Physics of Linear and Non-Linear Optical Waveguides
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Module – 01
Basic Optics
Lecture – 03
Poynting Vector, Maxwell's Equation in Dielectric Medium (Contd.)

Hello student to the lecture number 3 for the course is Physics of Linear and Non-linear Optical Waveguides. Today, we will going to cover the Poynting Vector and Maxwell's Equation in Dielectric Medium that we have started in the last class. So, this is a continuation of the previous class.

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Also, I like to in this class, I like to mention about the text book and reference reading. As a text book for this particular course we have two books. So, Optical wave guide theory by Snyder and love this is one book, you can read as a text book and another book Introduction to fiber optics by A. Ghatak and K. Thyagarajan very popular book in India.

So, you can also have a look to this book, as a textbook, for reference reading Fundamental of optical wave Guide by Okamoto is a good book, you can check that book. And, also the book by G P Agrawal Introduction to Non-Linear Fiber Optics, it is a very very important book in the field of non-linear fiber optics or non-linear wave guide.

So, when we cover the non-linear part, non-linear characteristics of the wave guide, then this book will be important and you can go with this book. And, finally, Optical electro electronics by A. Yariv it is also a very well known book. So, these are the text books and reference reading.

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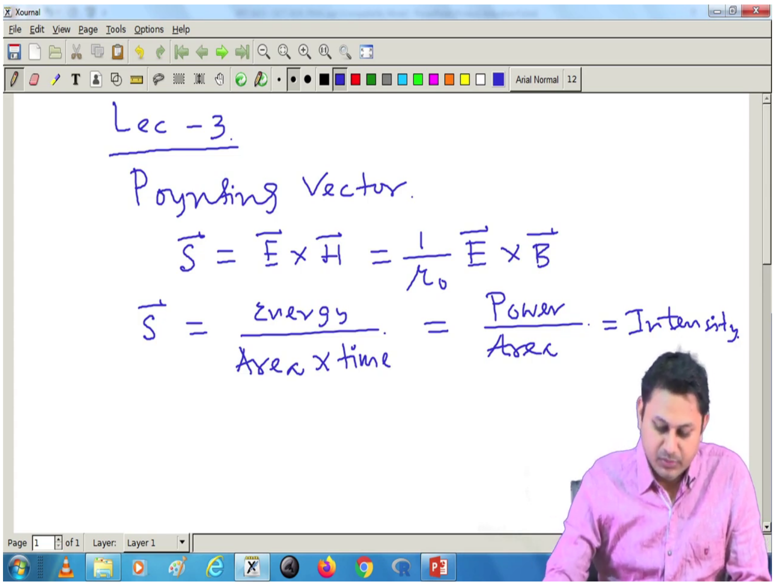

Today, we have lecture number 3 and in lecture number 3 we start with the poynting vector. So, we start with poynting vector. So, by definition poynting vector is defined in this way, S is equal to E cross H or 1 by mu naught, if I deal with free medium E cross B, vector B.

So, S which is our poynting vector is the energy flow per unit area, per unit time. So, eventually this is power divided by area, which is intensity. So, eventually it tells us about the intensity of the system and also the power flow and this is the recipe E cross H through which you can calculate the poynting vector. And, we will try to find out what is the value.

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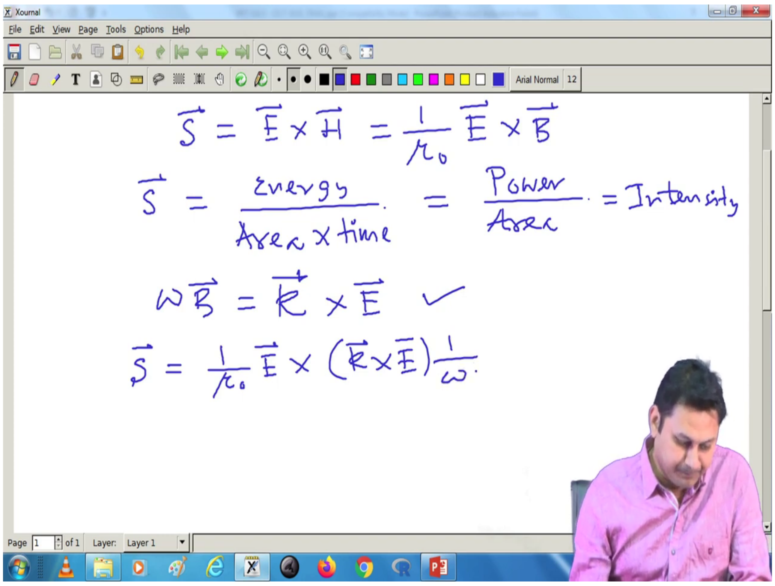

And, in order to find the value we directly use the relationship between B and E which we derived last day. So, we have the frequency omega multiplied by B is equal to propagation vector k cross E.

