Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 26
Modes in Slab Waveguide (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 26; and in this lecture, we will continue to our discussion which is Modes in Slab Waveguides. So, in the last class we draw the modes and we find. So, let me draw once again quickly how the modes will look like.

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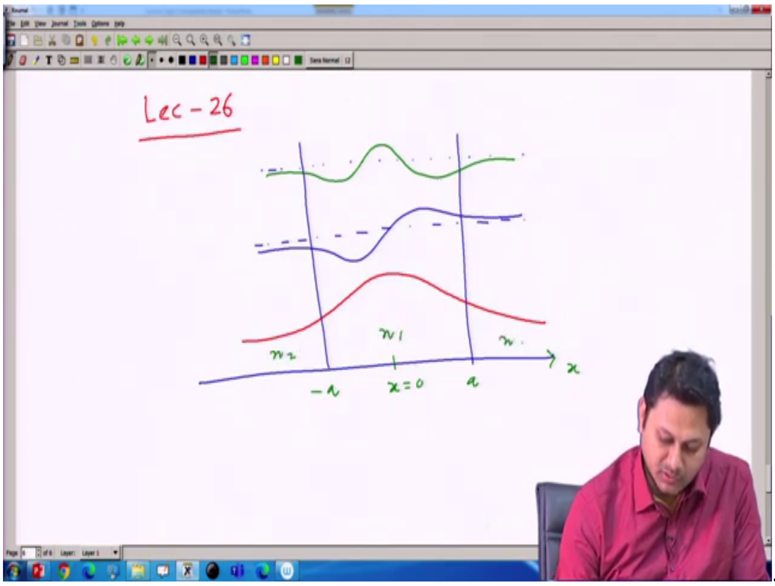

This was the fundamental mode. Then the next higher order mode which is anti-symmetric is in nature was like this and also I have I another mode which was something like that. So, these are the distribution of the mode.

So, this is at the point minus of a, this is at the point a, this is x direction and this point is at x equal to 0. Refractive index was n 1, this is n 2, this is the cladding part where the mode decays exponentially. And in 3D also we need to understand how the modes will propagate in 3D.

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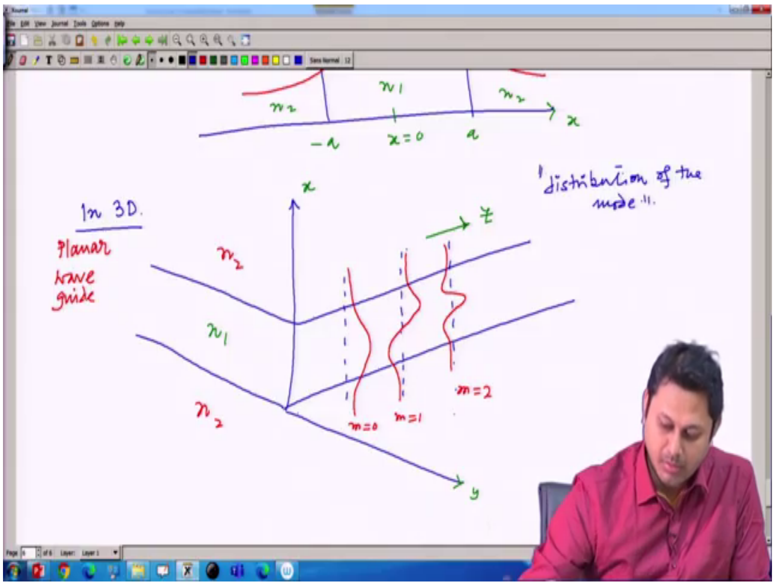

So, this is the 2D dimensional picture. So, in 3D how the modes will propagate? So, let me give you an idea about that. So, this is roughly the core part and waveguide is defined in this way. So, this is suppose the waveguide. So, I have a cladding region here, refractive index n 2, refractive index n 2 and this is refractive index n 1 in the core part.

