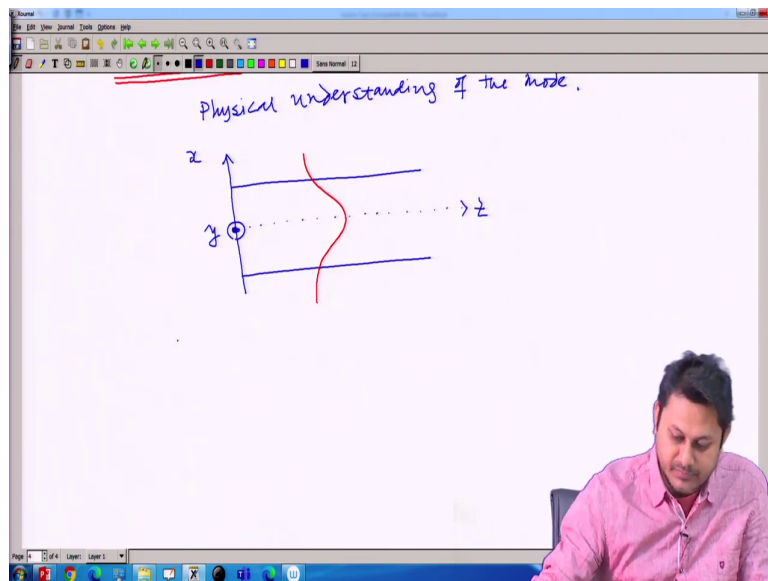


Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 03
Modes (Cont.)
Lecture - 28
Physical Understanding of Modes

Hello student, to the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number 28, and in this lecture we will going to understand the Physical Understanding of the Mode. So, lecture 28.

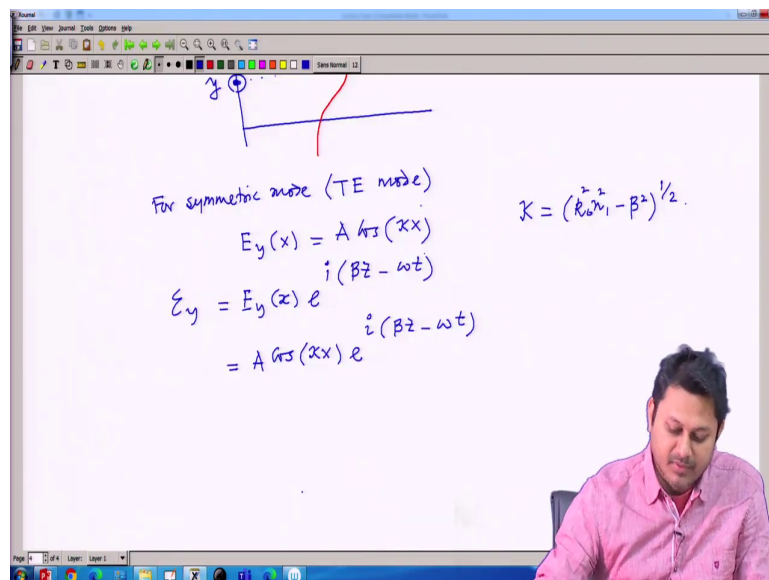
(Refer Slide Time: 00:30)



So, today we mainly concentrate on the physical understanding of the concept of modes. So, let me write it physical understanding of the mode. So, we already figure out the distribution of the mode for a slab waveguide. So, let me draw it once again the structure.

So, this is the structure of the slab waveguide, we have x axis along this direction and this is the direction along, which the modes are propagating. So, this is z and here if I have a origin so, y-axis is perpendicular to the plane of x – z and the fundamental mode is symmetric one and looks like this. This is the fundamental mode m equal to 0 mode.

(Refer Slide Time: 01:54)

