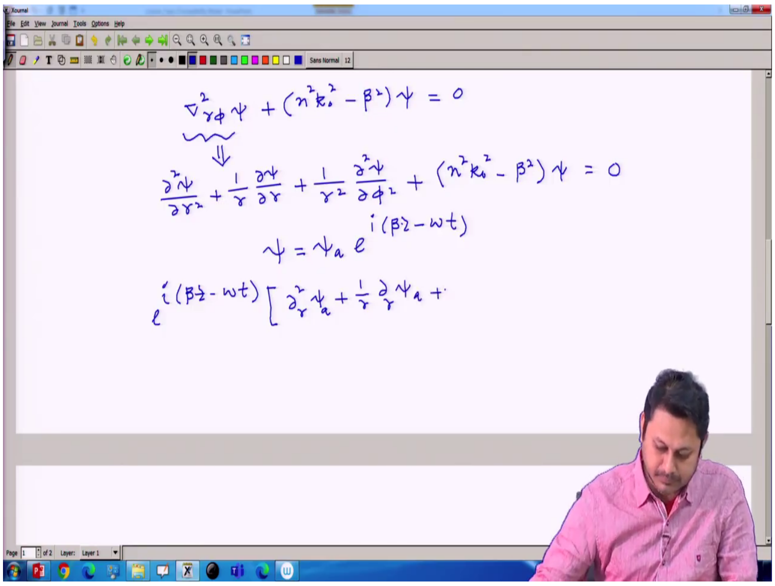

So, in expanded form it should be del 2 psi del r square plus 1 by r del psi del r, plus this is only this portion is this one, plus the rest of the term that we already derived ok. Now, psi is something like psi a e to the power of i beta z minus omega t, already we defined that. So, if I put this value here, then you can see e to the power i beta z omega t does not have any dependency of r and phi and here it is only psi.

So, I can write this equation in terms of psi a. So, it should be simply e to the power of i beta z minus omega t, and then it should be this plus, whatever, I have.

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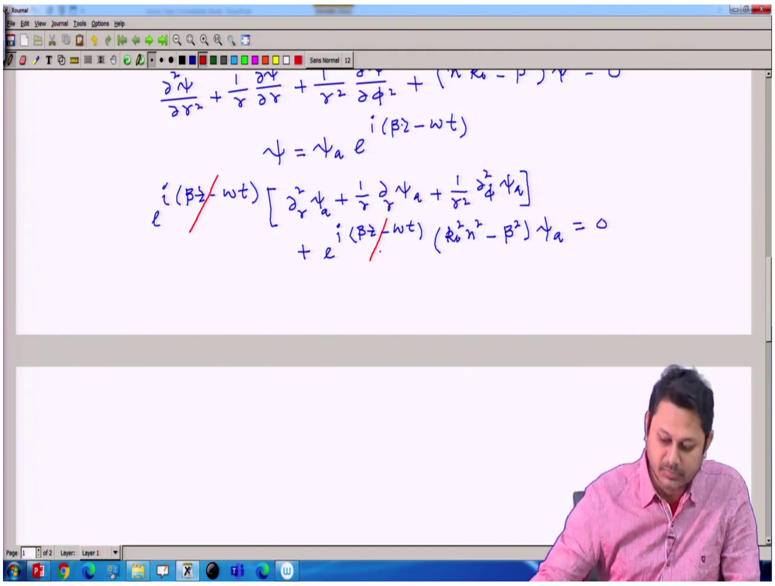

So, simply this term and this term will cancel out. And, eventually we have this ok.

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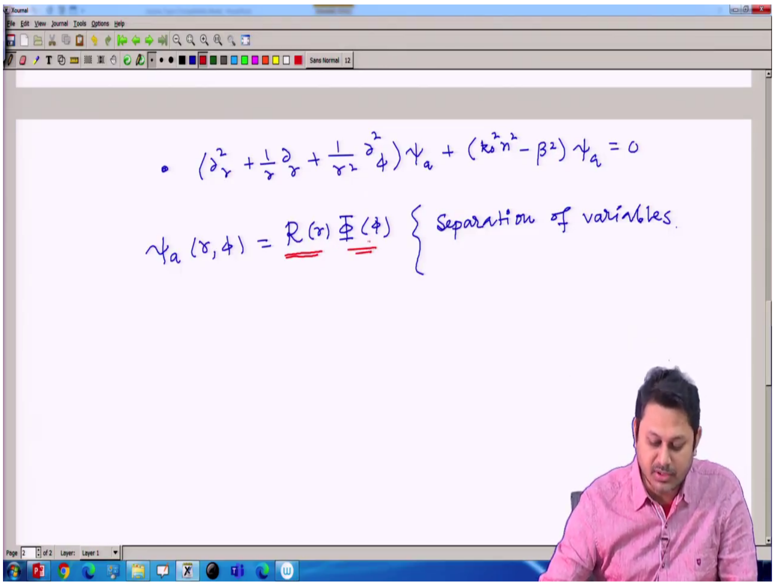

