**Physics of Linear and Non-Linear Optical Waveguides**
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**Module - 03**
**Modes (Cont.)**
**Lecture - 25**
**Modes in Slab Waveguide (Contd.)**

Hello student, for the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture number 25. And in this lecture also, we are going to continue with the concept of Modes in Slab Waveguide.

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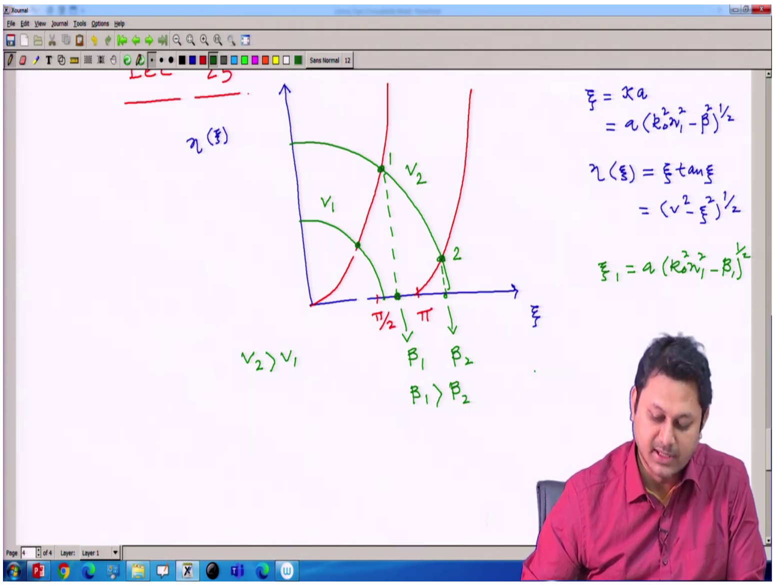

So, we have lecture number 25. So, let me quickly remind what we have done so far. So, we try to find out the transcendent solutions of the transcendental equations and this is my equation. So, let me write here. So, xi tan xi is equal to V square minus xi square whole to the power half; that is the equation I try to solve graphically. My xi is kappa a which is a multiplied by k 0 square beta 1 sorry; k 0 square n 1 square minus beta square whole to the power half.

And here, we plot xi and here, eta as a function of xi, where eta I define eta as this left -hand side and right-hand side both. So, xi tan xi and also which is equal to V square minus whole to the power half. So, these are the definitions I have written in the right -hand side. And now, I am going to plot that and if you remember, when I plot xi tan xi, it should be something like that.

So, here this value is pi by 2, this value is say pi. And when I have the value of v, then at some point I have one solution and if I increase that. So, there is a possibility that I can have two solutions; this is one solution, this is another solution. So, this is my say one V 1 and this is another value of V which is more than V 1.

So, my V 2 is greater than V 1, which corresponds to two solutions. One in one case, I have one solutions, In terms of xi and we already find that this solution can be written in terms of beta as well. So, this is my say beta 1 and another solution sorry for. For V 2 if I want to find out the solution, then just a minute. For V 2 specially if I want to find out, because for V 2 I am having two solutions.

So, here I have one solution which is beta 1 and here I have another solutions which is beta 2. Now, beta 1 from here I can see that beta 1 is greater than beta 2, because this is the solution in terms of xi not beta; beta extract from xi. From here, I can see that xi 1 is equal to a k 0 square n 1 square minus beta 1 square whole to the power half. So, if xi 1 is high, then beta is low. If xi 2 xi 1 xi in general. So, if xi is high here, xi 2, then the corresponding beta is low. So, that is precisely written here.