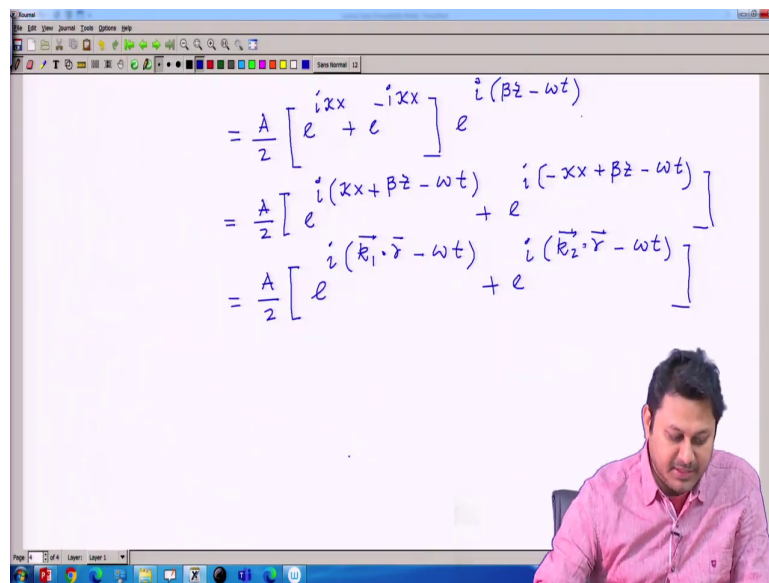


So, for symmetric mode this is essentially the TE modes. So, better to write TE mode; for symmetric TE modes, we figure out that E_y which is a function of X is $A \cos$ of $k_x X$,

where kappa was something like $k_0^2 - n^2 k_0^2 - \beta^2$ whole to the power half.

Now, if I find out the total field according to our notation it was E_y function of x e to the power of $i\beta z - \omega t$. E_y I already figure out, so, let me put this $A \cos$ of $kappa X$ e to the power of $i\beta z - \omega t$. So, that is the value of my E_y .

(Refer Slide Time: 03:51)



$$\begin{aligned}
 &= \frac{A}{2} \left[e^{i k_x x} + e^{-i k_x x} \right] e^{i(\beta z - \omega t)} \\
 &= \frac{A}{2} \left[e^{i(k_x x + \beta z - \omega t)} + e^{i(-k_x x + \beta z - \omega t)} \right] \\
 &= \frac{A}{2} \left[e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)} + e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)} \right]
 \end{aligned}$$

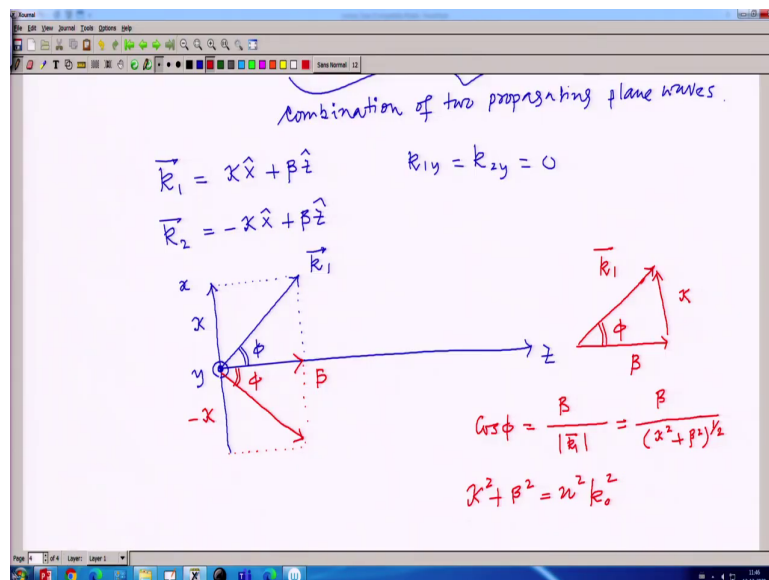
Now, I can write this in a plane wave form like this. So, $A \cos$ $kappa X$ I write in this way e to the power of $i kappa X$ plus e to the power of $-i kappa X$ that essentially gives me the $\cos$. And then additional term propagation term is e to the power $i\beta z - \omega t$.

Well, I can combine these together these two term together. So, it should be $A/2$; e to the power of i then I have $kappa X + \beta z - \omega t$. And another term I have as e to

the power of i minus of κX plus βZ minus ωt . So, I can divide the entire field into two part and this two part is basically two propagating waves.

So, if I now write in a more convenient way two plane waves in a more convenient way it should be $i \mathbf{k}_1 \cdot \mathbf{r} - \omega t$ that is the first plane wave. And the second plane wave is $i \mathbf{k}_2 \cdot \mathbf{r} - \omega t$ that is the second plane wave. So, I have two plane wave the combination of these two plane waves basically gives me the modes. So, something hidden in there.

