Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 27
Waveguide Dispersion

Hello student, to the new class of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number 27 and we will going to cover a very important concept called Waveguide Dispersion.

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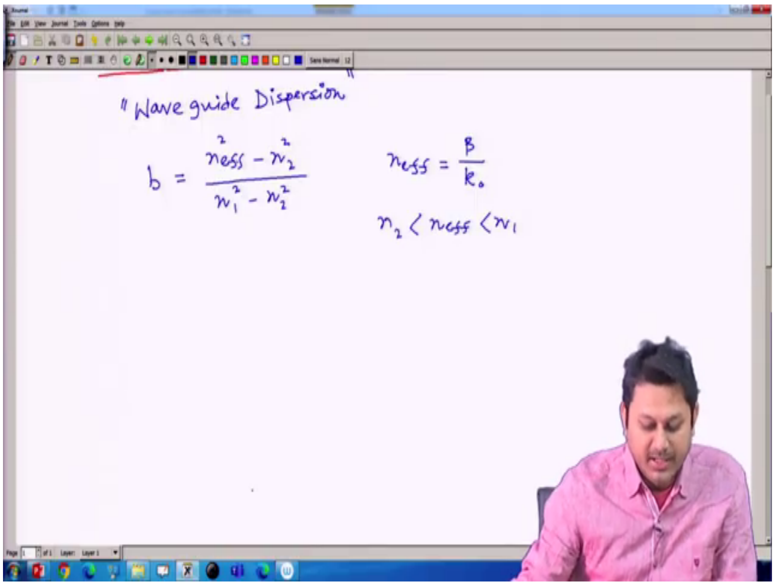

So, today we will going to cover something called Waveguide Dispersion. Well, we already defined the b parameter as n effective square minus n 2 square divided by n 1 square minus n

2 square, b is a normalized propagation constant and n effective is this. Well, normally and also n effective for guided modes should be in between in an n 1 and n 2. Normally n 1 and n 2 are very close because this is the refractive index of core and cladding and we make it very close to each other.

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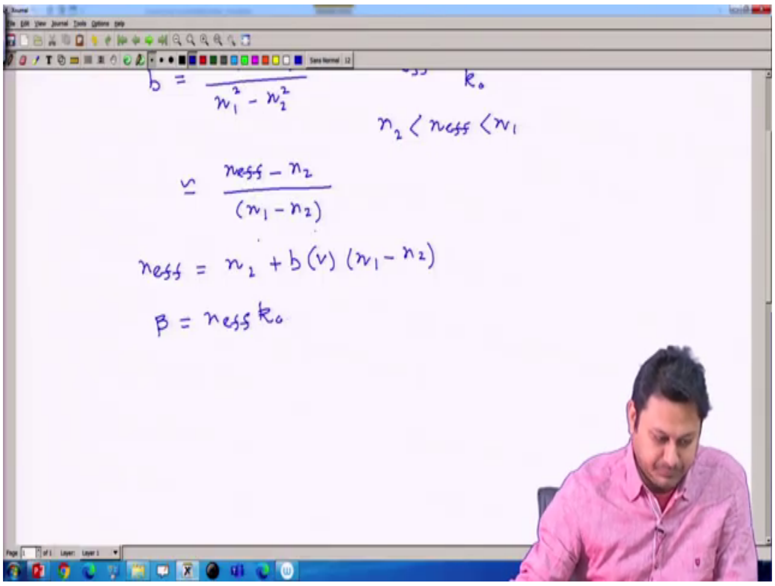

So, this equation can be approximated as n effective minus n 2 divided by n 1 minus n 2, well from that I can find out n effective as n 2 plus b function of v. In the last class we find that b is a normalized propagation constant and it should be a function of v, with changing v we find that b will going to change. Now, beta already I defined in terms of n effective it should be n effective multiplied by k 0.