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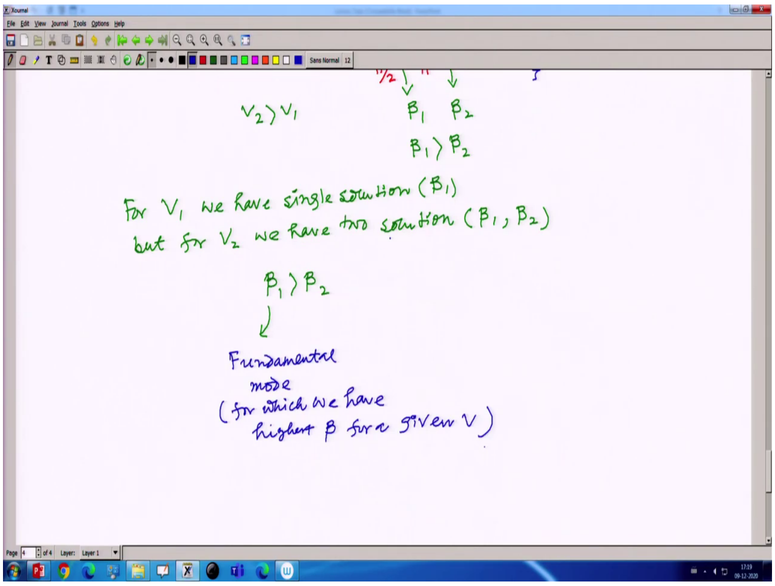


So, for V 1 we have single solution, but for V 2 we have two solutions; the single solutions corresponds to 1 beta say, beta 1. But for V 2 we have two solutions; one is beta 1 and another is beta 2 and beta 2 beta 1 is greater than beta 2 and this is corresponds to fundamental mode.

So, fundamental mode is a mode for which we have the highest beta 2. So, this corresponds to fundamental mode for which we have highest beta for a given V. When the V is given; that means, this radius is fixed. I can have more cutting points if I increase go on increase. So, this curve will continuously this rate curves which corresponds to xi tan xi will continuously changing we have another branch here and so on.

So, if I keep on changing the value of V by increasing this amplitude or increase increasing this radius; V is basically the radius of this circles, then what happened that I should have more cutting points. And the cutting points every cutting points, I have 1 beta and for highest beta we have the fundamental mode that is all.

Well, after that today what we will do, we will try to find out the another set of solution which is the anti-symmetric solution that is also interesting. So, we will study the anti-symmetric solution as well, so another branch of solutions.

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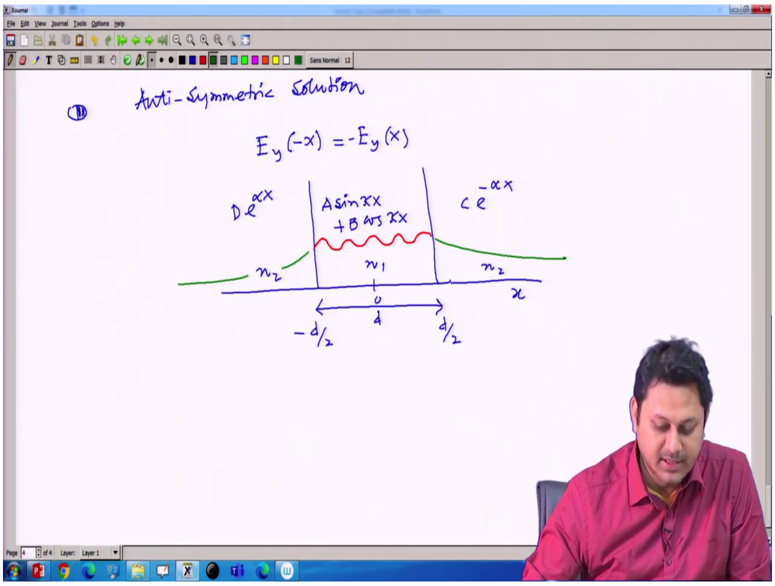


So anti-symmetric solutions or anti-symmetric modes so, for anti-symmetric solution we have the condition that my E y of minus x should be equal to E y with a negative sign of x; that is the condition. The general solution we know what is the general solution. So, the general solution if you remember in the core region let me draw once again. So, that was the structure of the waveguide.

