Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 24
Modes in Slab Waveguide (Contd.)

Hello student, to the course of Physics of Linear and Non-Linear Optical Weveguides. So, today we have lecture number 24, and we will continue with the Modes in the Slab Weveguides ok.

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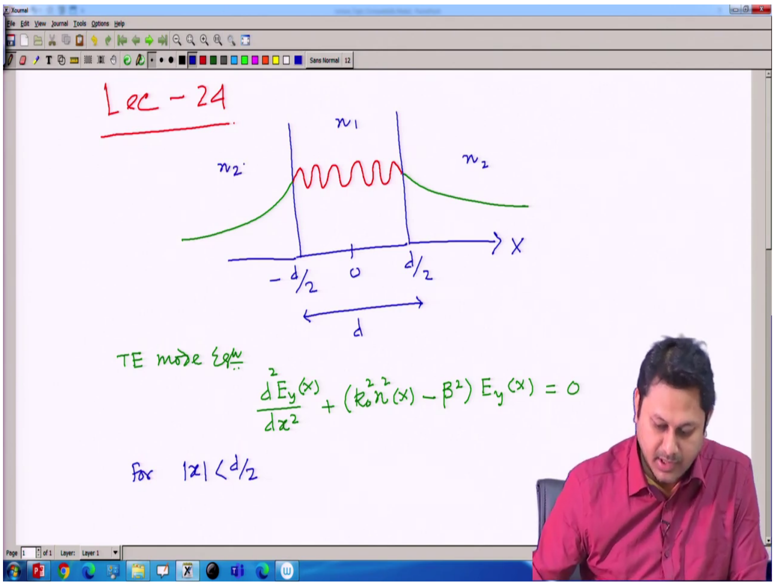

So, in the last class, we try to understand the modes in the slab waveguide. So, let me remind what we have done so far. This was the structure of the waveguide along this axis we have x, this is origin, this is d by 2 and minus d by 2, so that this is d this is the core part of the waveguide. And, we had a solution which is sinusoidal in this region and then exponentially decay in this side.

So, my equation was the mode equation was d 2 E function of x dx square plus k 0 square n square function of x minus beta square E y function of x equal to 0 that was the mode equation. And, in the region for mod of x less than d by 2 which is in the core region by the way the refractive index here in this region was n 1, it was n 2, n 2 this side.

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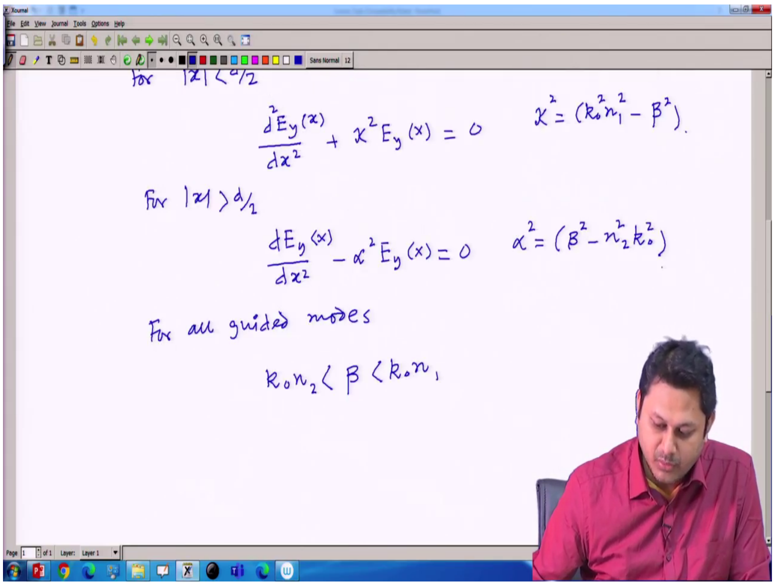

So, this equation one can write as this way. I can defined a variable here kappa square, where kappa square was k 0 square n 1 square minus beta square. For mod of x greater than d by 2 that means, at cladding region this equation modifies as where alpha square was defined as this way. Note that for all guided mode the restriction of the beta is beta should be in between this values.