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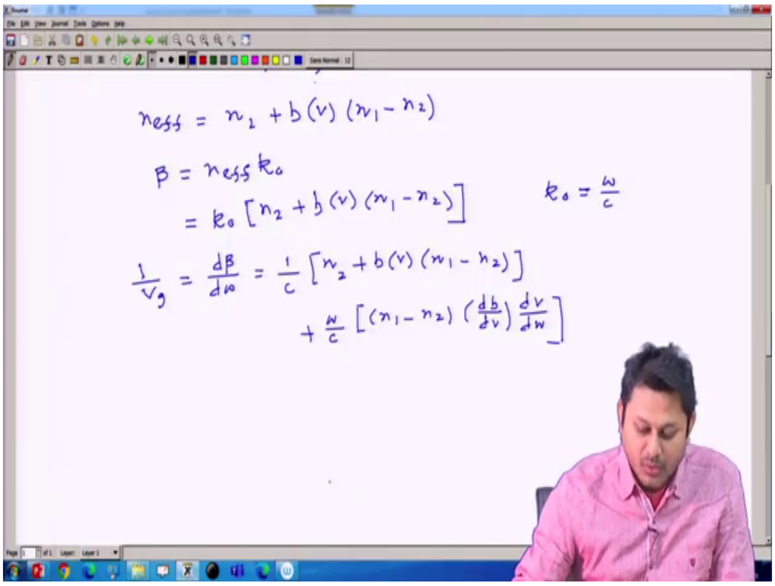

So, it is eventually k 0 multiplied by n 2 plus b which is a function of v, then n 1 minus n 2. Now the group velocity dispersion in order to calculate I can first calculate the inverse of group velocity which is d B d omega. Now, we know k 0 is equal to omega divided by c. So, I should have a omega sitting here psi I can write it as 1 by c because the first derivative that, then n 2 plus whatever the term I have and then I have second term making the derivative with respect to omega.

So, I can write it del b del v and d v d omega chain rule. Please note that when I make a derivative with respect to omega I consider the n 2 and n 1 is not depending on omega or I am not taking the derivative of that.

This is because in this calculation we try to find out the waveguide dispersion and the variation of n 1 and n 2 with respect to omega is already taken care in the case of chromatic dispersion. So, I will not going to take any this term here, only try to find out what is the waveguide contribution for this dispersion.

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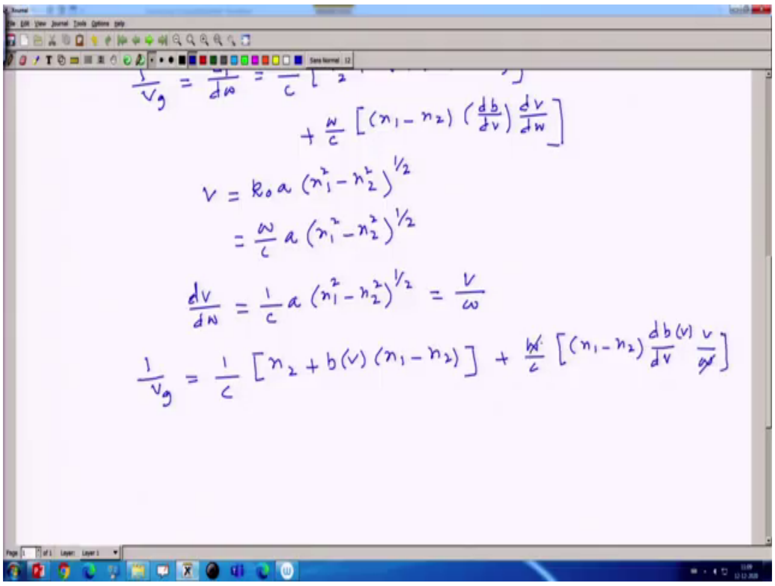

Now, v we know v parameter is defined as k 0 a n 1 square minus n 2 square whole to the power half by definition. Now I can write it as omega divided by c a n 1 square minus n 2 square whole to the power half. So, d v d omega this I can figure out and it turns up to be 1 by c a n 1 square minus n 2 square whole to the power half, I can simply write it as v divided by omega.

Well, then 1 by vg will be equal to 1 divided by c this term the first term I should write the first term multiplied by n 2 plus b function of v n 1 minus n 2. And then second term I put plus let me write it here omega divided by c then n 1 minus n 2 d b which is a function of v divided by d v and this derivative del v del omega which I calculate and it comes out to be v by omega. So, this omega and this omega is going to cancel out.