So, this is the core part this is origin 0; this is d by 2 this point is minus d by 2. So, that this distance is d. And now, I have two regions here, this is n 2; this is n 2 the symmetric waveguide this is n 1. The general solution if I write the general solution here, if you remember it was A sin of kappa x plus B cos of kappa x; that was the solution here and here we had a exponentially decaying solution.

So, suppose c e to the power of minus of alpha x; that should be a solution here. And D e to the power of alpha of x is a solution here. Mind it in this region the value of x is negative. So, I have an exponential decay. So, I have a sinusoidal solution here like this and then, this solution will exponentially decaying at this region and on this region defined by this values so, that we know.

So, now, for anti-symmetric solution, I can now reduce these this whatever the solution I have in the core region. And I know for anti-symmetric solution, the cos part will not there anymore. Because then, we will not going to have any anti-symmetric solution.

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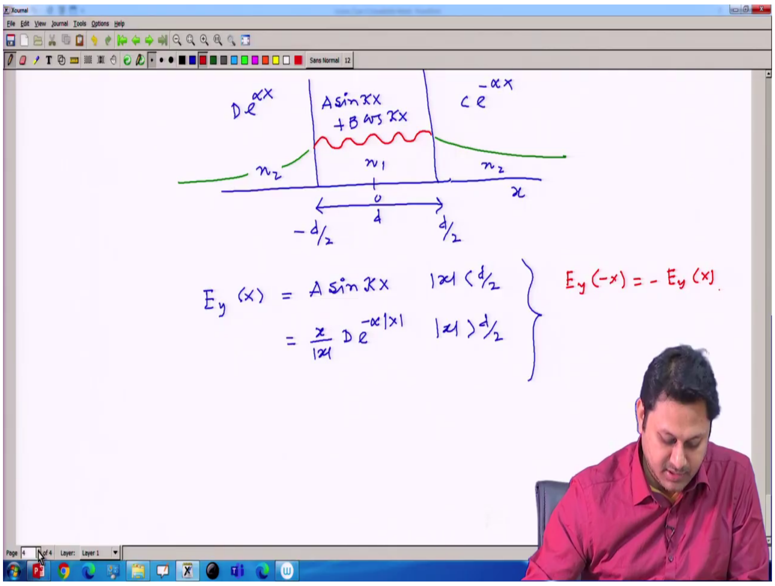


So, I have for anti-symmetric solution I have a general solution of the form say, E y x should be equal to A sin of kappa x that is for x less than d by 2. And I can also have a solution say of the form x divided by mod of x D e to the power of minus of alpha x greater than d by 2.

So, that I can have; so both these case solution following this condition that E y minus of x is equal to minus of E y x ok. So, that is the solution.

This is the way one can define the anti-symmetric solution and this x divided by mod of x take care of this issue. If you please try to do by yourself and check it that whatever the solution is given here; whether it is following the condition that is written in the red ink in the right-hand side of these two solutions. Well, again for these two solution these solutions, I have the boundary conditions.

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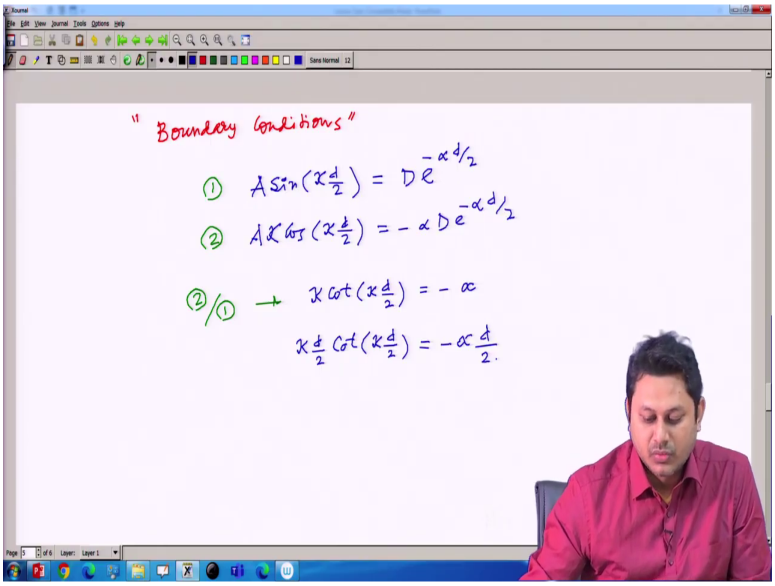