And along this direction I have y, along this direction obviously, it should be x and this is the direction along which the we have a propagation. So, it is z. Now, if I want to find out how the mode will look like then it should be something like that. So, this; so, the fundamental mode will look like this.

It will something like this. This is a fundamental mode. For higher order modes for higher modes this part I need to draw once again, something like this. This is for say m equal to 0, this is for m equal to 1 and if I go on to higher order modes, so it should be something like this. This is the next one say m equal to 2 and so on.

So, these are the way the mode should be distributed and it should look like this when it is propagating in a waveguide in the planar waveguide. So, this is in a planar or slab waveguide. This is the distribution of the mode; this is the distribution of the mode ok. Now, we will going to introduce an interesting parameter called b parameter, so which is dimensionless nature.

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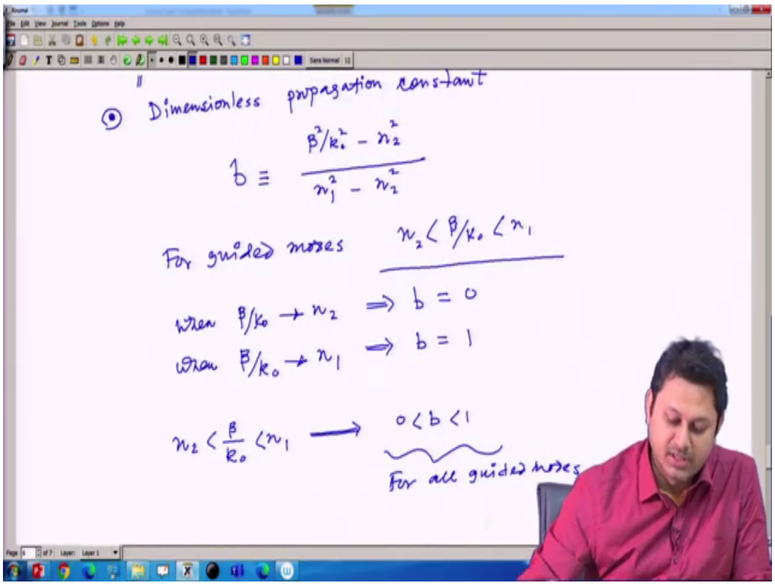

So, let me introduce in this way. It is called dimensionless propagation constant. The dimensionless propagation constant is b, we which you call the b parameter which is defined in this way; b is beta square divided by k 0 square minus n 2 square divided by n 1 square minus n 2 square. So, this is the way one can define the b parameter which is dimensional.

And now we know for guided for guided modes we have a restriction over b beta and we know that. So, beta divided by k 0 is less than n 1 and is greater than n 2. This is the restriction we have. So, if I put this restriction over here in this dimensionless propagation constant whatever the definition we have you can see that when beta by k 0 tends to n 2, we have the value of b is equal to 0 and when beta by k 0 tends to n 1, we have b equal to 1.

So that means, I can reduce this risk guided mode restriction which is in terms of beta divided by k 0 where beta is a propagation constant and into a straightforward relation in b. So that means, n 2 less than beta propagation constant divided by k 0 n 1 this restriction for all guided mode can be simply written in terms of b which is a new assign new defined dimensionless propagation constant since it is a dimensionless.

So, this restriction will leads to a new restriction over b which is much more simpler and this is for all guided modes, for all guided modes. For all guided modes this restriction one should have in terms of b. Now, we want to map the modes that I mentioned in the starting of the class. So, in order to map the modes what we will do?