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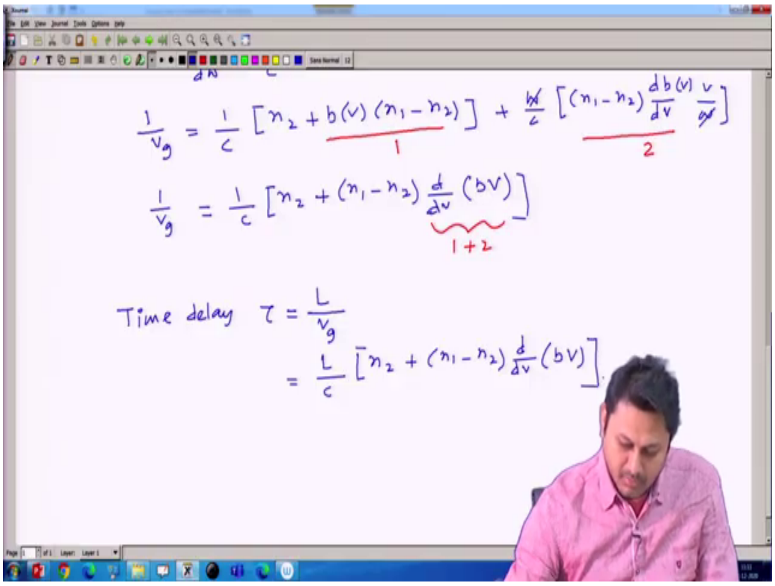

So, eventually I have 1 by c n 2 plus n 1 minus n 2, if I take n 1 minus n 2 then I will find that these things can be written in a compact way and this is del del v of v multiplied by v. Because if you make a derivative of this quantity b multiplied by v you will have this term and this term this is 1 and this is 2 this term is basically combination of 1 plus 2.

Well after that we will find so I now I know what is my 1 by vg. So, the time delay so the time delay which is tau in general defined as L divided by v g, so here it should be L divided by c multiplied by n 2 plus n 1 minus n 2 and then d of d v b multiplied by v bracket it. So, this is my time delay.

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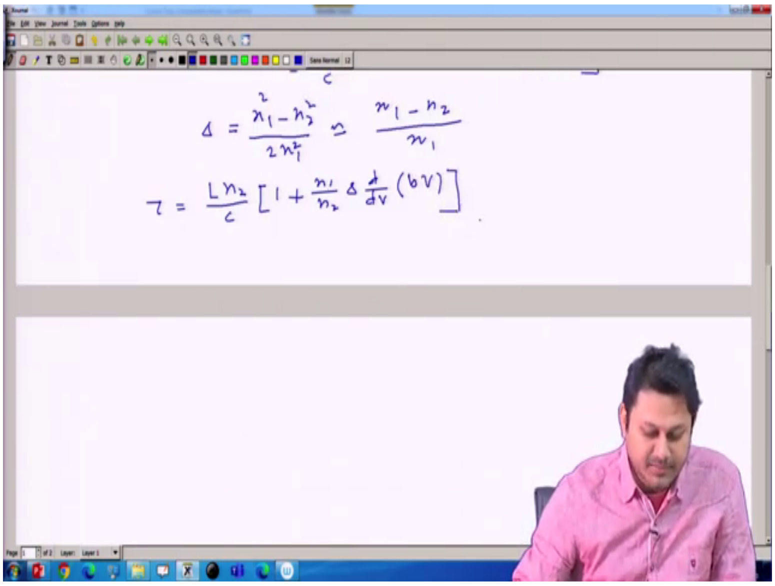

Now, I can slightly modify this expression, because we know delta is equal to n 1 minus n 2 because every time it is not I should not write n 1 minus n 2. So, this is square of that and 2 divided by n 1 this is n 1 square this is by definition. So, I can approximate as n 1 minus n 2 divided by n 1 that is my delta. So, I will going to replace this in the next line and then going to find the value of the quantity time delay in terms of delta.

So, eventually my tau will be L n 2 divided by c 1 plus I just take n 2 common and it should be L n 2 and then I have n 1 divided by n 2 delta and d of d v these quantities. Now if I want to find out what is the change of this delay with respect to delta that basically gives me the idea of dispersion.

