Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 29
Power Associated with a Modes

Hello student the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 29 and today we are going to learn about the Power Associated with the Propagating Mode ok.

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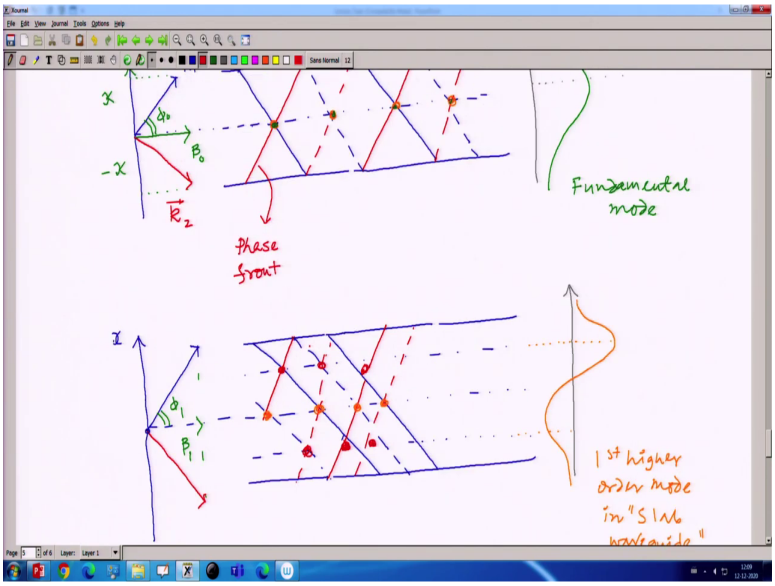

So, in the last class we have an idea that how the modes is propagating and for fundamental modes and higher order modes, how the interference play the interference of the phase one play a role to construct the structure of mode, etcetera.

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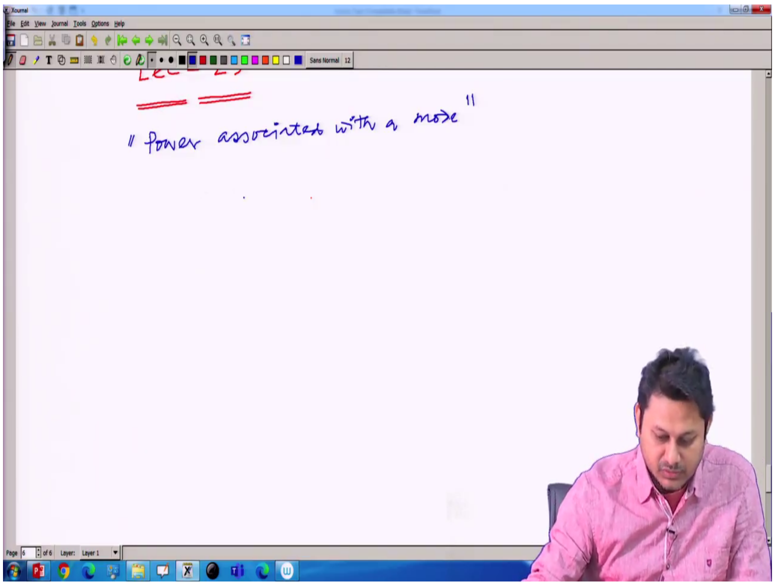

In today's class we will going to learn about the power associated, what is the amount of power we are going to calculate rather; what is the power associated with a mode. So, the amount of power associated with a mode that we will going to calculate. So, in order to calculate that, first let us refresh our concept of pointing vector.

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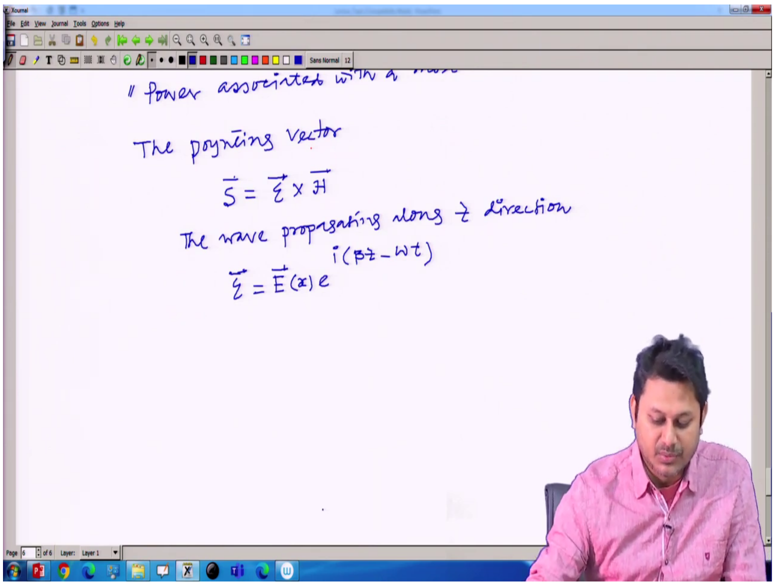

So, pointing vector; so, the pointing vectors S was defined as E cross H. So, the waves propagating along Z direction, the wave propagating along Z direction will have a form of E as this that we already find sorry. It is not E y, the general form is E, vector E is equal to E of x e to the power i beta z minus omega t. So, for the wave that is propagating along Z direction in the waveguide that is normally the case. We consider the direction of propagation as Z. So, electric field can be in this way.