(Refer Slide Time: 06:24)



So, this is basically the combination of two propagating plane waves combination of two propagating plane waves. Now, this propagation vector $\mathbf{k}_1$ and $\mathbf{k}_2$ is interesting. So, my $\mathbf{k}_1$ is κX plus βZ with sorry, βZ in terms of unit vector it should be $\hat{Z}$, $\beta \hat{Z}$ and my $\mathbf{k}_2$ vector propagation vector is minus of κX plus $\beta \hat{Z}$.

So, the Z component is same for both the vectors; only thing that is changing is the x component and one is plus and another is minus. So, if I plot it will be very easy and the component say k_y k_{1y} component and k_{2y} components, y components of both the propagation vectors is 0. So, now, if I plot this two wave vectors, so, this is my x-axis and this is my propagation z-axis, y is perpendicular to this. So, this is my coordinate system.

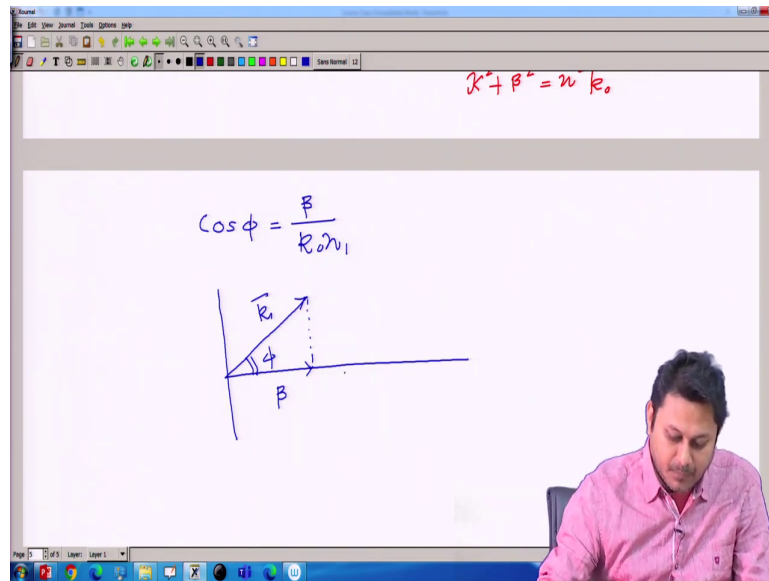
But, the k vector does not have both k_1 and k_2 does not have any kind of y components. So, if I have k_1 like this. So, this is my k_1 vector. So, the x component and z component is given say this angle is ϕ , say this angle is ϕ . So, my another component it should be simply in the opposite direction. So that, I can have the same z component this, which is both the case cases it is β and this is κ .

So, let me write it the proper way for blue I should write blue ink. So, this is my κ which is the y component and this component is minus of κ . Now, this is x component and this is also the x component of that and eventually this angle is ϕ as well. So, when a mode is propagating, so that means, two wave vector is propagating in this way, so that we can have the mode structure.

Now, here this is my β as I mentioned along this direction is my β , this direction is my k . So, this is my say k_1 , this is my β and this is κ . This is if I write in the β and κ so, these are the values and this angle is ϕ . Now, $\cos \phi$, what is $\cos \phi$? So, let me [FL]. So, $\cos \phi$ is β divided by k mod of k , in this case k_1 .

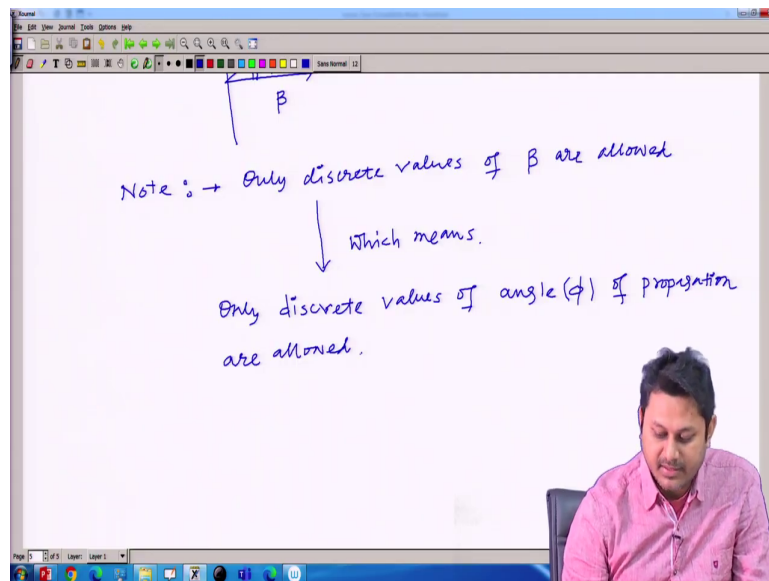
So, it is simply β divided by $\kappa^2 + \beta^2$ whole to the power half. But $\kappa^2 + \beta^2$ we know this is n_0^2 sorry, n^2 , k_0^2 refractive index multiplied by k_0 .