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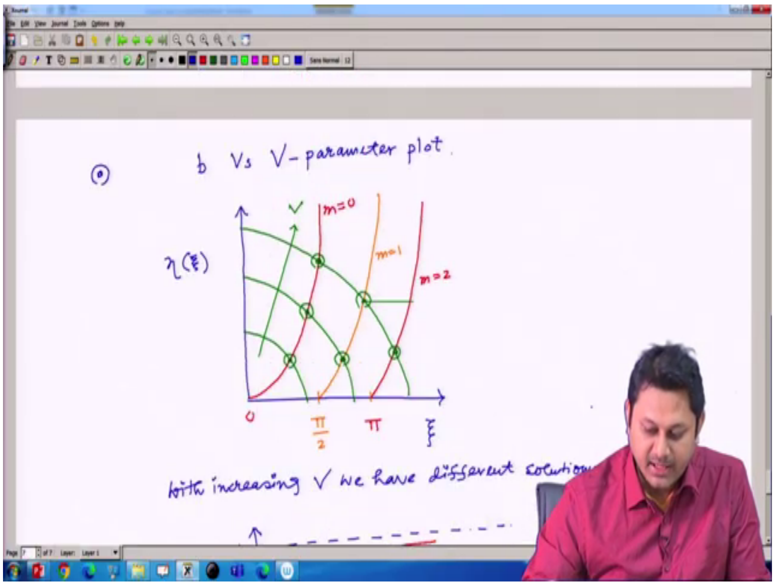

We will plot something called V versus b parameter plot b versus V parameter plot that will going to plot. So, that basically gives me a lot of information that is why this plot is important. So, how you plot that let us check. So, let me draw once again this transcendental equation that we have been drawing for last few classes. So, these are the plots let me draw another.

So, this one, where red is the symmetric one and this orange color is the anti-symmetric one. So, suppose this point at pi by 2, this point at pi, this is 0 and along this direction I have my xi and I plot all the all this tan xi and all this stuff. Actually I am plotting eta as a function of xi according to my notation.

Now, I am plotting the V parameter as well and every time I am having the solutions here; one solutions, again I have a solutions here, again I have solutions and again I have a solutions like this; three solutions and so on. So, with increasing V, so, what eventually we do along this direction?

We basically increase the value of V. So, let me do that in this way. So, along this direction I am increasing the value of V. So, with increasing V what we are getting? With increasing V we have different solutions. With increasing V, I have a different solution in terms of xi or different solution in terms of beta.

Every time when I have a solution here in terms of beta I can readily find out what is the b using this dimensionless definition of b. So, every time basically I will going to get a value of b.

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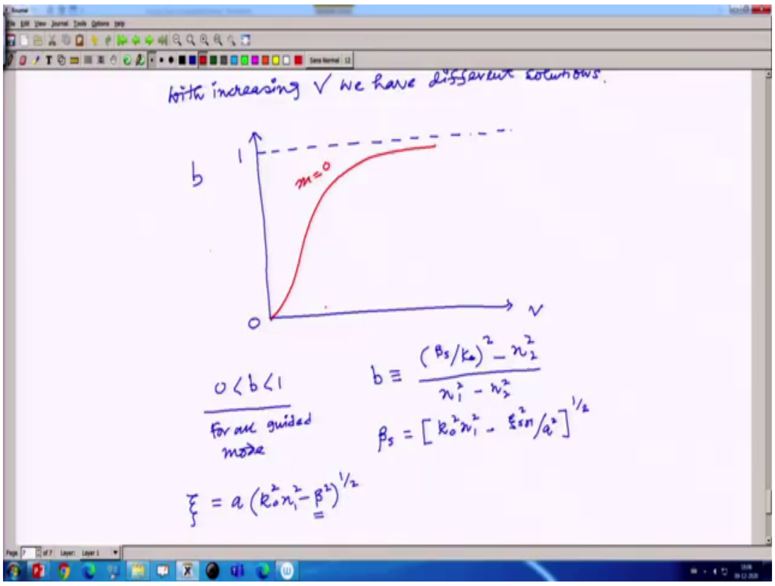

Now, if I plot these solutions in terms of b with V, so, along y axis I will going to plot b and we know that there is a highest value over b. b is restricted from 0 to 1 and along this direction I plot my V. So, this is 0, this is 1 and along this direction I increase the value of V that we have precisely done here in this figure.