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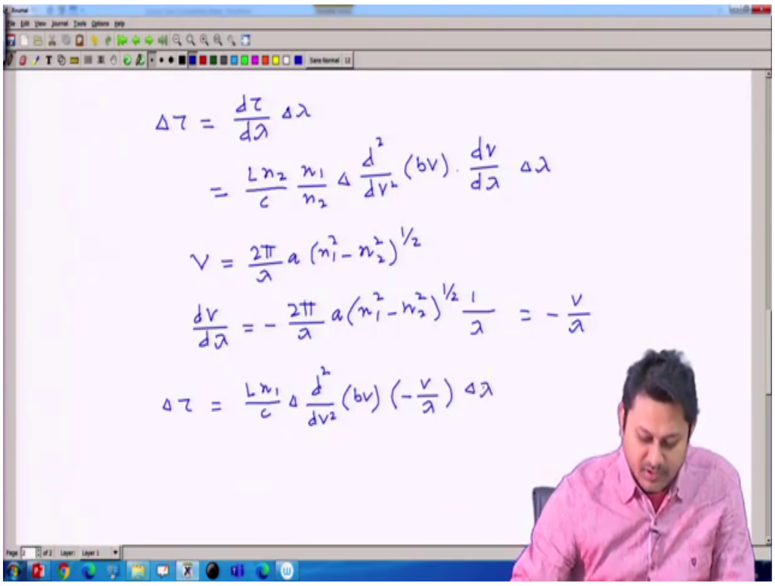

So, change of t which is delta tau equal to with respect to the wavelength, this calculation we already done in calculating the material dispersion I have to do that.

So, this quantity if I make a derivative with respect to lambda for this quantity, you can see that the first in tau the first term is not depend on lambda. I already mentioned that I am not considering n as a function of frequency or wavelength here when I calculate the waveguide dispersion. I take account these things in a when I calculate the chromatic dispersion.

So, here the first term is a constant and the second term the contribution of the second term I will have the contribution of the second term through this v, because v is a function of lambda now.

So, I can write it as L n 2 divided by c then whatever I have n 1 n 2 delta and now I have to make a derivative with respect to lambda. So, what I do here I make a derivative with respect to v first, so that I have the double derivative with respect to v then b v and then del v del lambda using the chain rule.

And finally, I have so here I am making a mistake it should be delta lambda. Well, again I can have the relationship v I know it is 2 pi divided by lambda a n 1 square minus n 2 square whole to the power half. See, if I calculate d v d lambda it should be simply minus of 2 pi lambda a n 1 square minus n 2 square whole to the power half. I have one lambda here so I put another lambda here it should be 1 by lambda square and then I can write it as minus of v by lambda.

So finally, I can put this here and my delta tau becomes my delta tau becomes L n 1 divided by c, then delta this double derivative should there as usual and then this derivative I just figure out it is minus of v by lambda and delta lambda. So, this is my delta tau now once we know my delta tau I can define the waveguide dispersion.

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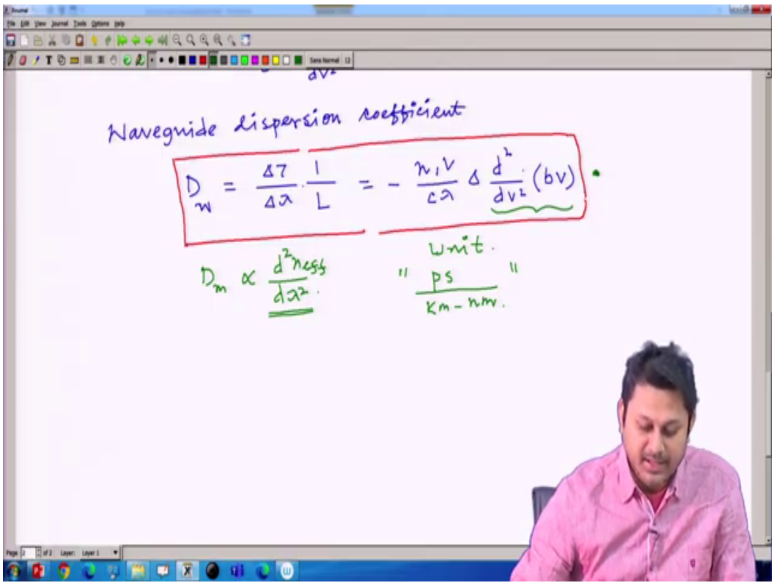

So, the waveguide dispersion coefficient the waveguide dispersion coefficient I write it as D w, for material dispersion we defined as D m here we going to define at D w, which is the same definition that rate change of tau with respect to the change of wavelength with unit distance.