So, I need to put the boundary conditions here as well. So, the boundary conditions and what are the boundary conditions? Again, the same old boundary condition actually. And I will not going to write explicitly I directly write. So, A of sin kappa of d by 2; at d by 2 what happened there is a continuity of these two solutions. Then it should be D of e to the power of minus alpha d by 2; that is one solution. Second solution, the derivative is also continuous at that point.

So, A of kappa cos of kappa d by 2 will equal to minus of alpha D e to the power of minus alpha d by 2, so I have two equations; boundary conditions using the boundary conditions, this is 1, and this is 2. Again, by making 2 divided by 1, I can have kappa cot of kappa d by 2 is equal to minus of alpha.

I can make it as kappa d by 2 cot of kappa d by 2 is equal to minus of alpha d by 2. Exactly the same procedure that we have used to find out the transcendental equation for symmetric mode we are just following the same thing. So, again I can write it as.

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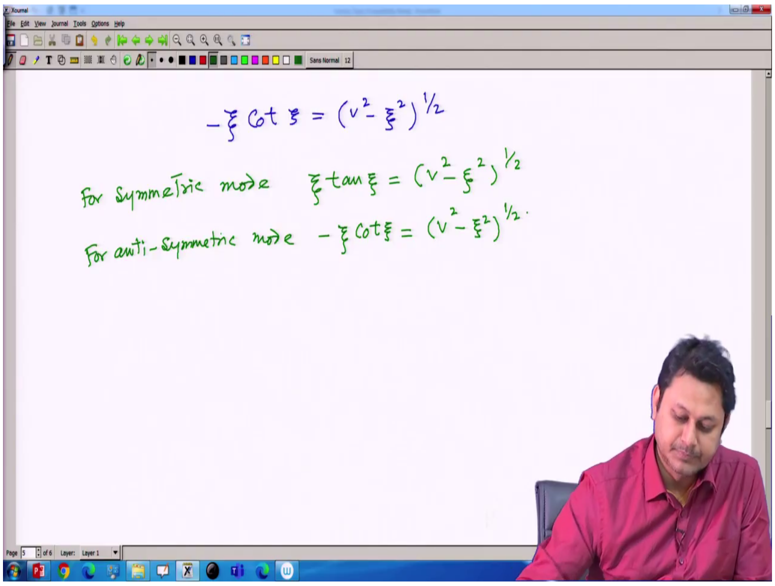


Chi cot instead of having sin; I now have cot. So, cot chi is equal to alpha d by 2. I can again write in a similar way and I can have V square minus xi square whole to the power half. So, right-hand side is exactly the same thing that we already find in the case of symmetric waveguide. For anti-symmetric waveguide, I am also having the same thing. Only thing is that instead of having tan, I have cot. So, I need to plot that that is all.

So, let me write it together. So, it will be easier. So, for symmetric mode, we have the transcendental equation xi tan xi is equal to V square minus xi square whole to the power half and for anti-symmetric mode. Now we have minus of xi cot xi I missed a minus sign here, because there is a negative sign sitting here.

So, that is why I should have a minus sign is equal to V square minus xi square whole to the power half. Well, these two solutions these two transcendental equations simultaneously present in a waveguide, when it is supporting both symmetric and anti-symmetric modes. So, when the waveguide is supporting both symmetric and anti-symmetric mode, then I need to plot both the transcendental equation together.