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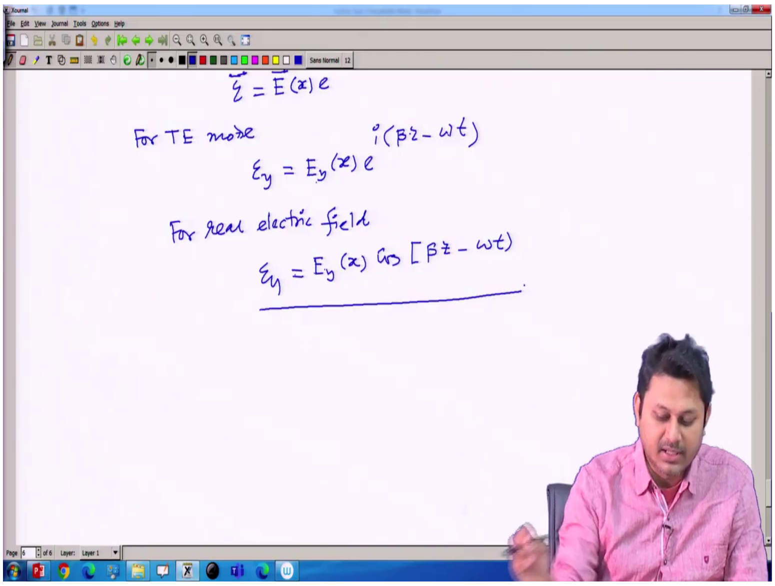

And for TE for TE mode, for TE mode we have E y as omega t. So, that is the, that is the structure of the field that we have during the propagation. Now, for real electric field because electric field we know that electric field and magnetic fields are real. So, for real electric field I can write this as so, for real electric field we have E y as E y x cos of beta z minus omega t.

So, I just replace this exponential thing to cos because the electric field is real that we always these things we can always do. So, this is the real electric field and this real electric field is carrying some kind of power and that we will going to calculate.

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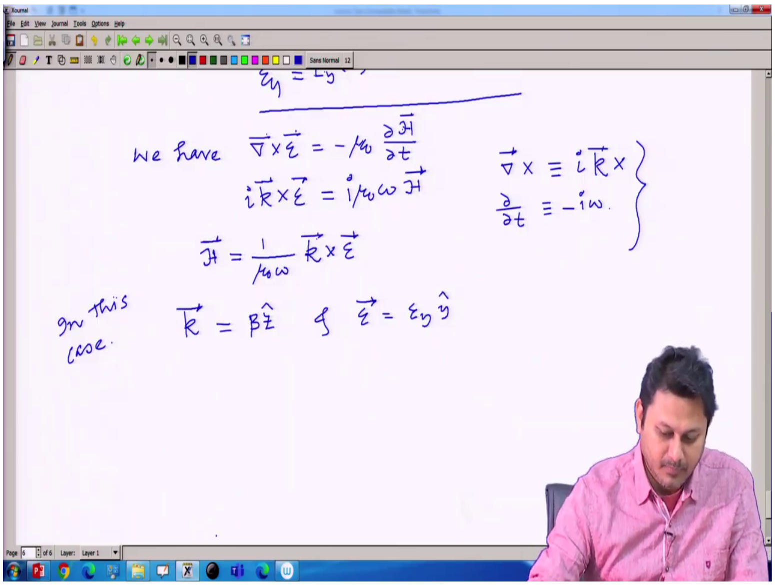

Now, if this is my electric field I can we have in our hand the Maxwell's equation which says that curl of E should be equal to minus of mu 0 del H del t. So, I can replace this thing as i k cross E is equal to i mu 0 omega. This because I know that curl of these things for plane wave is equivalent to i k cross that we learned earlier and del del t this operator can be replaced as minus of i omega for plane wave. These two identities we know and that is used here.

So, my H becomes when we have this my H becomes 1 divided by mu 0 omega and then k cross this one. Now, I know my k this is the general form, but in this case in our case, in this case for when the wave is propagating for the slab wave guide along z direction. So, the k if it is propagating along z direction I have beta Z. So, the form of the k should be something like this and my and also the electric field is for t mode we know is this one.