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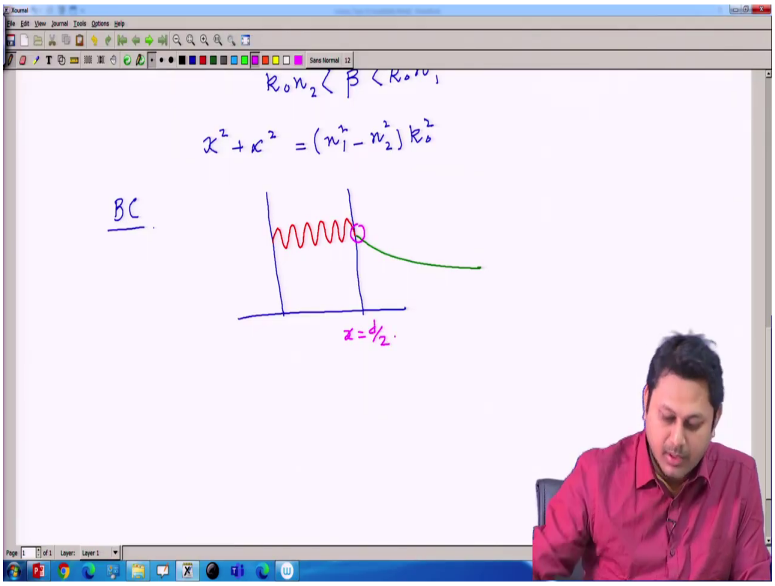

Also from here we can find one thing that my kappa square plus alpha square is n 1 square minus n 2 square multiplied by k 0 square. There was a boundary condition in the interface. So, let me draw once again. So, the structure was something like this – a sinusoidal solution here and then an exponential decay. So, this is interesting. This is the boundary actually at this point which is as x equal to d by 2.

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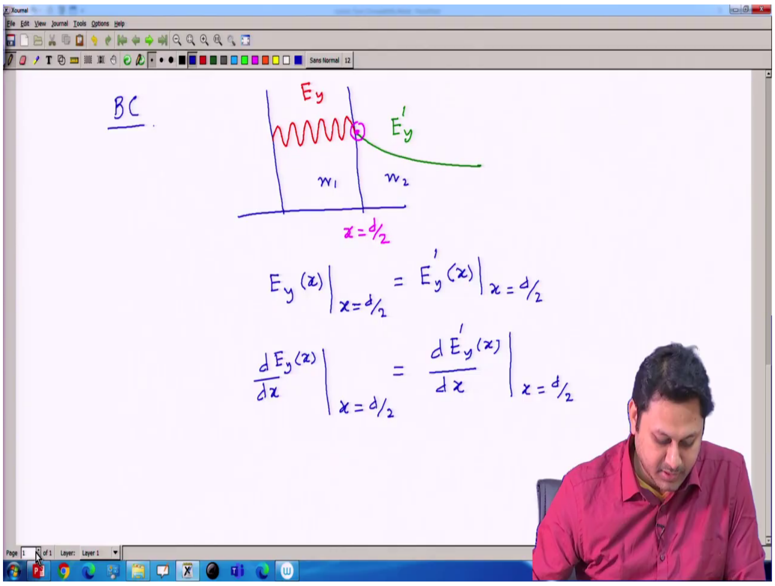

And, the boundary condition tells us E y if the solution if I write the solution as E y here and the solution here in the cladding region if I write E prime y. So, at the junction here in this point at the boundary, this is refractive index n 1 and this is refractive index n 2. So, at this point I had E x at x equal to d by 2 is equal to E y x, x equal to d by 2 the prime. That is the first boundary condition.