This is the expression; I have now right now. Well, I can now make a standard procedure which is called the separation of variable, that it is a function of r and phi. So, I can make it as a function of R r and big phi phi, which is called the separation of variables. I am making a separation of variables of that.

If I now have the separation of variable by making a function of R and phi like this and put this equation put this solution in the equation whatever we have. Then it is easy to show that I will have an equation like this.

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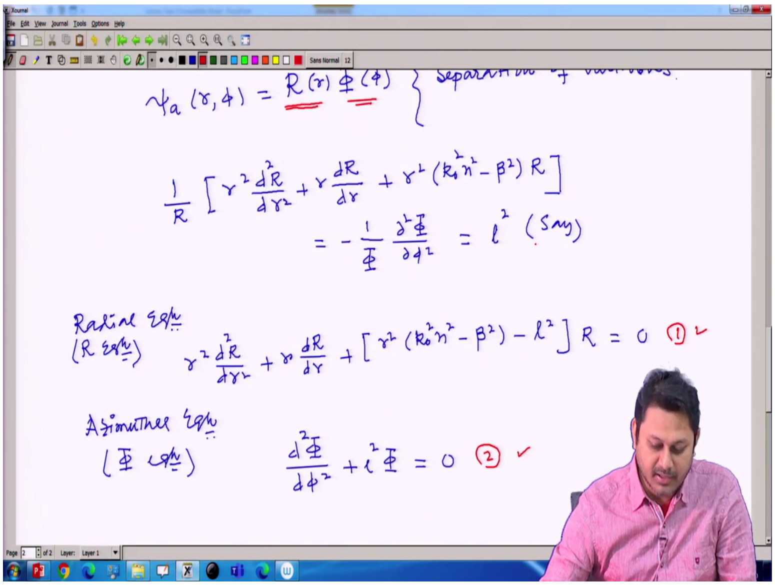

This is equation with respect to R and in the right hand side I can have an equation with respect to phi only. Now, the left hand side of the equation is now as a function of R and the right hand side is a function of phi only.

Now, these two are equal; that means; obviously, they should be equal to some constant and I write this constant at l square. I write this in as a l square. Because just after few minutes you are going to find, that it will form a very well known differential equation.

If, I write this constant at l square only for that, because it can in principle it can be any constant. So, I write this constant in spatial form to have a spatial differential equation in our hand.

So, now I have the radial equation; that means, the equation related to r as r square d 2 R d r square so, only the only the left hand side I am writing. Plus r d R plus R square k 0 square n square minus beta square, then minus l square multiplied by R is equal to 0 ok. This is my left hand side I multiply this R here.

So, then put it back and then I am having this equation. So, this is the equation radial equation I am having. On the other hand the Azimuthal equation or phi equation, this is R equation. So, the phi equation should be something like this, d 2 phi d phi square plus l square big phi is equal to 0.

So, now I have 2 equation in my hand, this is the radial equation and this is the phi equation or Azimuthal equation. And, if I solve this 2 equation, then I can have the value of R and phi. And if I have the value of R and phi, then I can eventually have the value of psi a, which is the field distribution. Well, now I need to put something some condition, because in our case we have the step index fiber.