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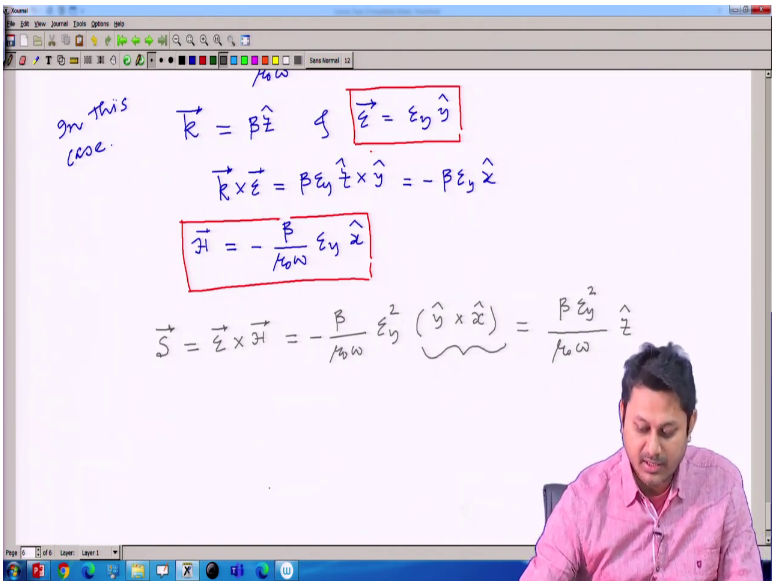

So, the k cross E is how much? Because it is beta so, beta E y it should be Z cross y. So, we have minus of beta E y and then it should be along x direction. So, this is using the vector identity I can have this. So, k cross E is this quantity. So, H is essentially vector H is essentially minus of beta then mu 0 omega and then we have E y and x unit vector.

So, I have my E, I have my H. So, what is my E? E I already have here for E mode and my H, I also calculate using the Maxwell's equation in this. So, my S is simply called E cross H not curl is equal to E, I already have and B, I already have. So, it should be minus of beta divided by mu 0 omega then E y square and then I have y cross y unit vector cross x unit vector.

So, eventually I have this quantity is nothing but beta E y square divided by mu 0 omega and this quantity is minus of Z and this minus is going to absorb. So, I have Z. So, my S which is the pointing vector is moving along Z direction. So, that is again not a surprising result, but important thing is that I now have a the value here which basically tells me what is the amount of power it should carry.

When the propagation constant of the mode is beta with a frequency omega and the electric field is E y square and form of the electric field we know. Then what is the value of the pointing vector what is the value of the power associated with the mode? One can calculate through this explicit expression of the pointing vector that is the code.

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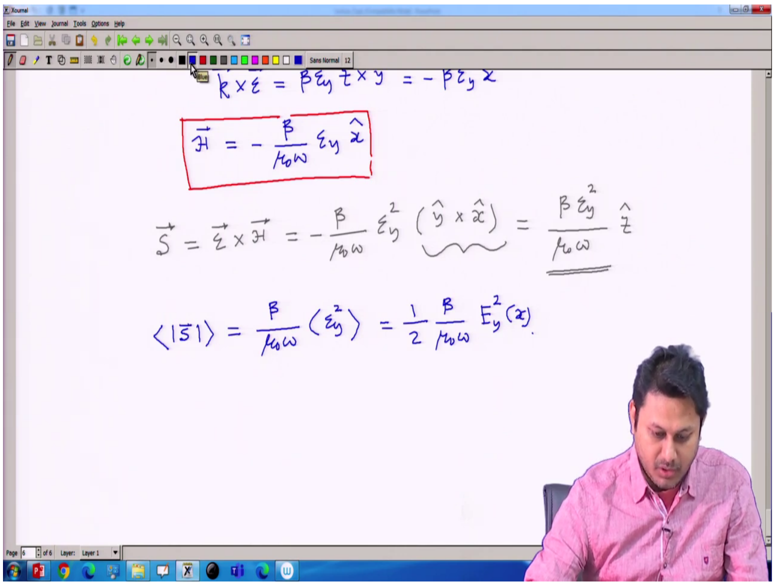

So, now, if I want to find out the average because, normally we do the average because this is a sinusoidal field. So, if I want to find only the average of whatever the expression we have so, I can simply have beta divided by mu 0 omega and the average of this quantity because this is now having a time dependent part.

And if I do the average over that I should have because it is a cos functions we know that when we make a cos function and it is a square of that. So, average of cos square whatever the function we have then it should be half of half is the value of the average. So, because E square if you find from here square it is E y square cos square and if I make an average of this cos square beta Z minus omega t.

Then I have half term and I have something which is x dependent the distribution and it should be like this. So, my pointing vector the average of the pointing vector should come in this form. So, what is the meaning of that? So that means it is the amount of I mean in terms of power.

So, the amount of power that is carrying per unit area is this amount because that is the definition of the pointing vector; amount of energy that is passing through per unit area per unit time. So, let me now write down the thing.