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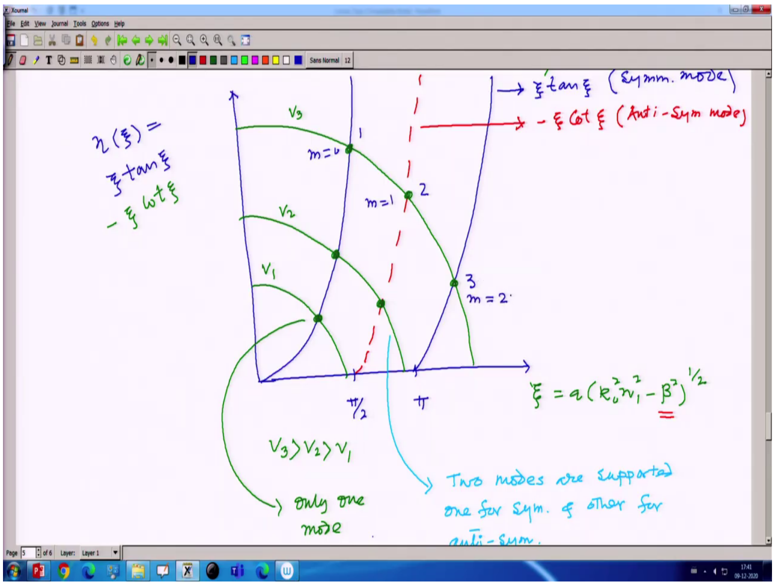


Let me do that. So, I am plotting here eta xi which is either xi which is equal to either tan xi or minus of xi cot xi; both I am plotting and also, I am plotting the right-hand side. So, I am also plotting the V square minus of this circle. So, everything I am plotting here.

And this side is the variable and only one variable is in our hand and that is xi. Increasing xi means I am increasing the value of beta, because this is xi. So, let me write it what is actually the xi. So, xi is again this one. So, what I am doing that, I am changing the value of beta to find the solution. So, this is my variable here.

By increasing xi means, I am basically decreasing the value of beta and try to get the solutions. Now, I draw the two solutions. So, by say by green, I should draw by blue rather I should draw the symmetric one. An anti-symmetric one is also there it should this is value say at pi by 2 and this value should be at pi. Anti-symmetric one, I draw with a red line with a dotted curve. So, this will give me the anti-symmetry.

So, this is for this is for xi tan xi which is coming for symmetric modes and this one is for minus of xi cot xi which for anti-symmetric modes. Now, I am drawing the value of V. So, now, you can see that I have this value; one cutting point here. I have this value; one cutting point here and an additional cutting point here, this is for one for symmetric beta another for anti-symmetric beta. Now, I am go on increasing. At some point, I can also have three solutions; one is here, another is here and another is here.

So, this these are the discrete solutions every time I am getting say, this is for V 1, this is for V 2, this is for another we say V 3; where V 3 is greater than V 2 is greater than V 1. And every time when I have a V 1 which is the which is the lowest value of V, I have only one solution and only one mode is there.

So, the for this case. So, let me define that for this case, I have only one mode. So, for this value the waveguide is supporting only one mode, because only one solutions are there only one solution is there. For this case for this case for this case, I have two solution two modes, two modes are supported; one for one for symmetric and another for anti-symmetric.

So, one for symmetric and other for anti-symmetric so, two solutions are available here for this value of V 2 one for symmetric and another for anti-symmetric. And also, for V 3, I have three solutions; one for two for symmetric and one for anti-symmetric and so on.

So, with this curve I can map the number of modes and not only that, I can also have the value of beta and all cases I have the value of beta and once I have the values of beta. Then I know what is the propagation constant of the mode that basically characterize the mode. Now, we are going to draw the mode distribution, because that is important.

Because so far, we are talking about the mode symmetric mode, anti-symmetric mode, but we never plot what is the values I mean how the mode will going to look like. So, we will going to. Before concluding this class, I will going to show how the mode should look like ok. So, let me draw this quickly. So, this is mode distribution.