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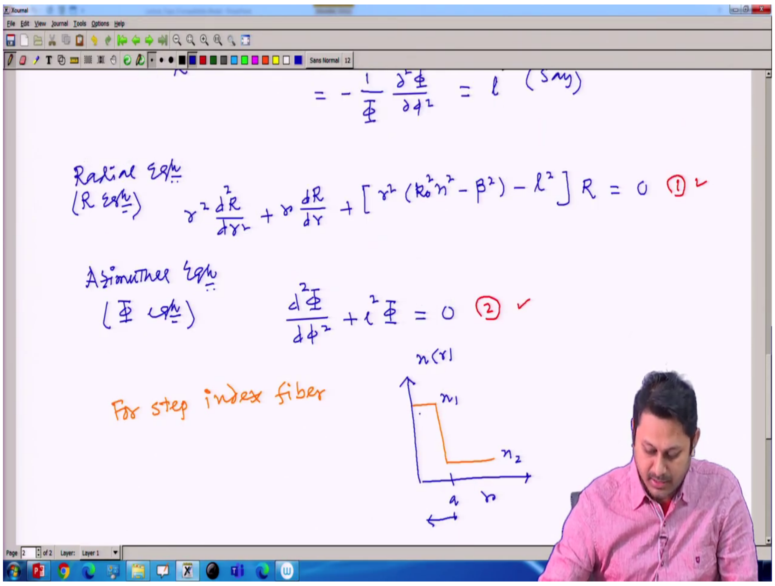

So, I we have so, let me write it for step index fiber, what is the geometry of the step index fiber. So, if I now ok, let me draw in a more correct manner, because, now I am dealing with r so, beta to remove that this portion. And, so this is the refractive index as a function of r, this is my r, and this is a, this distance is a, which is the core part.

And, the refractive index is n 1 here and n 2 here. So, for step index fiber, this is the refractive index profile. So, once we know this is the refractive index profile I will going to put this information here in this equation.

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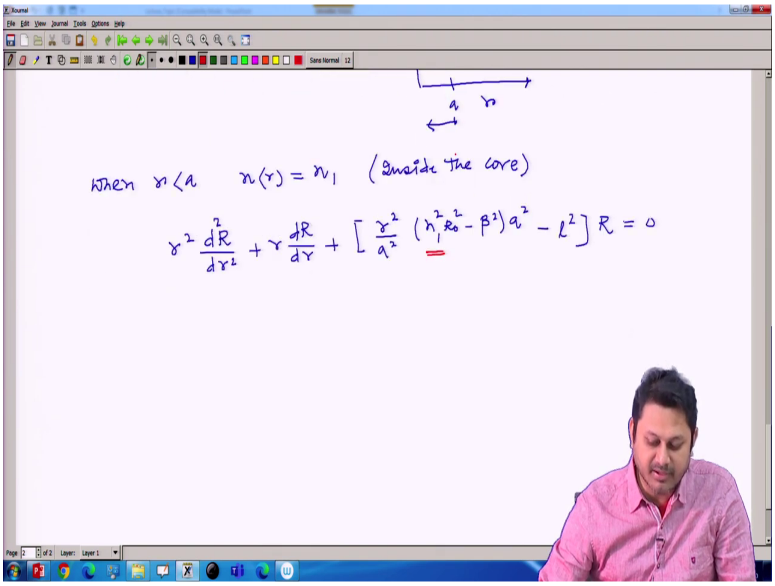

So, when r is say less than a, function of n r is simply n 1 a constant so; that means, this is inside the core. So, inside the core region I have this and I will put this into equation. And, if I put this into the radial equation it should be simply this. Now, I write here this term in this way, note it. I will put the n 1 here, because my I know in this equation, this n is now replaced by n 1.

And, I write this r square multiplied by k 0 square n square minus beta square as r square divided by a square, and then I multiplied another a square here. There is a specific reason for that it will make my calculation a little bit simpler.

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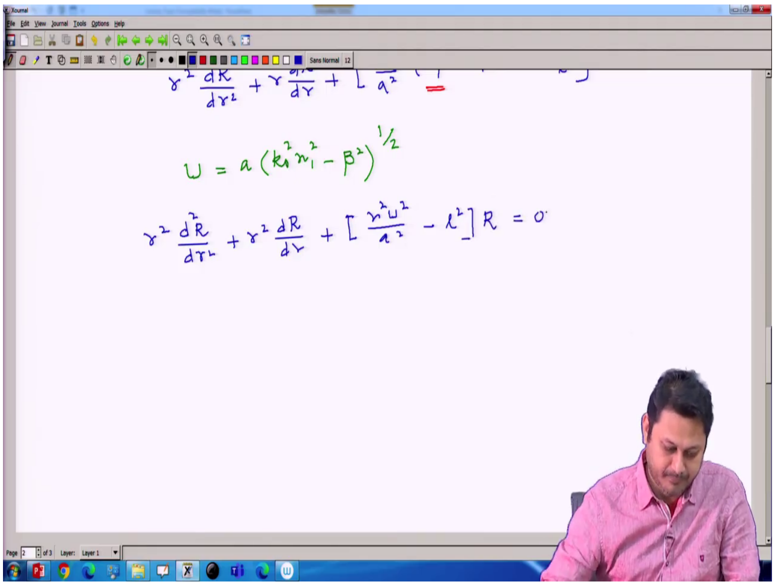