So, this quantity eventually minus of n 1 then v divided by c lambda and then I have a delta and I have d 2 d v square and b v. So, this is basically the mathematical form of the waveguide dispersion and in order to find out what is the waveguide dispersion the important thing that we now need to calculate is this quantity.

So, this is now become very important this derivative you may remember that when we calculate the material dispersion I have a double derivative for material dispersion D m it was proportional to the quantity n effective lambda square. So, during that time this quantity was very very important, here also we have a similar kind of quantity, but in terms of b and v and now I need to find out what is the variation of this quantity b multiplied by v as a function of v and make a double derivative of that, that basically gives me that what is the value of the waveguide dispersion.

And unit for this is if you find it should be picoseconds per kilometer nanometer, as usual which we already find for material dispersion. Because eventually these two dispersion are measuring the same property of the propagating wave in a dispersive medium that is why their unit has to be same.

Now, the next thing is how to find out this because I do not have any idea about the b v how the function of b is varying with v, however I have a picture for that.

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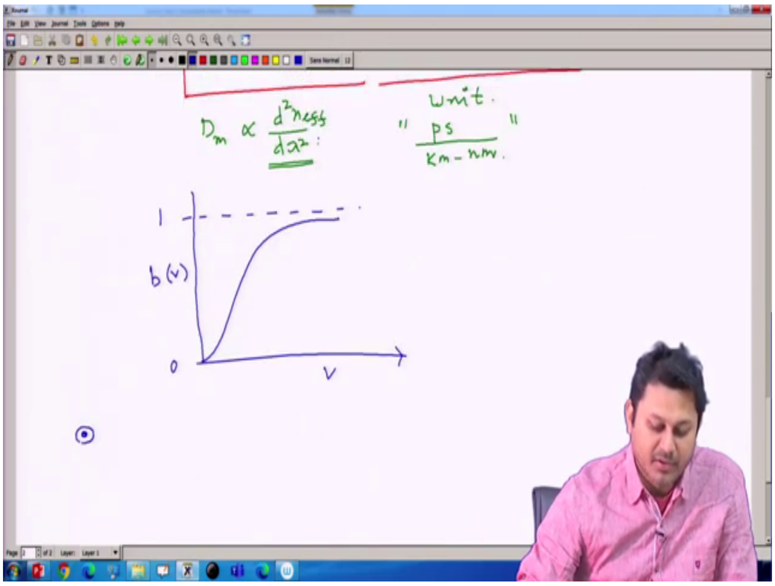

So, last year I draw that if I draw a picture b as a function of v for say fundamental mode it should be something like this that was some sort of picture we have. So, this is along this direction I have a b mind it here b is a function of v. So, I should write it here better to write in this way b which I plot is now function of v.

So, this plot we already find now if I if somehow I know what is the form then I can calculate the value of d omega D w because it is very much depends on the double derivative of this quantity. So, there is the way to find actually.

So, one way that you calculate all the b as a function of v and then make the double derivative with respect to this and then you find what is your dispersion material this waveguide dispersion for a given value of v. Now, this can be simplified analytically at to some extent we can do that using some empirical formula.