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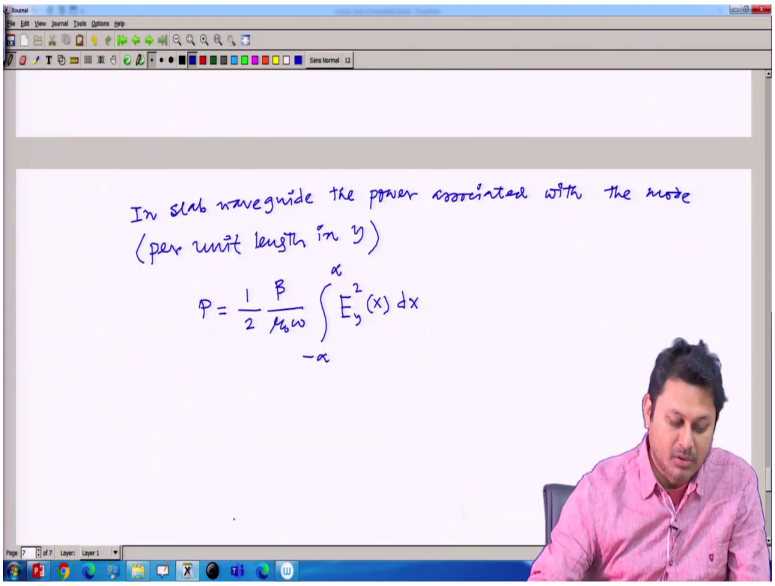

So, in slab waveguide, so, if I want to find out what is happening; in the slab wave guide, so, in slab waveguide, in slab waveguide the power associated with the mode with the mode per unit length in y that is important because it is a three dimensional thing. So, and y we take a extended up to infinity.

So, per unit length of y if I want to calculate what is the power then P should be equal to half of beta and then mu 0 omega and then it should be integration to the inter mode field what we have which is simply E y square function of x dx. This is the amount of power one should have that is associated with the mode per unit length of y.

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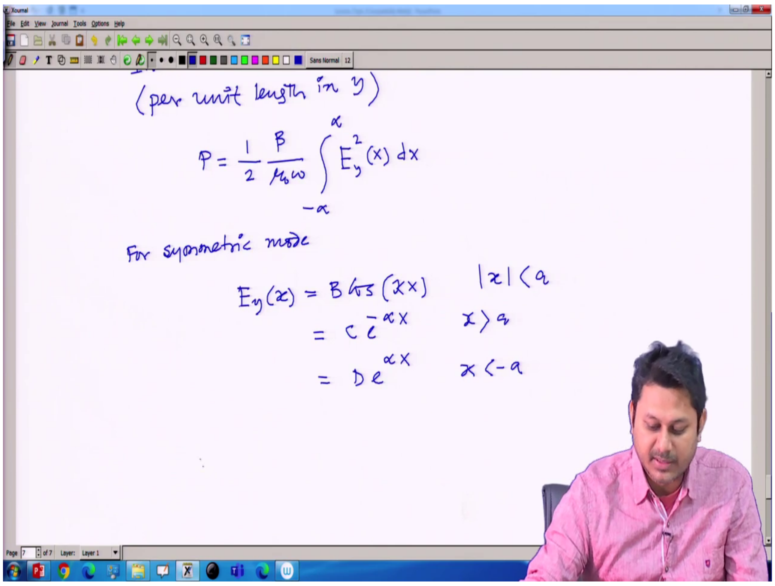

So, for symmetric mode: Now, for symmetric mode; so, symmetric mode E y we know what is E y for slab waveguide E y as a function of x for symmetric mode is simply say some constant some constant say A 0 in our notation it is B. So, better to written with whatever the notation we used earlier.

So, it is B the peak value and cos of kappa x in the region x is less than a. And for outside that it was C e to the power of minus of alpha x and D e to the power of alpha x depending on the value of the region. So, when x is greater than a, when x is less than minus a, these are the two regions where we have expected and exponential decay. So, that was the distribution field distribution.

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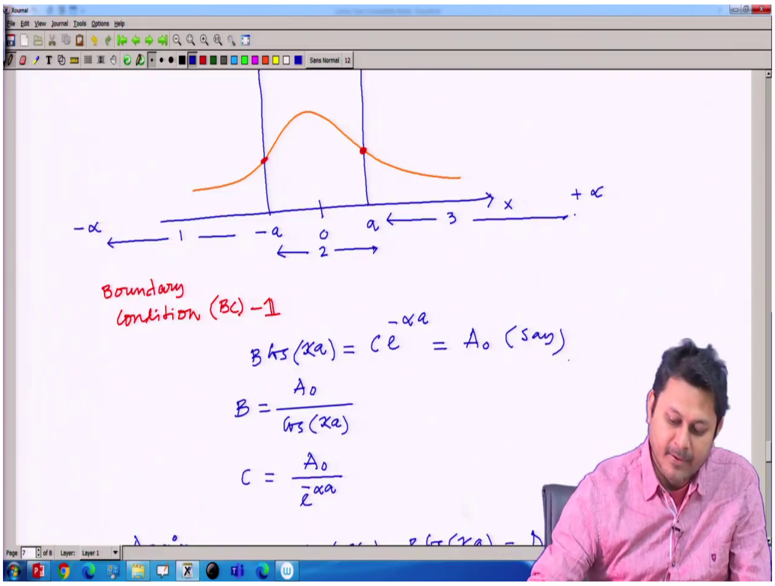

So, once again let me draw the structure and the field. And I had the mode distribution like this something like that. This point is a, this is minus a, this is 0 and this axis is my x axis. So, now, I need to find out what is the amount of power that mode should carry because this mode distribution is known and expression is known.