So, I will going to use this equation to and put it here to find out, what is the expression of S in terms of electric field E. So, S then will be 1 by mu 0 vector E cross and in place of B, we will I will put k cross E 1 divided by frequency omega.

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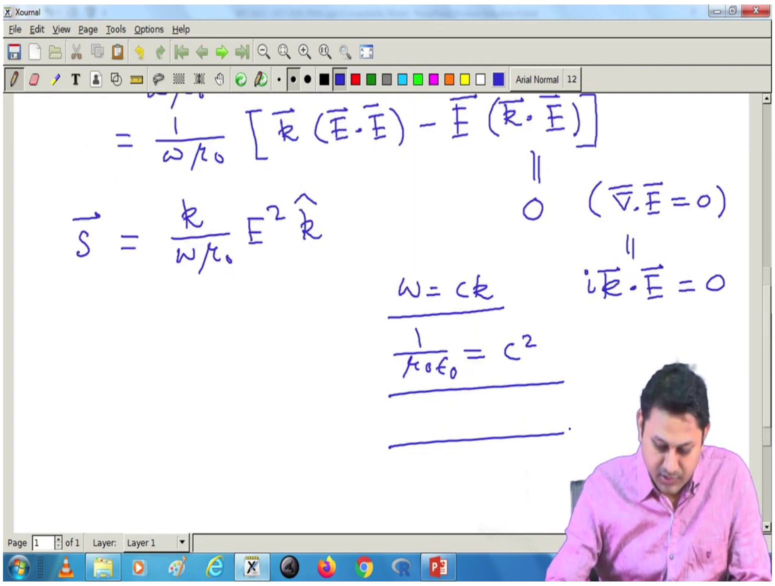

So, it is eventually; so, it is eventually S is equal to 1 divided by omega mu 0 vector E cross k cross E, it is like a cross b cross c. And, we will going to use this vector identity to resolve this and we know what is the vector identity for a cross b cross c. It is b c a, c a b this is the formula we normally use and if I have b c a. So, it should be k E dot E minus vector E k dot E; vector E k dot E.

So, these value, this value k dot E we know is 0 in free medium. Because, we have grad dot E is equal to 0. So, equivalently I can write i in place of grad operator I can replace i if it is a plane wave solution lastly we derived that, so this term is 0.

So, I have simply k divided by omega mu 0, then E square and the k unit vector that should be the value of my poynting vector S; S should be k omega mu 0 E square and the unit vector k. That means, the poynting vector is along the direction of the k.

Now, I can simplify this because omega is equal to c k that we know this is a relation. So, from that and also 1 by mu 0 epsilon 0 is C square. So, I use these two equation to find out; to find out the value of the a in a more compact form; value of the S in a more compact from poynting vector S in more compact form.

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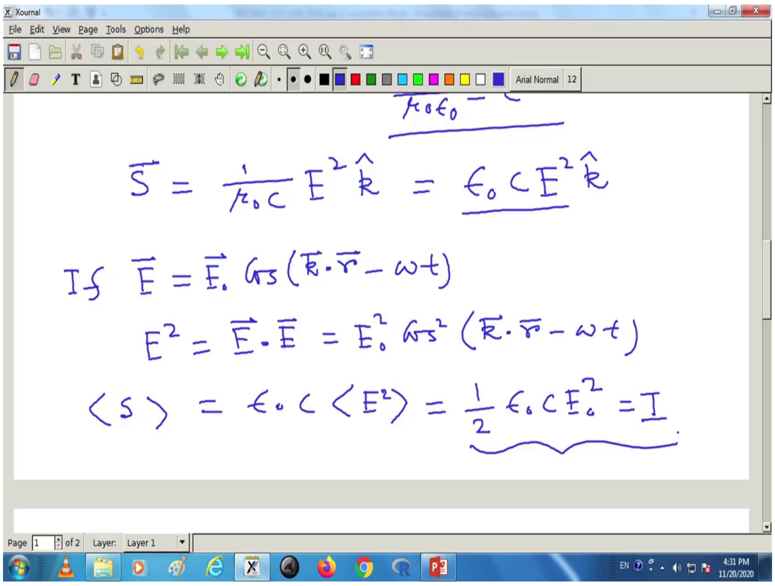