And, second boundary condition the derivatives at x equal to d by 2 is also equal so that the function is smooth here as well. The first boundary condition tells us the function is continuous, second boundary condition tells us the function is smoothly varying at the boundary.

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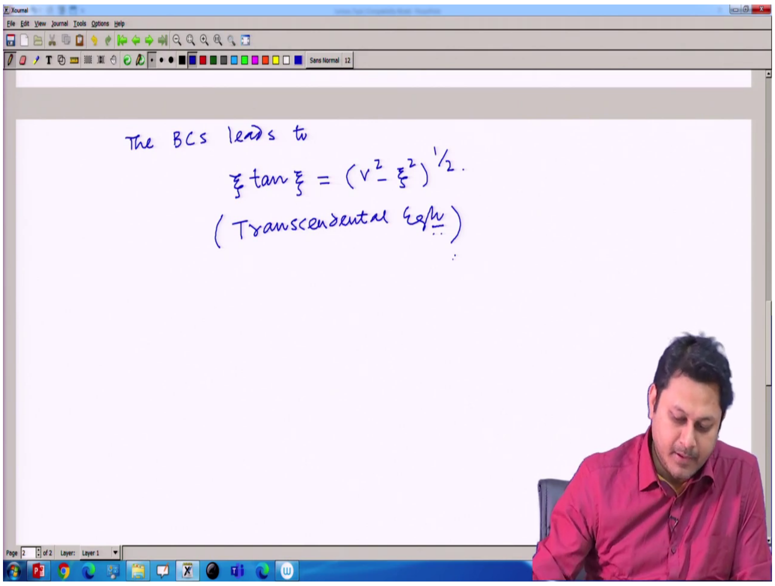

After putting these two boundaries, we find an equation something like this. These two boundary condition, the boundary conditions leads to this equation. This is a transcendental equation and one can have the solution only when we plot these two function graphically, and then the cutting points are the solution. So, there is in the transcendental equations we do not have any closed form solutions.

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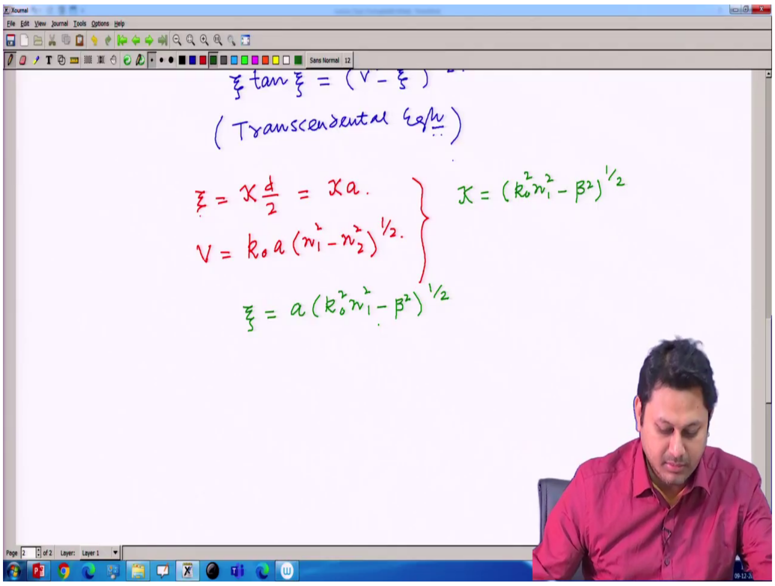

By the way and also define the xi; the xi is a parameter that we defined as xi was kappa d by 2 we defined as kappa a and v parameter is defined as usual with k 0 a multiplied by n 1 square minus n 2 square whole to the power half. So, these are the two parameters we define.

Mind it, my kappa is k 0 square n 1 square minus beta square whole to the power half. So, xi is essentially a into k 0 square n 1 square minus beta square whole to the power half and if we if you see dimensionally it is a dimensionless quantities, xi is a dimensionless quantity.