But, before that we need to understand because these constants are very important. B, C, D, I need to execute this constant and we know the boundary conditions. If I have certain boundary conditions here then I can execute the constant. So, if I took this boundary conditions. So, boundary condition 1; so, boundary condition in short BC 1. So, what is the boundary condition 1 here?

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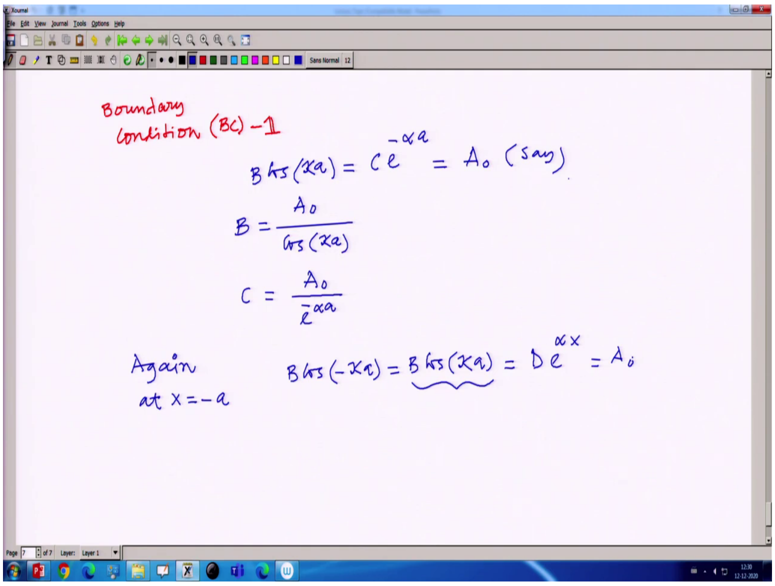

The boundary condition 1 here is simply that if I have the continuity condition here so, this is B of cos kappa a is equal to C e to the power of minus alpha a. And these two are constant because and this constant can be written in a new name say some name say A 0. This is one boundary condition and after putting this boundary condition I can write the B and C in terms of a single constant A 0.

So, my B becomes A 0 divided by cos of kappa a and my C becomes, A 0 divided by e to the power of minus alpha a. These are the two values of A and B I have. Again in the left side I also have the same boundary condition. And if I put the same boundary condition again at X equal to minus a, what we have we have?

We have B of cos of minus kappa a, which is eventually B of cos of kappa a is equal to D of e to the power of alpha x. And now B of cos of kappa I already defined in the earlier case is a. So, this is nothing but A, A 0 rather.

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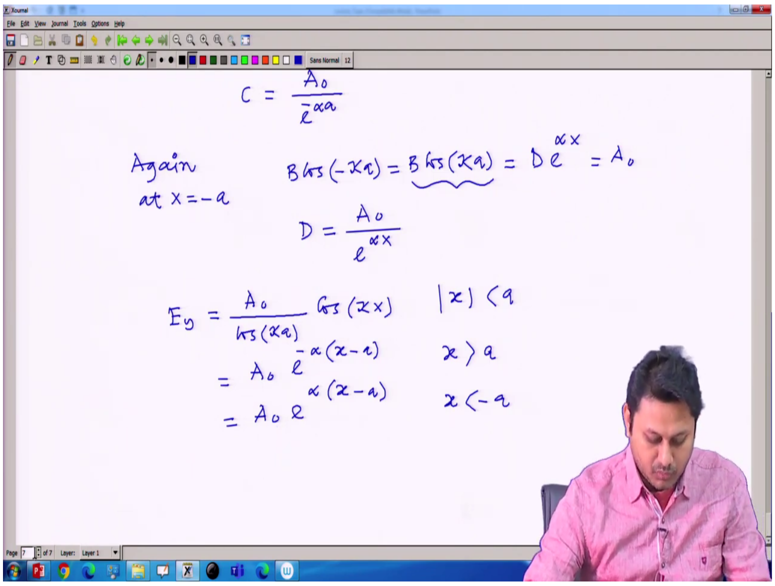

So, I can have the value of D from this expression as A 0 divided by e to the power of alpha x. So, I have all the values B, C and D which are the constants of these fields in terms of a single constant A. Now, if I plot that, if I now write that the final field then final field E y will be equal to A 0 divided by cos of kappa a and then cos of kappa x.

This is for mod of x less than a. On the other hand I have A 0 e to the power of minus of alpha x minus a and A 0 e to the power of alpha x minus a, this is for x greater than a and this is for x less than minus a. So, these are the three fields. Again I rewrite in terms of a single unknown constant A 0 a single unknown constant A 0. Now, what I do is this.