(Refer Slide Time: 12:31)



Hence, my $\cos \phi$ is β divided by $k_0 n_1$, nothing special with that. But, important thing that we need to understand here is that only discrete. So, let me draw it once again. So, what happened for this? So, I can correlate with the ray optics and the wave optics here, this angle is ϕ . So, this is my k vector k_1 vector and this is my β and $\cos \phi$ is β divided by k_0 .

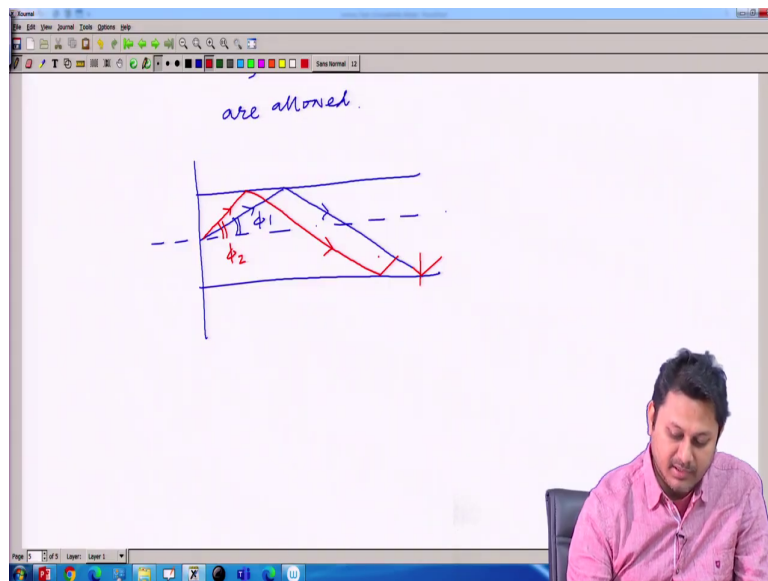
(Refer Slide Time: 13:26)



So, I can write, note: - only we find already find that only discrete values of beta are allowed. So, we already figure out in the previous when we calculate the mode and all these things that the beta value depending on the cutting point of this transcendental equation from that we can extract the value of beta and that is discrete in nature. So, that means, beta value is not continuous it is discrete.

Now, if it is discrete that means, which means only discrete or selected angles are possible. Discrete values of angle phi, angle phi of propagation are allowed right, only discrete angle of phi or propagation is allowed.

(Refer Slide Time: 15:32)

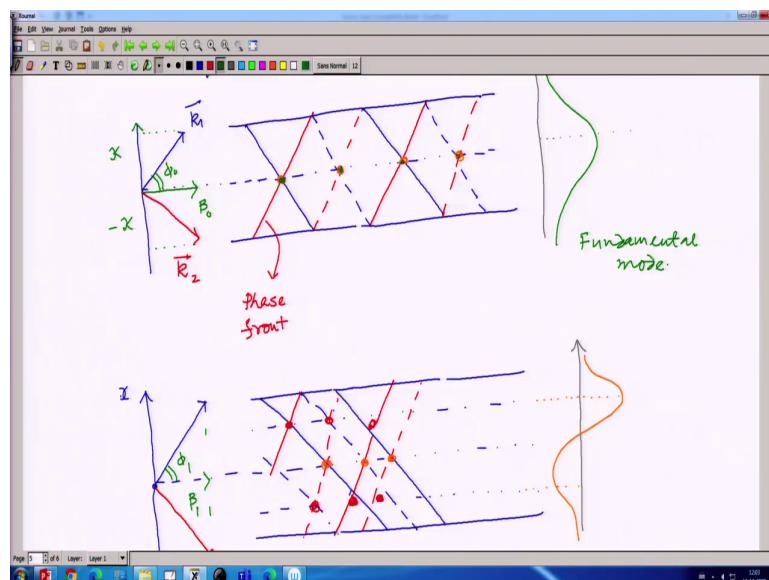


So, that means, when the ray is propagating and that is not a very new thing or surprising thing we know that and we also prove that in a different way when we deal with the ray optics. That if I launch a light it can propagate and this is a certain angle say ϕ_1 . And the next slide that is going to propagate is maybe some another discrete angle say ϕ_2 .