Now, we introduce a variable, a parameter, a u which is a into k 0 square, n 1 square minus beta square. Note it U is a constant and a dimensionless quantity, so, better to write it is to the power half. So, this is something which is dimensionless. And, if I now write this equation in terms of U, then I should have this.

So, r square d 2 R d r square plus r square d R plus this portion you can see this is my U square. So, I can write it as r square, U square divided by a square, and then minus l square R equal to 0.

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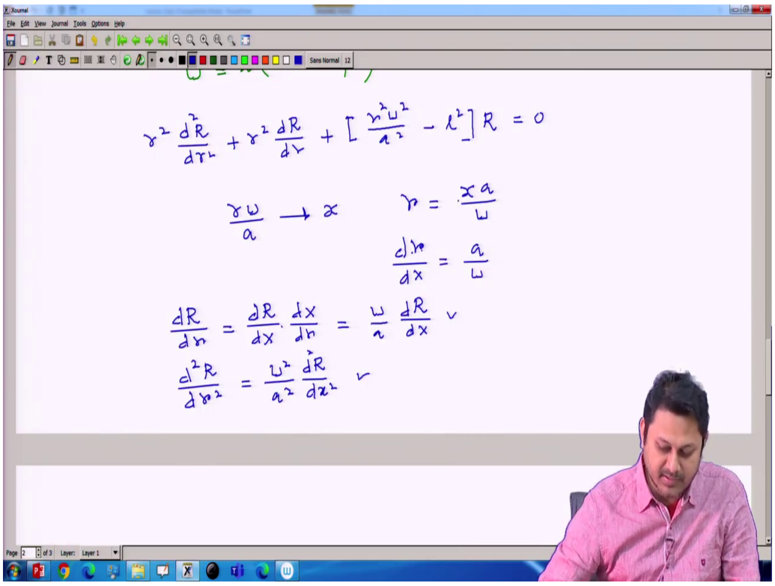

This quantity r U divided by a we write it as x I rescale it as x, then my r becomes simply x multiplied by a U. So, my d R d x should be equal to a by U. So, d R d r, so this is a variable of r, now I change this variable to x. So, this quantity I can now write as using the chain rule as this, which is U by a ok, here it should be a smaller.

So, it should be U by a d big R dx. In the similar way, if I write the second derivative with respect to R, then it should be simply U square divided by a square d 2 R d x square. So, this transformation I have this is one transformation this is another, which I will going to put in this main equation. If I put everything into the main equation instead of r I put x a divided by U, this double derivative with respect to small r I put this value and so on.

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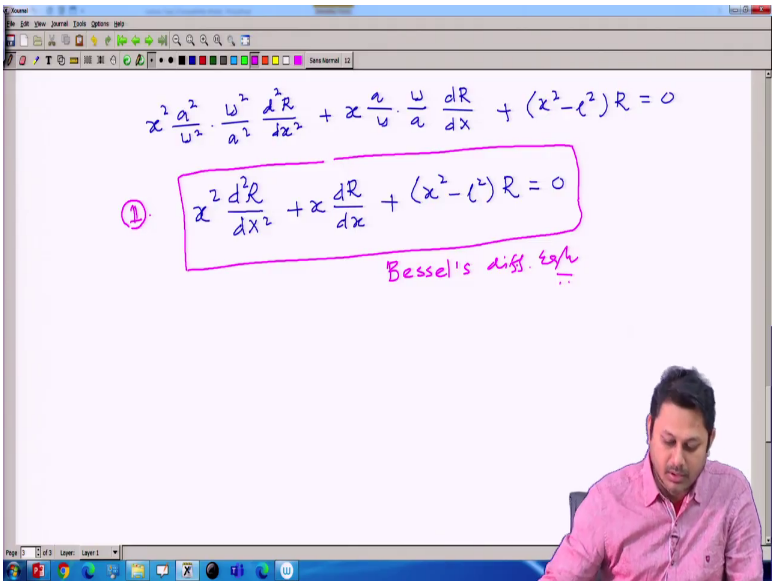

If I do, then I will going to have something like x square a square divided by U square. Then, U square by a square d 2 R d x square plus x a by U, then U by a, d R d x, I am writing all the term explicitly, so, that you can understand what is going on here. And, x square divided by.