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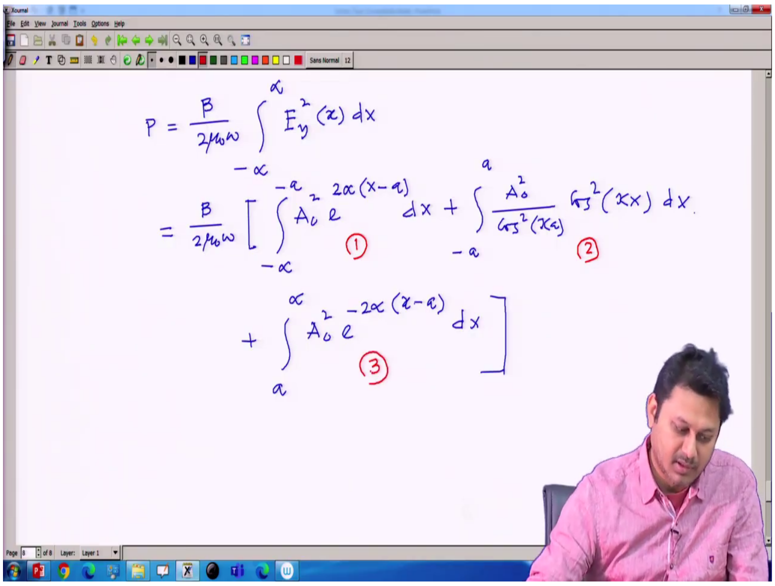

Now, my power P is beta divided by 2 mu 0 omega integration of minus infinity to infinity E y square which is a function of x over dx. So, these I can write, I can divide into three parts. So, I can divide these into three parts. So, this is part 1. So, from minus infinity, so, this is goes to minus infinity to a.

So, I have part 1 here this is 1 part up to here. This is minus a 2, this is part 2 and last part is here goes to infinity plus infinity. So, this is part 3. So, the entire integration can be divided into 3 part; 1, 2, and 3 and if I write this. So, let me write in bracket. Integration of minus infinity to minus a, this is the first part.

When I write the first part, it should be A 0 square e to the power of minus no not minus 2 alpha then x minus a then dx. That is the first integral.

Second integral this is the one. Second integral is minus a to a, I have A 0 square divided by cos a square kappa a because this is the new constant this is B basically B square rather cos square kappa x dx that is the second part of the integral. And the last part of integral plus a to infinity A 0 square e to the power of minus 2 alpha x minus a dx.

So, all the 3 parts are; this is 1, this is 2 and this is 3; 3 parts three integrals we need to execute. So, 1 and 3 this parts are easy because if you I mean if I go to minus infinity, so, the integral will be A 0 e to the power of minus alpha x minus a divided by 2 of alpha. And when I put this boundary conditions that minus a and infinity. So, in minus a it should be 1 and infinity it should be 0.

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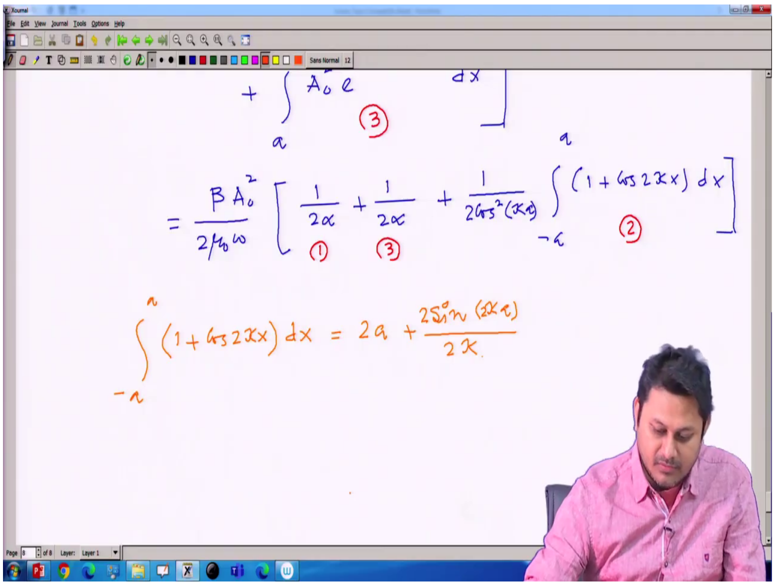

So, I can execute that quickly for both the cases and I can write. So, it should be beta divided by 2 mu 0 omega. I can execute this very quickly and A 0 is common for all the cases. I can put A 0 out of that. So, A 0 square I can write. And then this integration is as I mentioned is 1 by it should be 1 by 2 alpha for this case and here also if I execute it should be 1 by 2 alpha.

And then I have the third. So, this is the results of integral 1, this is the result of integral 1, this is the result of integral 3 and result of integral 2, I need to calculate. Again this is not a very hard thing to calculate. So, I can write it 1 divided by cos square kappa a. And if I put a 2 sign here then cos square I can write is minus a to a 1 plus cos 2 kappa a and then dx. This is my equation the integral 3.