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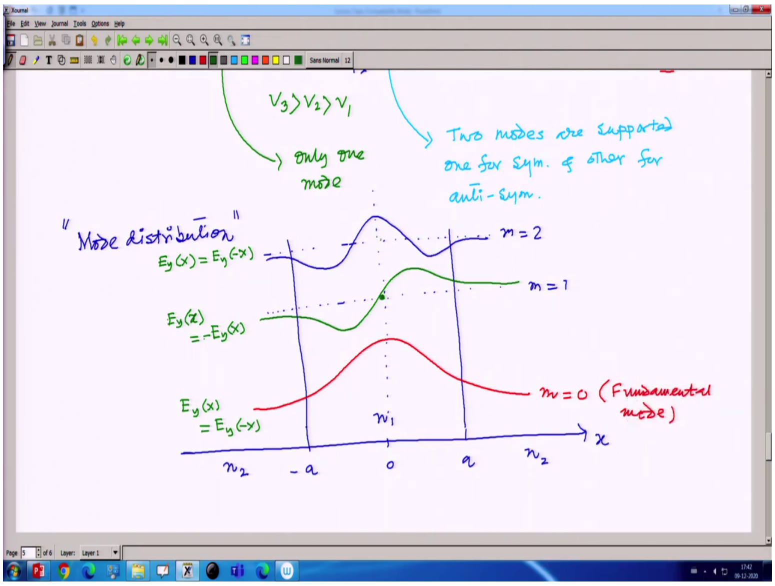


So, this is roughly the waveguide structure; like a potential well in the quantum mechanics. Exactly it is a similar kind of problem. So, I have this point O that is origin and this is d by 2.

So, according to our notation let me write it. As a, this is minus of a and along this direction is my x direction and mode is distributed over this x direction.

Refractive index here is n 1. Here, is refractive index say n 2. Here, the refractive index is n 1 and refractive index is n 2 this is the cladding part. So, now, if I draw the modes so, the fundamental mode will be something like this. I have an exponential decay here and I have a maxima here. So, this is my mode number 1.

So, I can write it as according to the notation we call m equal to. So, 0th mode or fundamental mode this is fundamental mode. And also, symmetric mode this is symmetric. So, this is the axis of symmetry. So, this is the origin actually. So, I should draw that; so that, it will help me to draw the other side of the modes. So, if I fold this so, there will be superposition.

Now, I should have in our next mode and in order to have for next mode. So, let me draw this thing. And next mode should be anti-symmetric, because here, if you look carefully; if I increase the value of V, then for this case I have one mode and as soon as I increase the value of V, I have another mode here, but this mode is cutting the dotted line which corresponds to the anti-symmetric solution.

So; that means, the second mode has to be anti-symmetric and this mode will be something like this. So, let me draw in a different color. So, I should have a distribution like this. So, this is my anti-symmetric mode. If you look carefully, you will find that ok. This should pass through exactly at zero point. So, it should roughly pass through at zero point. So, this is the zero point. So, it should pass through exactly at the zero point.

So, if you look carefully you will find that if I have. So, here the condition E y x is E y of minus of x; this condition is valid. In this case, E y x equal to minus of E y x this condition is valid. So, this is a symmetric mode this one is anti-symmetric one. I can go on with the higher order modes.

So, let me draw the higher order modes before the conclusion of the class. So, the next mode according to our picture there is a third cutting point here. So, this is I am drawing the three modes. So, let me define here. So, this is 1 this is 2 and this is 3. So, if I defined is as m equal to 0th mode m equal to 1 and m equal to 2; this is the way we define the number of modes.

So, now, this is for m equal to 0 this is for m equal to 1. And for, m equal to 2 I can have here from this figure I find we should have a symmetric mode and the symmetric mode will be this. So, this is for m equal to 2 and you can see here also, it is symmetric over x, so E y. So, here also the condition E y x is equal to E y of minus of x. So, I have this one.

So, with this, I should conclude my class here. And in the next class, we will try to understand this mode distribution with another parameter called b parameter and how one can map all the modes that will be discussed.

So, thank you for your attention and see you in the next class.