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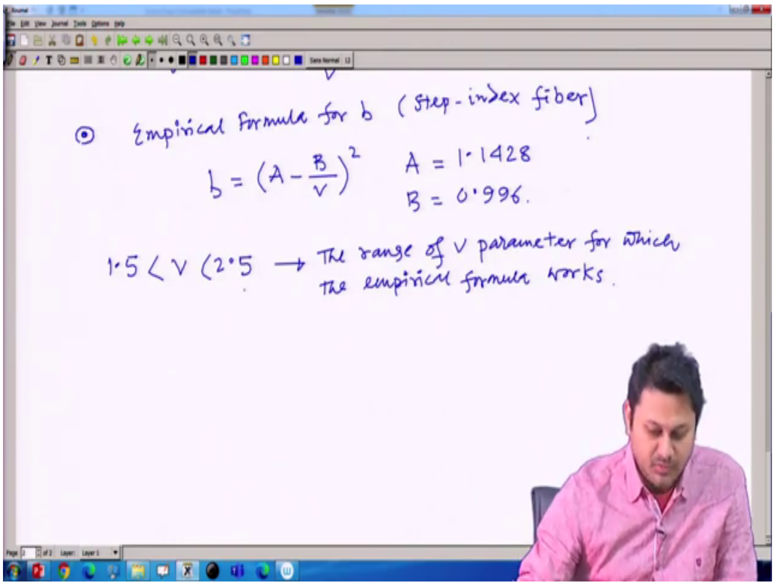

So, we can introduce the empirical formula for b, this is for say step index fiber true for, step index fiber and this empirical formula reads like this, where a b are constant and the value is precisely given as 1.1428 and b is 0.996 these are the two constant value of the two constant and b is x the form of b as a function of v is given in explicit manner, so that I can do the derivative.

By the way this empirical formula is not true for all the v there is a range. So, the range of v is this at this range the empirical formula works well. So, this is basically the range of v parameter for which the empirical formula works. So, this is a range of v for which the empirical formula works we need to keep these in our mind that it should not work for all the range of v.

But anyway we can now with this range what we can do we can able to find out the value of the dispersion coefficient, because now I know these functional what is the functional form. So, if I calculate this quantity del del v.

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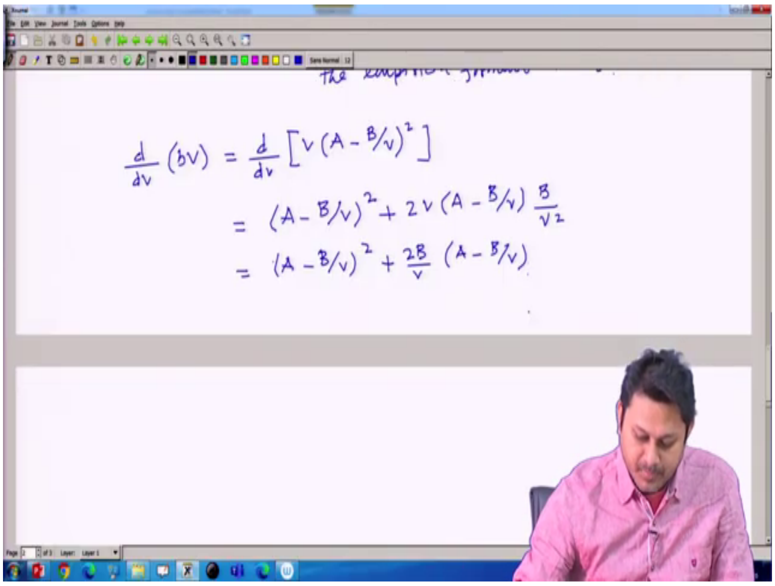

So, as a function of this so eventually I will calculate d the function of v multiplied by v b is a function of v is given it is A minus B divided by v and then whole square of that. So, it is simply whole square plus first derivative of that and second derivative of this.

So, it should be 2 of v of A minus B by v and then I have B of v square with a negative sign, but this negative sign will be absorbed by this one. So, eventually if I simplify then it should be A minus B divided by v then square of that and 1 we will going to cancel out. So, it should be plus of 2 B divided by v and then A minus B v.