So, first two integral I already calculate, the last integral I need to calculate. So, I can simply calculate here maybe I can use a different color. So, integral minus a to a 1 plus cos 2 kappa x dx, it should be simply 2a and then the integration of the cos it should be sin. So, I should have sin and if I put this two I could I should have a two of that. So, 2 sin kappa of a.

There should be 2 here 2 kappa of a divided by during the integration 2 kappa term will come out. So, this will be the value.

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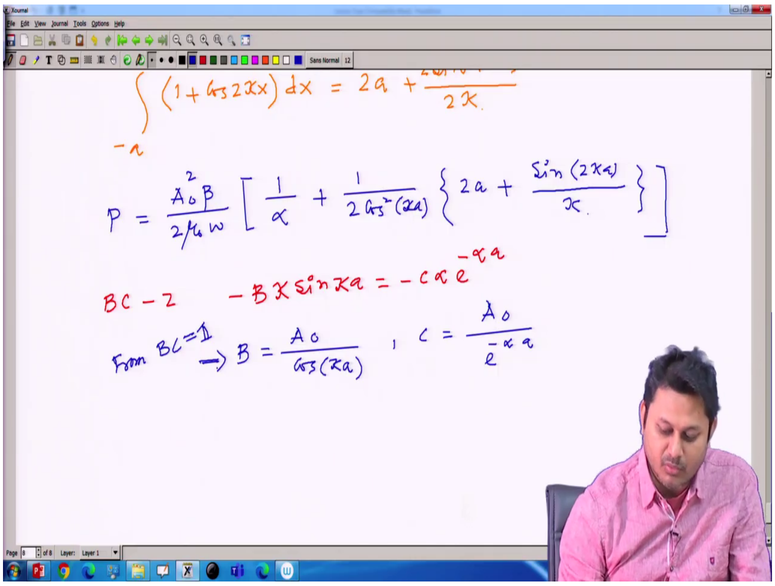

So, eventually my power will be A 0 square beta divided by two mu 0 omega 1 by alpha because 1 by 2 alpha plus 1 by 2 alpha I can write 1 by alpha and the rest of the term is like that. So, it should be plus 1 by 2 of cos square kappa a which is already there and then I have the integral value, which I calculated here 2a plus sin of 2 kappa a divided by kappa.

So, precisely that is the result, but one can have a more compact result out of that and that you can use by using that you can find the using the boundary condition. So, let us do that quickly.

So, my boundary condition 2 suggests, so, BC 2 suggest minus of B derivative of is continuous.

So, minus of B is sin Ka is equal to minus of C alpha e to the power of minus alpha a. Now, we already have B as A 0 of cos kappa a, this is we already have. And C is how much? A 0 from the boundary condition 1 I already have this. So, from BC 1, I already have this quantity, the value of A and B and C.

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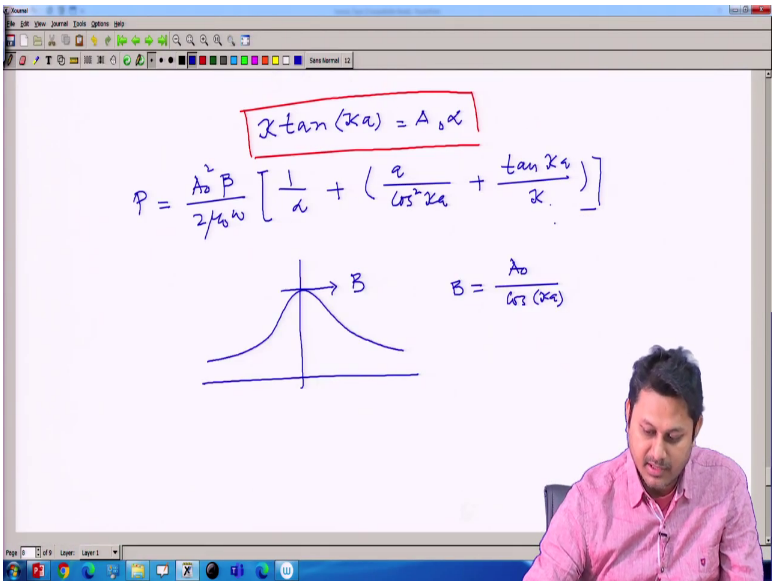

If I put this I simply have kappa of A 0, A 0 will cancel out minus, minus will cancel out. So, a kappa tan of kappa a because when you put kappa or B here then sin divided by cos term will appear is equal to A 0 alpha e to the power minus a e to the power of e to the power minus of alpha is going to cancel out. So, this additional condition I am having now.

So, once we have this additional condition I can make use of that in this equation, in these equation and have may have some simplified forms. So, let us find out.