I just increase the value of b every time the radius of this circles will going to increase and every time when I increase the value of b I will start getting the solutions. For this curve I have three solutions here and so on. So, for m equal to 0 that means, only this set of curve.

So, this is my fundamental modes. So, that is why I write it m equal to 0 this 1. So, this is for say m equal to 0, this one for m equal to 1 and this branch again for m equal to 2 and so on.

When I plot b as a function of V, this plot will look something like this. This is a very interesting kind of plot it is something like this.

Now, b let me write it b is in between for all guided modes this is the condition. Now, ok this now b is also. So, all the solutions if I write it should be the beta solutions divided by k 0 square minus n 2 square whole divided by n 1 square minus n 2 square. And beta solutions is this one; whole to the power half because xi using the xi I can I can find out the solutions. So, xi if I remember once again xi was a sorry. So, if I extract the beta out of the solution it will be like this.

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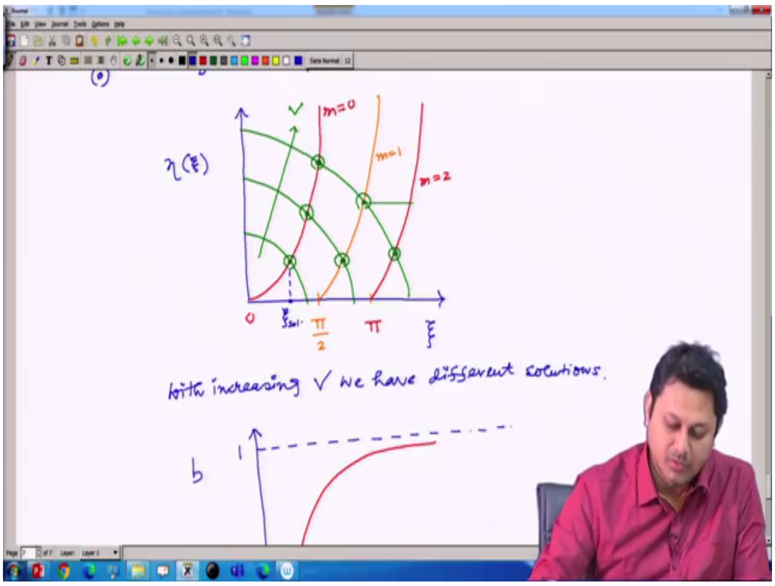

So, every time if I change the value of V and I captured the solutions these are the solutions xi s solution. From xi solutions I know what is the value of beta s, from beta s I can also calculate what is the value of b and I put the points in this beta V plane. So, every value of V, I have one b; every value of V, I have another value of b and so on. And if I start plotting then these are the solutions these are the these are the this is the way the curve will look like.

Again I can have for this is for say one branch m equal to. So, I should write it as say for m equal to 0. So, another solution will start appearing because after increasing the value of V there is a point pi by 2 according to this plot over that if I increase the value of V then it will now have a second solution here. And I have a second branch for b as well.