So, it should be simply; it should be simply, 1 divided by mu 0 and then k by omega I can write at c; k by omega i right 1 by c and then E square and k unit vector and again mu can be replaced by c square if epsilon 0. So, epsilon 0 c 1 c will cancel out and we have E square k mod vector. So, this is the term I have this is basically the relationship with the S vector with the amplitude of the electric field E.

Now, we know that the electric field if electric field is sinusoidal in nature. I can write is a plane wave solution form as this cos of k dot r minus omega t. This is the solution of E we know, this is the plane wave solution we know. And, now if I want to find out the average value because this is changing with respect to time. So, if I want to find out the average value. So, what is E square? So, E square is E dot E. So, I have E 0 square cos S square k dot r minus omega t, this is the value of E square.

Now, if I want to find out the average value of the S; average magnitude of the S. So, I need to do the averaging on the right hand side as well. So, it should be epsilon 0 c the average over E square. And, E square we know it is changing in this way E square cos square k dot r minus omega t. If, I do the average of cos square k; k dot r minus omega t. I simply have a half term, because we know the average value of cos square term is half.

So, eventually I have this quantity which is my intensity. So, in this straightforward calculation we find the relationship between the poynting vector with the intensity. Poynting average value of the poynting vector is essentially the intensity.

So, this is a very important expression because this gives us the idea how the intensity is related to the peak value of the electric field E 0. Well, now the next in the next part we will going to calculate, we will going to find out Maxwell's equation in a dielectric medium.

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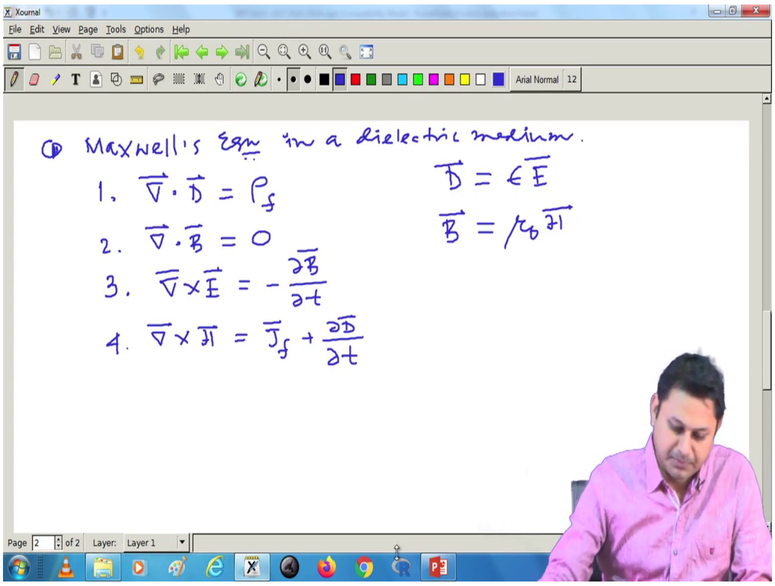

So, we will going to find out what is the Maxwell's equation in dielectric medium and then from that we will going to derive the Maxwell's wave equation. So, if I write one by one, so in the dielectric medium the Maxwell's equation, we will have this form.

Grad dot d is equal to rho f, rho f is the free volume charge density. Grad dot B is equal to 0, curl cross E is minus del B, del t. It will not going to change inside the medium, in the medium, something if something is changed that is the equation 1 and 4, which is containing the source terms. And, finally, curl cross H is equal to J f plus del D del t.

Now, we should remember that the vector D is epsilon E and if the system is non magnetic B is mu 0 H, for non magnetic systems. Now, we will going to make use of this 4 equation to find out the wave equation, how the how an optical wave is propagating inside a dielectric system.