So, in between ϕ_1 and ϕ_2 there should not be any angle that we are going to propagate and that we already figure out when we try to find out in the concept of ray optics inside the waveguide. So, certain angle are allowed and this also here we can prove these things by using the symmetric mode concept.

So, in the mode picture from the mode picture also we can extract the same information that sudden there are certain angles for which the rays will going to propagate.

(Refer Slide Time: 16:46)



Now, we like to understand another important thing let me draw that. So, far what we have, so say this is along z direction and I have my say k vector in this direction k 1 and my k 2 is this direction. So, this is my k 2 vector and this is my k 1 vector and with the combination and this is suppose I write it as beta 0 fundamental one along this direction. I have kappa and this is minus of kappa that was the structure I figure out.

So, now these two wave vectors will going to propagate inside a waveguide and let me draw this waveguide first. Suppose, this is the waveguide, I request you to please understand the concept because my drawing may not be very correct because I am drawing that by using my hand. So, obviously, there is some restriction limitation of this drawing.

If you want to find the concept you can go through the books where this concept are given, but qualitatively you can understand whatever the figure. I show, qualitatively you can

understand exactly what is going on actually physically. So, when these two k vectors are moving like this inside the waveguide for a guided ray, so, for k_1 I can have the phase front moving like this. So, one phase front is here which is perpendicular to the k_1 vector.

In a similar way, I can have a phase front that is moving perpendicular to the k_2 and I can have something like this. And, then I have the say I have another phase front say here, say like dotted one. Because the phase front is also moving is a sinusoidal wave. So, I have a peak and then I have a dip.

So, it is something like this, then again I have a phase front like this and I have this and so on. So, when these two when these phase fronts are propagating again I have let me draw let me complete this I have one phase front like this, another it should be like this. So, when it is propagating these phase fronts are propagating so, let me draw this right this is a phase front.

For k_2 if the phase front is in red color and for k_1 the phase fronts are in blue. But important thing is at these points I have the constructive interferences, at this point every point and these points is in such a way that it falls exactly in the middle of that waveguide.

If that is the case then what I will going expect at the output is something like this. So, this is my x-axis and the output the field distribution let me draw it in different color will be like this I have a maxima here. Because I have all the interferences constructive interference sitting here in these points and as a result I will have.

So, this is my fundamental mode and this is the way one can correlate the mode distribution in terms of β and all these things. In a similar way you can also have the field distribution of the second asymmetric mode. Now, the calculation will be same, only thing is that instead of $\cos$ you can have a $\sin$ here for anti symmetric mode, but the rest of the thing will be same.

Now, for that let me draw once again this picture. So, I have a draw the z-axis first. Suppose this is a z-axis my k_1 vector is this one and my k_2 is this one, and here I have the value of

say another value β_1 . Previously I have β_0 and now, I have β_1 mind it this β_1 should be less than β_0 obviously, because this is a higher order mode.

And this angle say previously it was some angle this angle say ϕ_0 . And, now, I have another angle another mode is propagating with another angle with ϕ_1 . If that is the case and the rest of the part are same along this direction I have x and obviously, y is perpendicular to this. I am not drawing that.

Now, again I have the waveguide in my hand. So, this is the waveguide I am having. So, let me draw it properly otherwise better to write this way, ok. So, my z -axis should be in between. So, my drawing should be very precise, otherwise I am going to miss the it should be roughly at the middle of this. My drawing is again I am saying my drawing is not very much perfect, but you should understand the concept that is important.

Drawing and all these things you can find very good drawing you can find in good books that is not the issue. But the issue is whether you understand the concept or not, and I am trying to do that only to make you understand the concepts. So, here again for I have the propagating suppose I for red I can have like this and then another.

So, a front like this, and I can have a wave front. I can have a wave front for this as this. So, my drawing is again not very much correct. So, it should be something like this. So, this wave front is something like this. For here again I can draw a front sorry, it should be red color. So, mind it the color.

So, the red color would be something like this. It should be perpendicular. My drawing is not correct, but the important thing you should note here, that here I am having a interference. Here I am having interference. In the similar way I can have an interference here also, I can have an interference here also.