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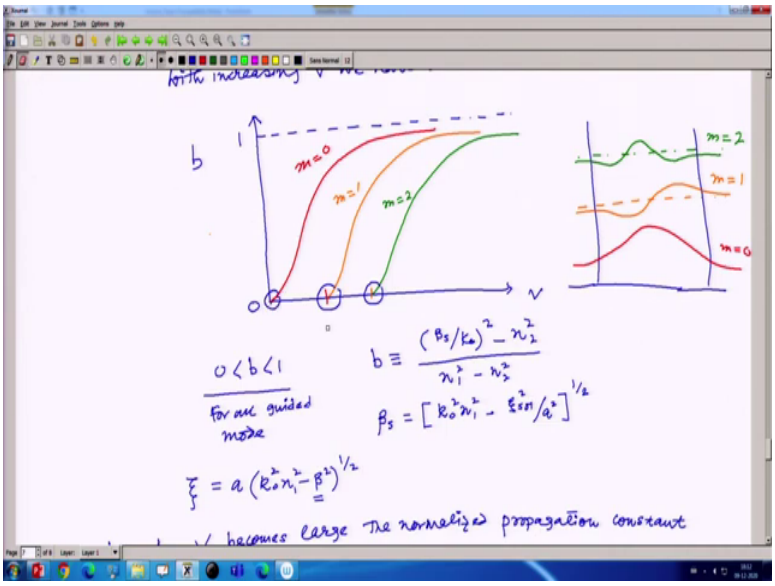

If I draw the second branch of b it is it should look like this. In a similar way I have another branch third branch and so on every time it because the V there is a restriction over b which is less than 1 and greater than 0, so that means, it should not go beyond one value.

So, this is for m equal to ok, I should put the same color in order to reduce the confuse confusion. So, let us put at m equal to 1 and this is for m equal to 2 and so on these are the branches. Now and different modes are associated with. So, if I draw here side by side, so, it is interesting. If I draw that side by sides how the mode will look like? So, ok let me erase this.

So, this is the fundamental mode, red one for m equal to say 0. First order mode it is like this m equal to 1 and again I have a symmetric mode like this. This is for m equal to 2. So, for all these modes I have the relationship between b and V and it is given here. Well, I now try to find out few information from this figure. So, let us write it one by one.

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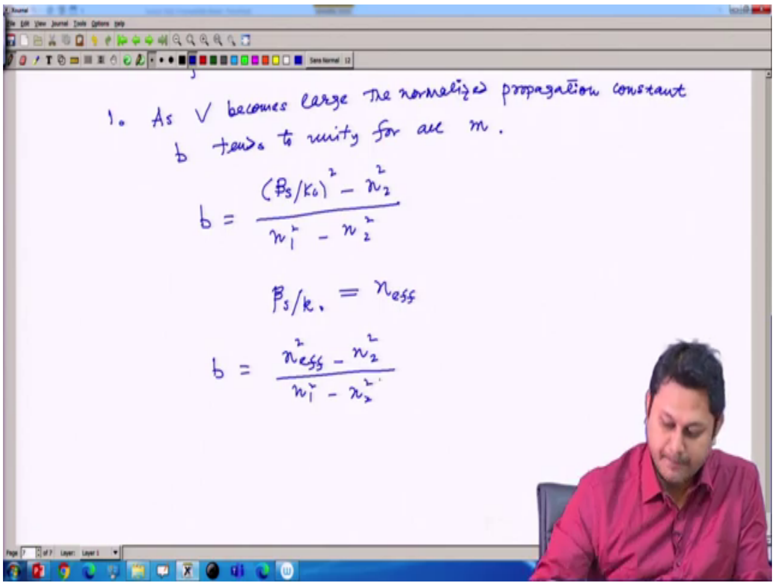

So, first what I note that so, which is trivial as V becomes large. The normalized propagation constant normalize propagation constant b tends to unity for all m. So, whatever the modes we have m 1 2 3. So, on if I increase the value of V, so b will march to the unity. Next ok. So, b by the way b is beta solution divided by k 0 square minus n 2 square divided by n 1 square minus n 2 square.

And we now defined something very interesting which is called the effective refractive index. So, beta is divided by k 0 is called the effective refractive index of the mode. So, when the mode is propagating through the system it will experience effective refractive index and I can define in this way. So, b is eventually n effective n effective square minus n 2 square whole divided by n 1 square minus n 2 square.