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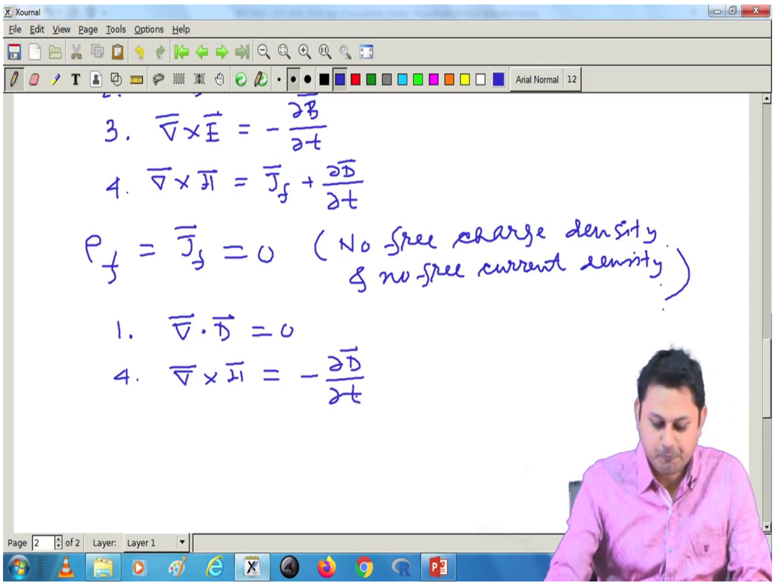

And, in order to do that first we consider that rho f and J f these things are 0. So, there is no free charge density and no free current density. So, the free charge density of free current density are not there. In such case in this equation 1 and 4 is modified. However, equation 2 and 3 will not going to modify. So, equation 1 will be simply grad dot d is equal to 0 and equation 4 will be curl cross H is equal to minus of this quantity ok.

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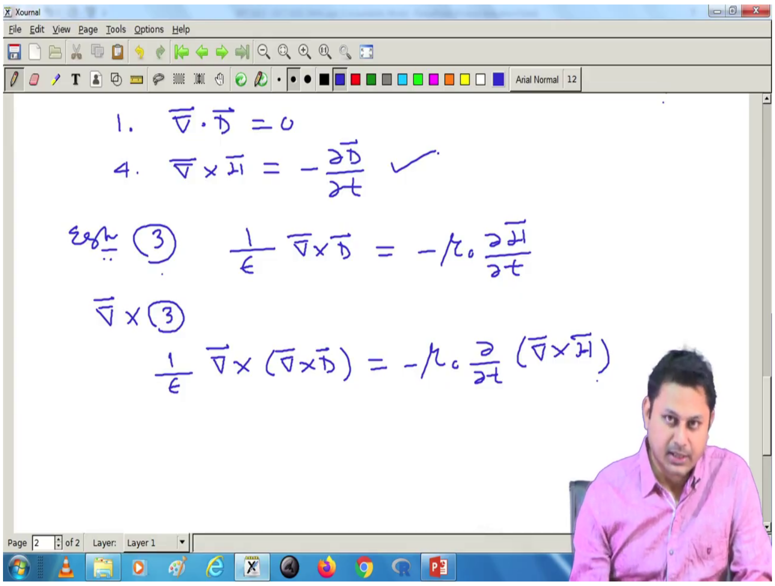

So, equation 3 also can be written in terms of D and H, because it will be useful in our calculation. Equation 3 I can write it as this, I just replace the value of E and B in terms of I replace the value of E and B here. The value of E and value of B, I just change in terms of value of D and value of H. Well one thing you should note here when you do, that epsilon is not a function of space. Then, only you can take this epsilon out of the curl operator.

So, the system is isotropic and homogeneous at the same point. Well now what I do? I make a curl of equation 3. If, I do that then it should be 1 by epsilon curl of curl of D this quantity in the left hand side is curl cross curl cross t in the right hand side, it should be del del t, then curl cross H. Now, curl cross H from equation 4 I have this one. So, I will going to use this equation here.