It should be this line should be here. So, again my so, here I have interference. But, in the middle part what I have is the in the middle part every time, I am having the destructive

interferences if you look carefully. So, now, again I should draw that. So, from this point I should have a dotted line, oh it should be red color ok, it is red.

So, in the middle part again I have a destructive interference here and here also I am having the constructive interferences, blue line. So, if I draw that it should be something like this. So, I have a constructive interference here. I should have a constructive interference, so, this point should be here and then I have a construct. So, this point again should be somewhere here.

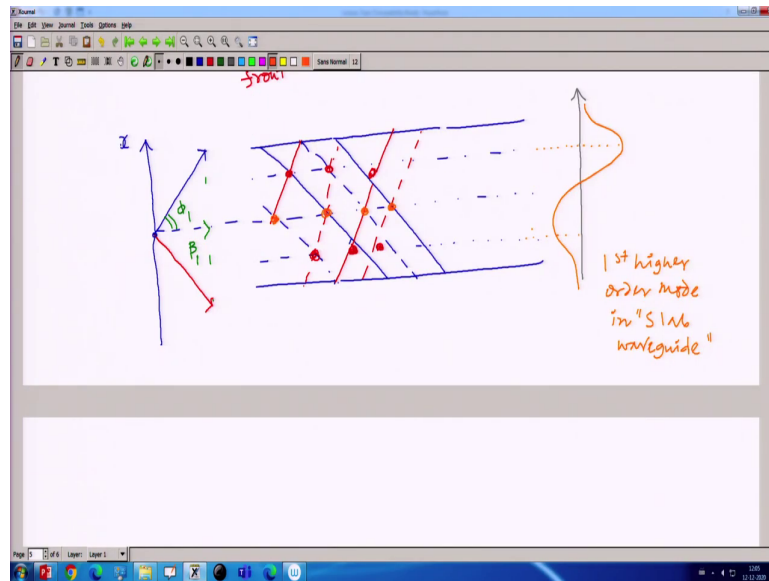
So, my drawing is little bit improper, but the important thing is that these wave fronts will move in such a way that every time when it is crossing here. And here it should form a constructive interference and in other case it will form a destructive interference. So, let me draw this constructive interference. So, if I now draw the constructive interference it should be something like this. And then in the middle part there is 0 and again, I can have a constructive interference. So, I have that.

So, these are the two regions where we have a constructive interference and as a result I have a maxima, here in these points and here I have a destructive interference middle every time. So, I will going to have a minima here. So, this is the way these modes are propagating. Today I do not have much time.

So, in the next class, I will try to draw this in proper manner, so that you can realize exactly what is going on. But also you can do that in your hand by using the scale and properly find out the points where you have a construct, where you need to have a constructive and destructive minimum.

Accordingly, you constructive and destructive interference based on that you have a maxima and minima, and based on that you can draw the lines like that. So, this is something I like to mention that this is conceptually very very important. So, this one is the for fundamental mode. So, this field I will have for fundamental mode, this field distribution and this field distribution is coming as a higher order mode first higher order mode.

(Refer Slide Time: 30:15)



So, first higher order mode in slab waveguide. So, in the slab waveguide with these simple drawing; however is not so simple for higher order modes. I again I suggest you to please go through the references the books I mentioned in the very first class. In this books these figures are nicely given with proper dimension and all these things.

Here my drawing is not that good the perpendicular thing is not correct. So, that is why I am not having the correct points, but at least I make you understand what is the concept of the modes when it is passing through in a medium in a waveguide having the refractive index n_1 and n_2 . And these β_1 and β_2 these are the propagation β_1 and β_0 are the propagation constant.

And, certain discrete angle for which it is propagating and when it is propagating what happened the phase front will also propagate. And they will go to make a constructive and

destructive interference during the path and as a result I have a overall mode distribution. And, this mode distribution is coming in this way: if you go higher and higher order modes then what happened? That you will have more complicated structure like this.

But, the basic concept is same if you draw properly every time you are going to get the concept of mode. So, with this note I like to conclude today's class in the next class we will study more on this modes how the power is what is the amount of power that should that a mode should carry will be covered.

And, that is also important that you need to know that how the power is following and what is the value of the power a mode going to contain during the propagation. So, with that I like to conclude here.

Thank you for your attention and see you in the next class.