So, this is the equation I am having after transferring everything in terms of x. So, you can see this U square a square U square, a square, a by U, U by a this term will going to cancel out and I will have a very well known equation in my hand.

This is a very well known differential equation, and I believe by that time most of you have already recognized this differential equation, this is the Bessel's equation. After putting all this effort, I find the radial equation can be represented in terms of Bessel equation. So, the solution once you have this Bessel's differential equations, Bessel's differential equation.

Once, we have the Bessel's differential equation after putting all these variable and all these terms, then the next thing is will be simpler because we know what is the solution. So, we are going to use this solution directly, because now the solution of the differential equation of the radial part is known that is. So, let me write is this equations equation 1, another part is still there, which is the Azimuthal equation.

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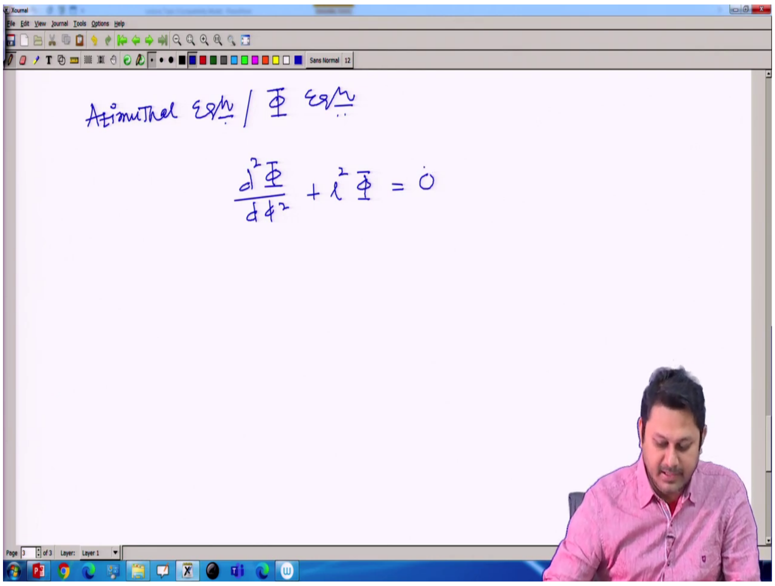

So, let me go back to my Azimuthal equation or my phi equation. This equation is little bit simpler and easy to solve. Because I know the structure of this differential equation and this is this, readily I have the solution and if I write the solution it should be a sinusoidal solution.

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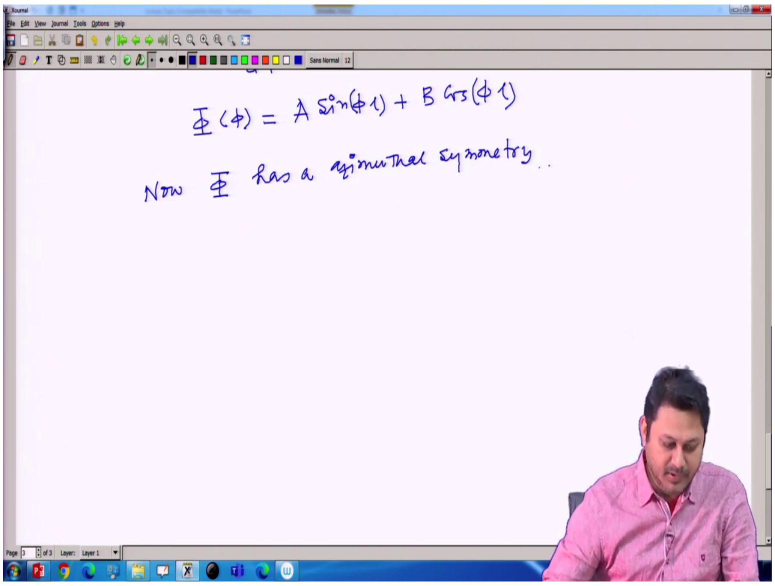

So, it is a function of small phi and I can write it as a constant A, then sin phi l plus another constant B cos of phi l. You can note that this equation has a phi symmetry. So, when now phi has a azimuthal symmetry, what is the meaning of azimuthal symmetry? So, if I look carefully through the fiber structure.