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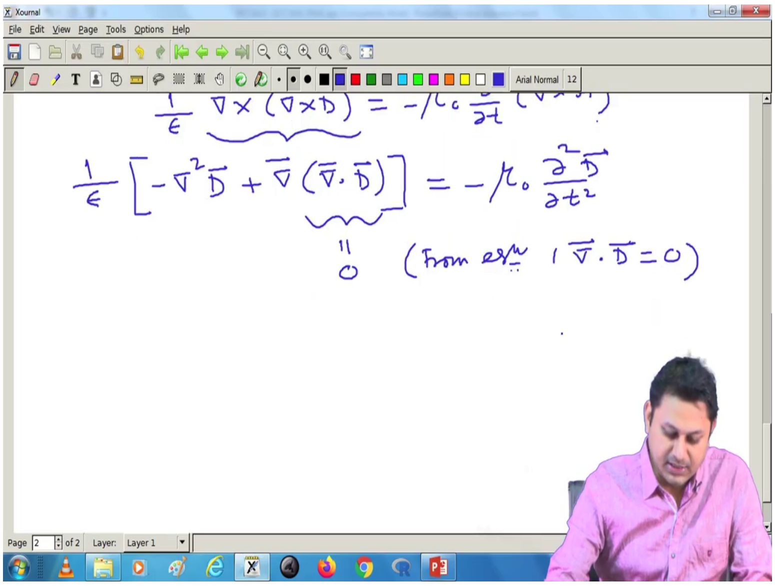

And, if I do I can have in the left hand side 1 by epsilon curl cross curl cross D again this is a well known vector identity and it should be minus of Laplacian operator D plus gradient of D this. And, in the right hand side it should be minus of mu 0 del del t square square because curl cross h is minus of del del t. So, I should have so from equation. So, I should have this curl cross, curl cross, so it is D.

So, now so, here this equation is plus. So, I make a mistake here so, this equation is plus because it is J plus these things. So, then this minus sign is there. So, finally, I can see that this term will be 0, because from equation 1 we have dot D is equal to 0, because the rho f is 0 the free charge density is 0.

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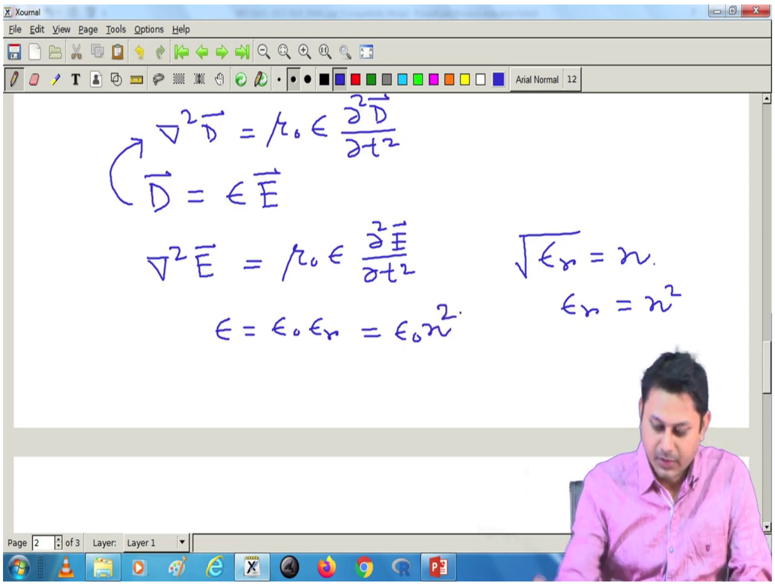

So, then we eventually have square D is equal to mu 0 epsilon to D del t square. Well, now if I want to write this entire equation in terms of electric field E, then I need to replace again because D is epsilon E. So, I can replace this thing here to make it to make the entire equation in terms of to write this entire equation in terms of E.

So, I can have that grad square E is equal to mu 0 epsilon del E del t square. Now, epsilon which is a susceptibility can be written as epsilon 0 multiplied by epsilon r the relative susceptibility. And, this epsilon r is root we know that root over of epsilon r is basically the refractive index of the system. So, epsilon r is equal to n square. So, we have epsilon 0 n square, we can put this equation; put this equation here in the wave equation.

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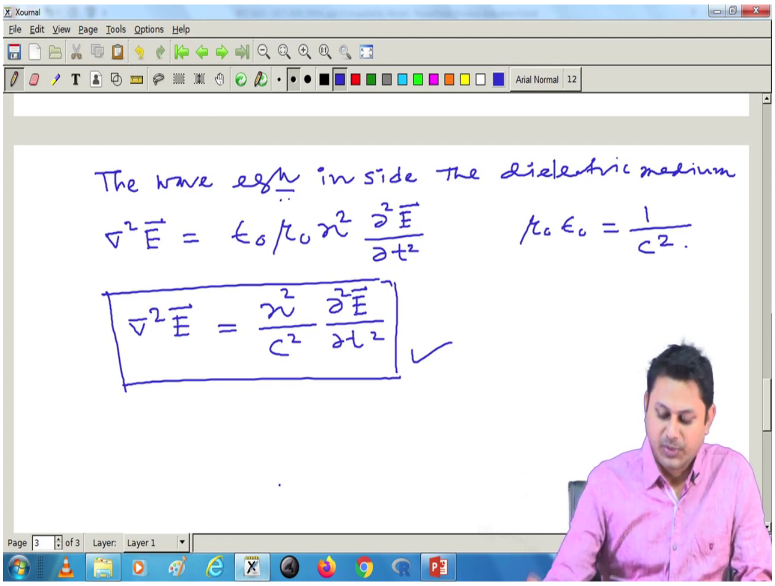