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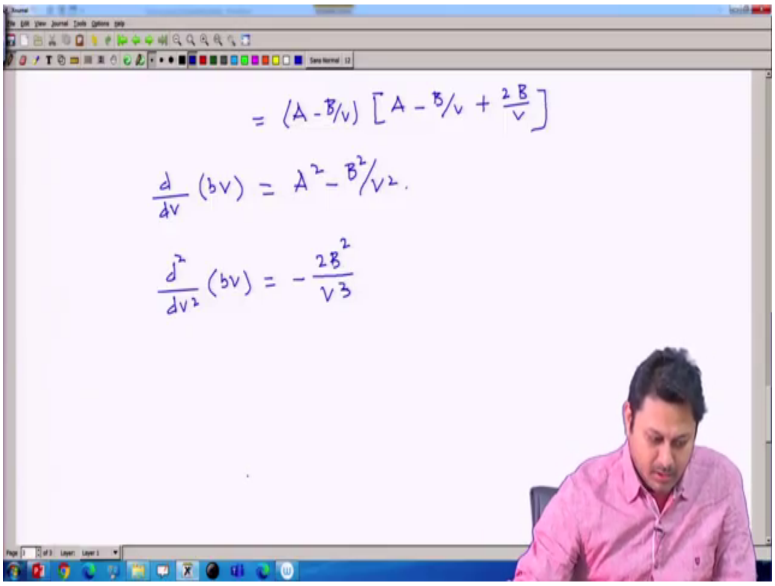

If I take A minus B v common A minus B by v common then I should have A minus B by v plus 2 B by v. So, it is eventually A square minus B square by v square straight forward calculation. So now, I have this quantity, so left hand side what I get d 2 v 2, sorry this is the first order derivative. So now, I need to calculate the second order out of that.

So, second order I can calculate by just making so this is first order. So, I find this is b v this I find figure out. So, the second order from this the second order I can calculate that d 2 d v square b v which is equal to minus of 2 B divided by v cube B square divided by v cube. Well, after that if I want to calculate this then there is a multiplication of v, so I need to multiply the v for that. So, v d 2 d v square b v is minus of 2 B square divided by v square.

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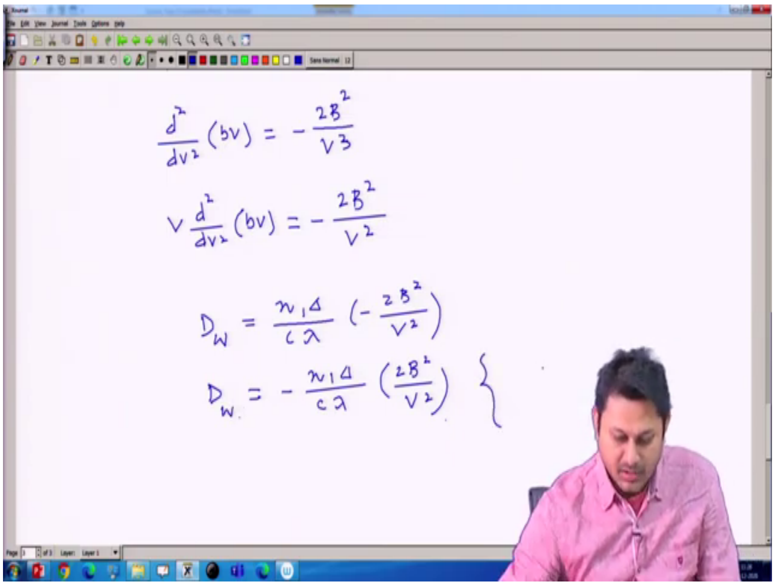

So, now we have my material waveguide dispersion as n 1 delta divided by c by lambda and v that quantity which is minus of 2 B square divided by v square this. So, this is eventually minus of n 1 lambda c lambda let us write it as simply 2 B square divided by v square. So, this is my D omega for this particular range of v, so for range of v so I already mentioned that the range of v is 1.5 to 2.5 for this range.