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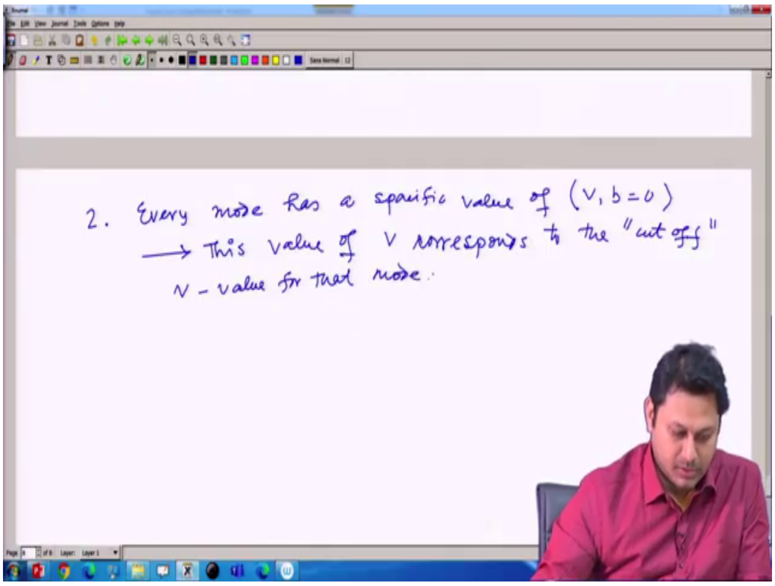

After that I can have so, 2 what I find that every mode has a specific value of V, b equal to 0. So, for every mode you can see there is a point where I have I am talking about this point actually these points are the points where b equal to 0 for a given v these are the points.

So, let me erase this I just wanted to show you these points otherwise it looks very ugly here. So, these are the points where I can say that this is 0. These points are always there for all the modes. So, every mode has a specific value of V, b, 0; this value actually this value of V; this value of V corresponds to the cut off V value for that mode.

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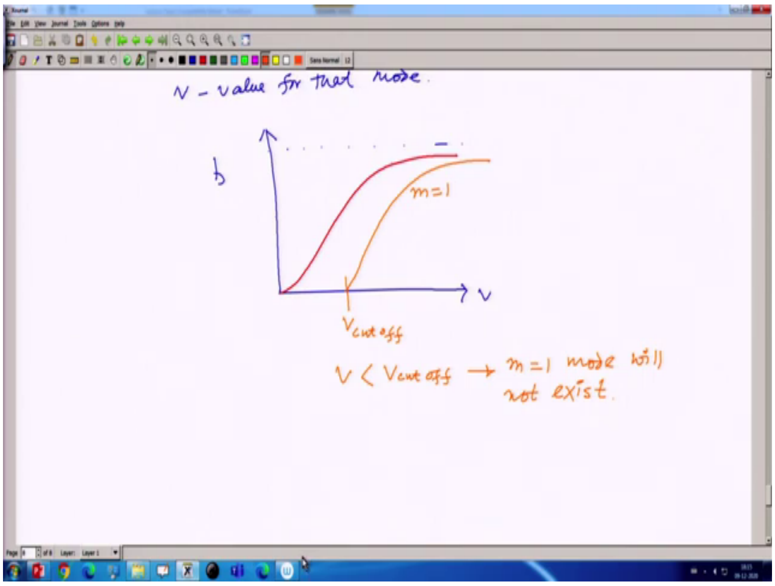

So, let me quickly draw and try to understand already I mention that. So, this is b and if I draw then this is one mode. So, for this mode the cut off there is no such cut off all the modes are allowed, but for the next mode it starts from here and going like this. So, along this direction I have a V and this is for m equal to 1.

For m equal to 1 mode I have a cut off value. So, this is V cut off for m equal to 1. If I have a value V less than V cut off; that means, m equal to 1 mode will not exist. So, if the value of V is less than V cut off according to this figure. So, m equal to 1 mode will not exist at all.