So, the wave equation finally, comes out to be; the wave equation inside the dielectric system dielectric medium will be grad square E equal to epsilon 0, mu 0, because mu 0 is already there and I replace the epsilon 0 as epsilon as epsilon 0 multiplied by n square n square, d 2 E the vector sign dt square. And, again mu 0 epsilon 0 is equal to 1 divided by C square.

So, eventually I have equation that we will going to use later C square is d 2 E d t square. So, this is the wave equation for a dielectric medium a system, when the electromagnetic wave is propagating inside a dielectric medium. So, this is the equation one can expect one can have. And, you can see that n is now associated with this equation. So, refractive index will going to play an important role.

And, obviously, in the free system, in the free medium n is 1. So, this equation will be simply grad square E is equal to 1 by C square, d 2 E d t square, but here since the refractive index are there. Because, the medium will going to a medium we will going to give some kind of resistance to the propagating wave in terms of refractive index. So, this refractive index the information of the refractive index should be here in the equation.

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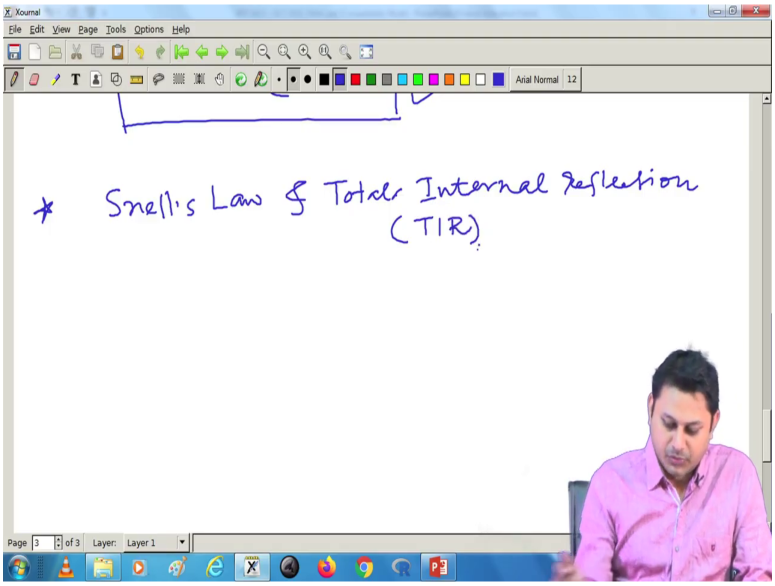

Well after that, we will now try to understand few thing which is Snell's Law and the concept of Total Internal Reflection; total internal reflection or in short TIR. So, we all know the Snell's Law nothing special.

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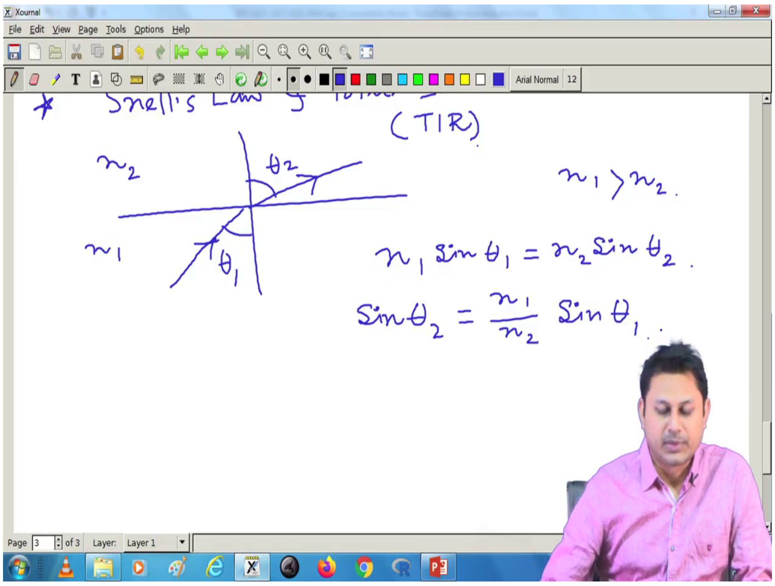

So, let me so, this is suppose we have two medium; one is n 2 another is n 1 such that n 1 is greater than n 2. So, in such medium if a light is coming in this way. So, it is refracted in this way. So, I have angle here theta 1 and I have angle theta 2.