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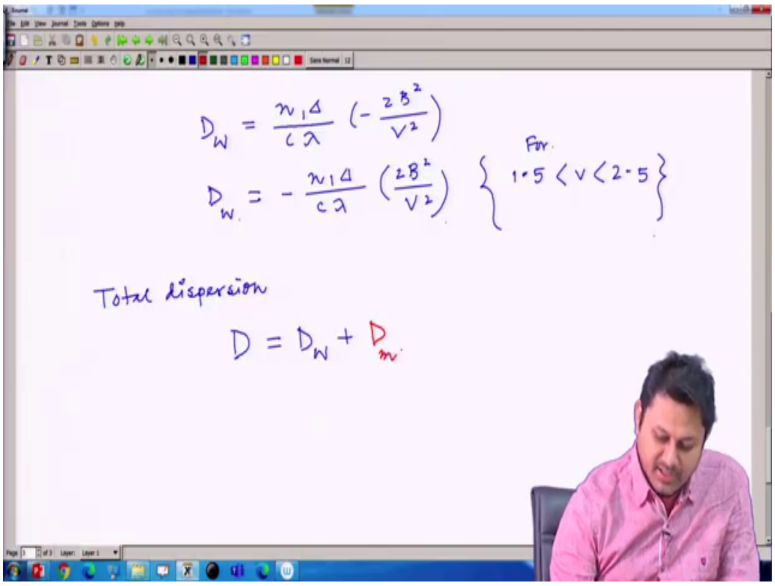

At least this is my 1.5 to 2.5 this equation works well. So now I have 2 dispersion, so let me now so the total dispersion eventually, if somebody want to calculate the total dispersion I can write D as a total dispersion and now it should be the combination of two dispersion one is waveguide dispersion and plus another is the material dispersion, which we already calculated in the previous class. If I add these two then I will go to get the total dispersion, now what is the material dispersion?

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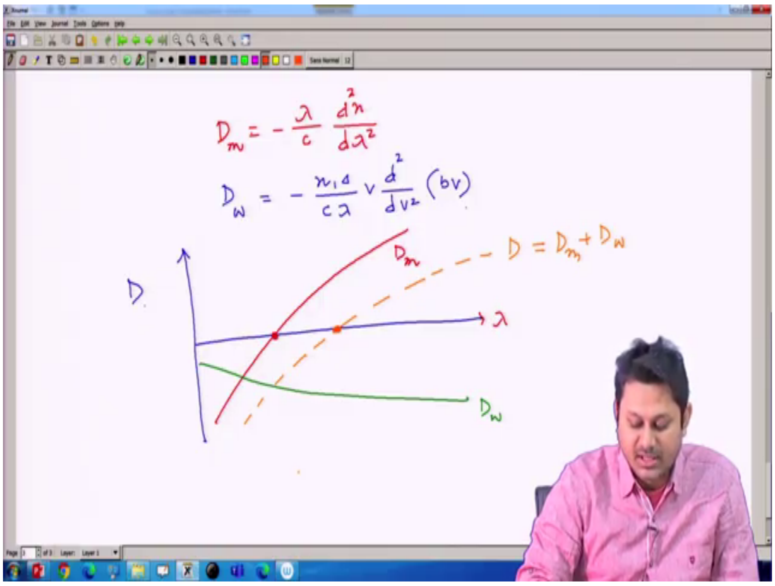

So, let me write it once again the expression. So, material dispersion is minus of lambda c d 2 n d lambda square that was the expression of the material dispersion. And for waveguide dispersion we just define the expression which is minus of n 1 delta divided by c lambda then v in general d 2 d v square and b multiplied by v. So, these are the two expressions for material dispersion and waveguide dispersion.

Now, if I add this two thing then I will get the total dispersion. So, what happened that I now plot the total dispersion D, so the material dispersion normally follows the curve like this some point it has a 0 and it is basically the way we calculate the material dispersion is something like this. So, along this direction I have lambda.

On the other hand the waveguide dispersion if I plot it should be something like this. Now the total dispersion, the total dispersion let us make in a different color is the combination of these two.

So, I should have something like this so this is my total dispersion this is the total dispersion D which is the combination of D m plus D w and you can see that by changing if the material is same. So, material dispersion is not going to change, but changing the geometry of the waveguide we can manipulate the total dispersion.

For example, here this 0 wavelength can be shifted from this point to this point by just changing the waveguide geometry and if I change the waveguide geometry if I modify the waveguide geometry. Then what happened that we can very nicely modify the dispersion and I can have my required dispersion total dispersion as per my choice.

So, this is a handy technique to properly find the required dispersion. So, one can manipulate this by just changing the geometry of the waveguide. So, with this note today I like to conclude. So, in the next class we will learn more about the modes.

So, thank you for your attention and see you in the next class.