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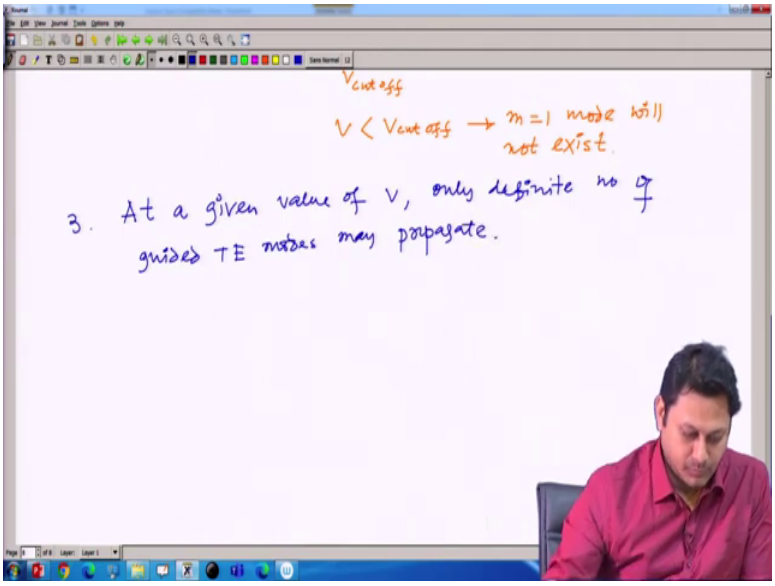

Then quickly 3rd point at a given value of V only definite number of guided TE modes may propagate.

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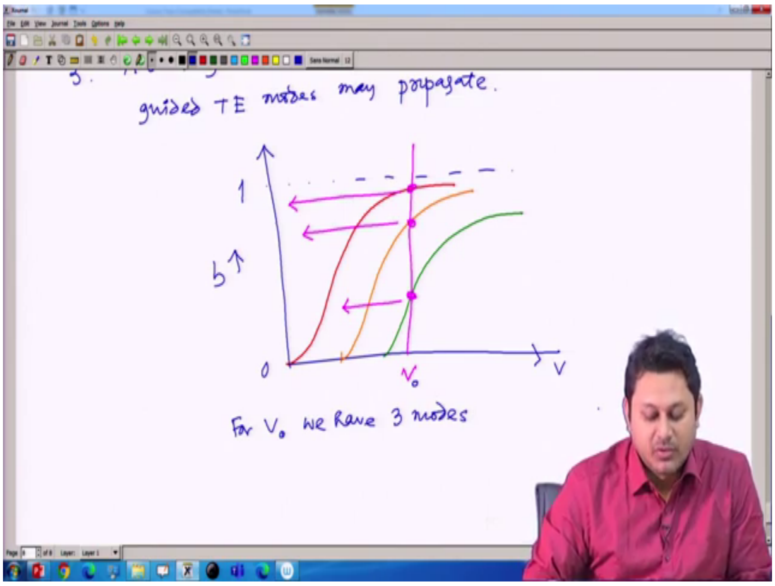

So, also this is quite understood that this is the figure this is 1, this is 0, this is b and I have this modes, I have this mode and so on. So, now if I have a some value V suppose here if I draw that this is some value V. So, for this value I have discrete cutting points and these are the solutions and these are the values of b. So, this corresponds to 1 b, this corresponds to 1 b and this corresponds to 1 b. So, these are the modes that will follow.

So, for this for example, for this V value for the for this V value for say I write it as say V 0. For V 0 we have 3 modes that will allow to propagate which is obvious. If I now reduce this value then I should have only two points for a given V. So, these two modes are allowed and so on ok this is along V.

So, with this node I like to conclude; in the next class we will try to understand more about the dispersion property based on the values of this b and V which is called the web guide dispersion. How the web concept of waveguide dispersion one can extract from this expression that we will going to discuss in the next class. So, with that note, I like to conclude, so.

Thank you for your attention and see you in the next class.