So, now, Snell's law says that n 1 sin theta 1 is equal to n 2 sin theta 2, this is simply the Snell's law. And sin theta 2 it is for our I can write this sin theta 2 in this form n, 1 divided by n 2 into sin theta 1. Why, I am writing sin theta 2 in this form? Because I am going to use this form in our next class when, we try to understand something called evanescent wave.

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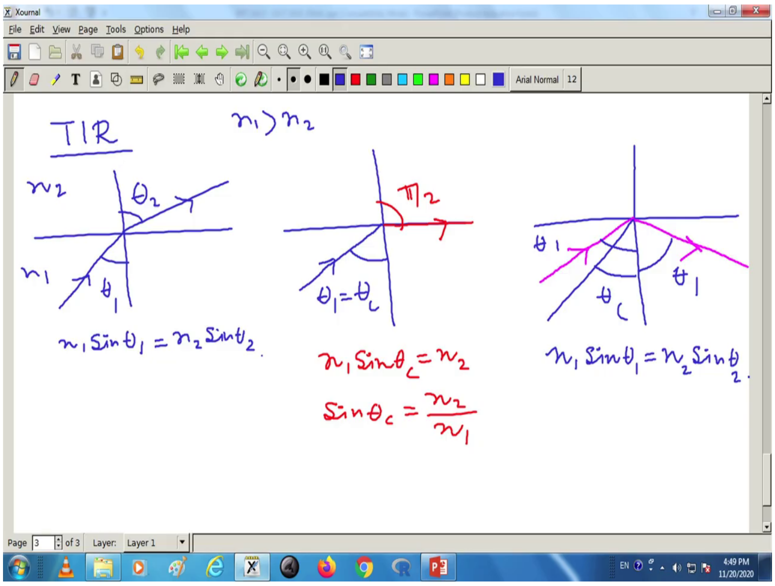

Before concluding today's class, let us quickly understand what is the meaning of total internal reflection. So, the total internal reflection if I have so, let us consider three condition; I have n 2 n 1 and n 1 is greater than n 2. So, this is simply when the light ray is falling here.

So, I have angle theta 1 and angle theta 2. And, it is following the Snell's Law like; n 1 [music] sin theta 1 is equal to n 2 sin theta 2. In this case if I now increase the angle of theta 1 so; obviously, this angle theta 2 it will going to change and at some point this angle will be 90 degree.

So, this angle is a spatial angle theta 1 is a spatial angle theta c this is called the critical angle. At critical angle what happened that this ray will be like here and this angle will be pi by 2.

So, we have a spatial equation and this equation is n 1 sin theta c is equal to n 2, or sin theta c will be n 2 divided by n 1.

And, now if I launch the light more than the angle of the critical angle suppose this. Then, this ray will be reflected back and this is the phenomena called total internal reflection. So, this is the angle and this is my original angle theta c.

So, this is my theta c and I launch the light with an incident angle theta 1 that is greater than theta c. And, this angle will be now theta 1 as well, because it behaves like a reflection. So, here also in principle since it is coming through the boundary condition, this equation, this Snell's Law is coming through the boundary condition.

In principle the equation n 1 sin theta 1 equal to n 2 sin theta 2 should valid. And, if it is valid then we will going to see in the next class that the angle theta 1 will be giving us a imaginary value. So, from the imaginary value we will try to find out what is what will be the fate of this wave. So, some wave will pass through this boundary and which we call the evanescent wave this evanescent wave eventually die out. So, that part we will going to see in our next class.

So, with this note I will like to conclude. So, today we mainly cover the concept of poynting vector then try to understand the wave equation in a dielectric medium. And, then very briefly we understand what is the meaning of Snell's Law and total reflect total internal reflection. So, in the next class we start with total internal reflection and try to find out more on what is going on there, when the incident angle is greater than the critical angle.

Thank you for your attention.