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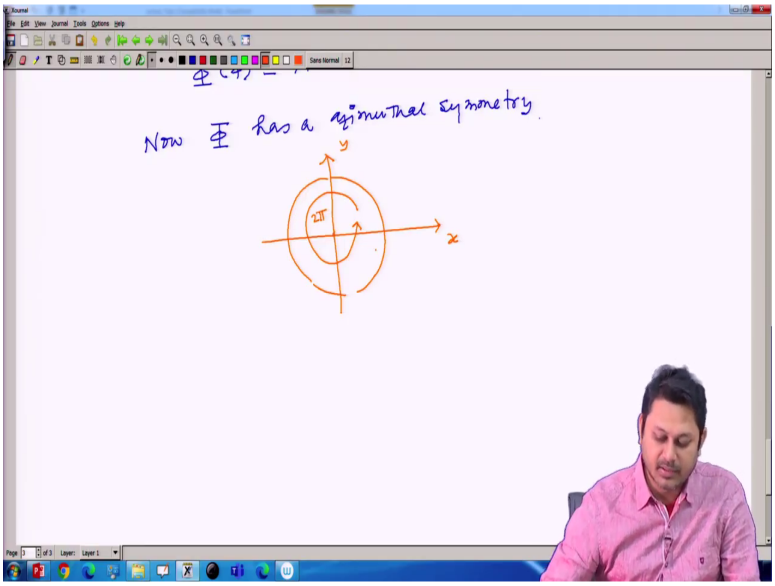

So, the core part if I draw the core part. So, this is the fiber core x and y. And, whatever the solution I have here if I go to 2 pi if I make a round of 2 pi, then the solution will remain unchanged.

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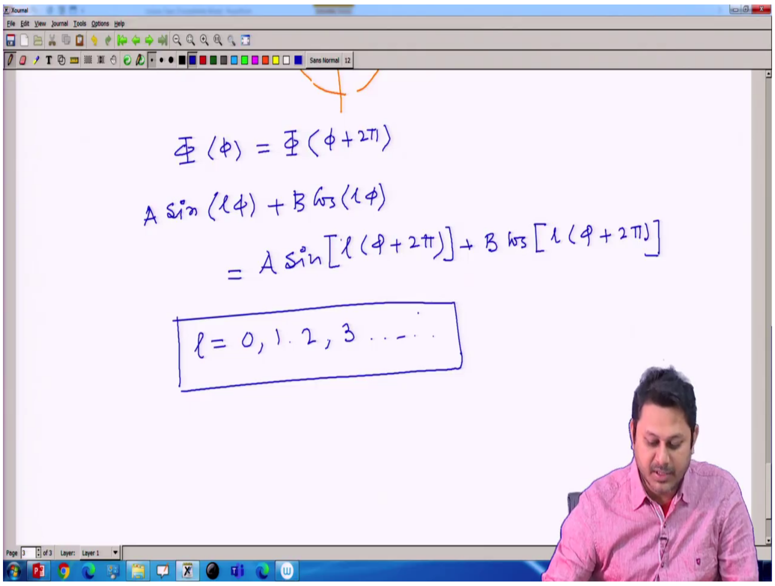

So; that means, phi of phi should be equal to phi of phi plus 2 pi. So; that means, A of sin l phi plus B of cos l phi, should be equal to A of sin l phi plus 2 pi plus B of B of cos. And, it should be, it should be the same thing once we have l is equal to say integer 0, 1, 2, 3 and so on.

If, the ls are integer, then this phi symmetry will follow. So, I have a very important information from this set, that l has to be the integer value having 1, 2, 3, 4. It is restricted to some integer value, discrete integer value, it is not a continuous if this is not a constant with continuous values, it is a discrete values and this discrete values are 0 1 2. So, eventually l is a integer.

Now, I do not have much time to you know solve this radial equation, the next part is I will going to solve this radial equation. So, in the next class we will start from this Bessel's differential equation and try to solve, the equation try to put the solution of the equation which are the Bessel solutions. And, put certain boundary condition to find out that, what should be the form of exact solutions. So, with this note I would like to conclude here.

So, thank you for your attention and see you in the next class for the solutions.