So, my P if I put all this value my P will be A 0 square beta divided by 2 mu 0 omega 1 by alpha plus. I just simplify, I just put this second boundary condition second condition which is this one to the equation and I have this one plus a divided by cos square kappa a plus tan kappa a divided by kappa. So, that we have.

Now, for fundamental mode this quantity this is a maxima and this is B according to my notation. So, it is better that everything I can have in terms of B then it is it should be the in terms of the amplitude peak amplitude. So, that we can do because B is A 0 divided by cos of kappa a. So, that in it information I am going to use now.

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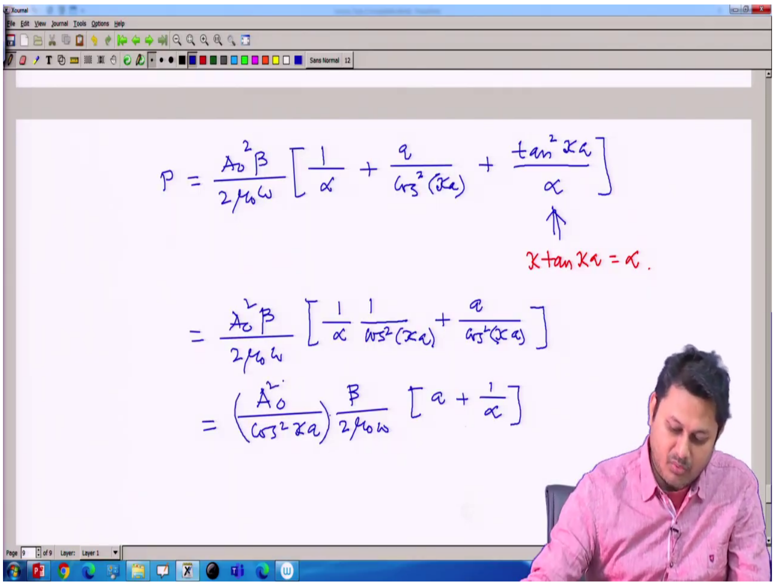

So, now my P will be A 0 square beta divided by 2 mu 0 omega 1 plus alpha plus a cos square kappa a. I split it into two part and then I can write it as tan square kappa a divided by alpha I just replace this condition. So, kappa I can write from this equation. So, this I can find by using the equation kappa tan kappa a is equal to alpha.

Once I have this then I can write it as A 0 square beta divided by 2 mu 0 omega and then I can have 1 by alpha common. 1 if I have 1 by alpha common then it should be 1 plus tan square kappa a and 1 plus tan square kappa a is eventually 1 by cos square kappa a. So, this is the first term and the second term we already have as a divided by cos square kappa a.

So, I can take this kappa cos square kappa a common from both the sides and I have A 0 square divided by cos square kappa a multiplied by beta divided by 2 mu 0 omega as a plus 1 by alpha. So, that is more compact form and this quantity now this should be A square. Now, I can write as a peak amplitude.

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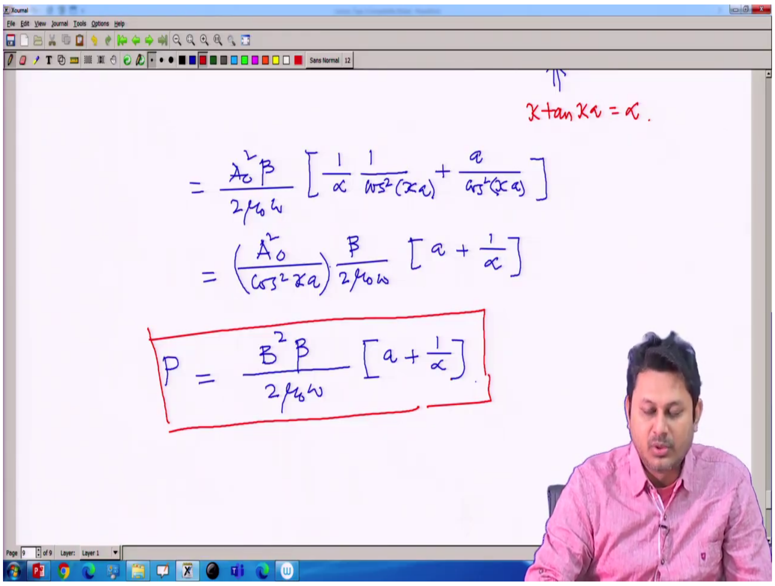

So, my power eventually have a more compact form which is B square beta whole divided by 2 of mu 0 omega and then a plus 1 by alpha. So, this is the power associated with the mode when it is propagating inside a medium having a propagation constant beta.

And if this alpha and a are the parameter that one can define for the wave guide and then you can readily calculate what is the value of power and which obviously, should be proportional to the peak value of the fundamental mode peak value of the symmetric mode, which is B and it is proportional to B square.

Well, with this note I like to conclude. So, thank you for your attention. See you in the next class. In the next class maybe we are going to start a new topic which is the Modes in Optical Fiber, which is very very important to understand.

So, thank you and see you in the next class.