Well, this is the expressions that we derived in the last class. So, up to this we have done in the last class. So, today we are going to solve this transcendental equation that is the goal here today.

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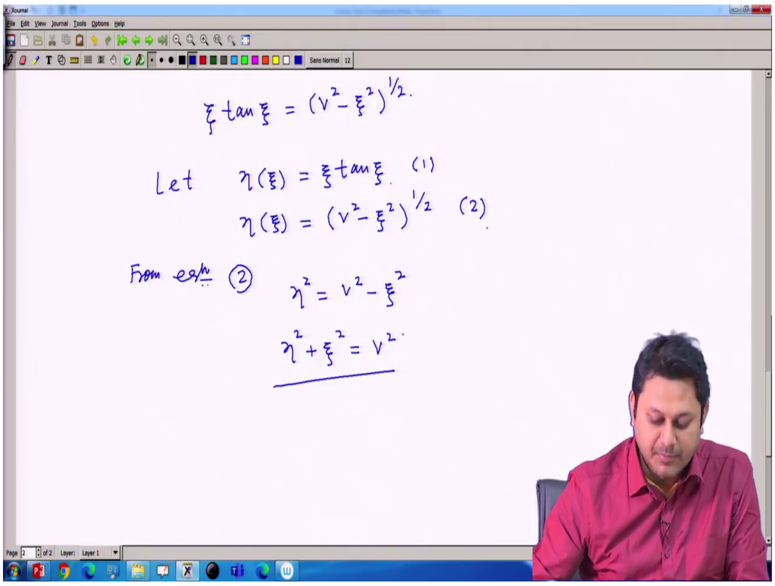

So, we have an equation in the form xi tan xi equal to v square minus xi square whole to the power half. So, what we do to solve? So, let us define a parameter. So, let xi eta which is a function of xi is xi tan xi. Eta is also define as this because my transcendental equation says xi tan xi is v square minus xi square whole to the power half. So, this is my equation 1, and this is my equation 2.

So, from equation 2, I can write eta square is equal to v square minus xi square or eta square plus xi square is v square. So, I have a expression in terms of eta and xi, mind it, v is a constant, v is constant.

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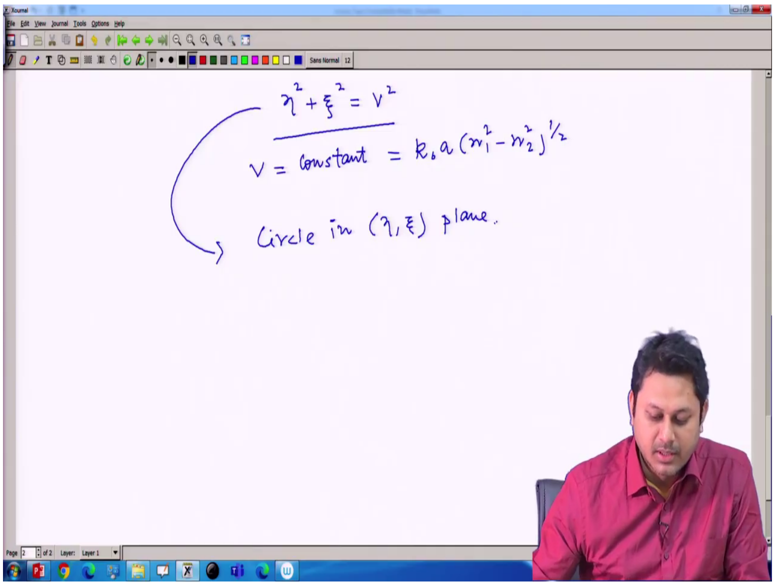

Because v is constant which is equal to k 0 a n 1 square minus n 2 square whole to the power half. So, all the terms here k 0 a n 1 square minus n 2 square are constant, it is not depending on anything. So, that is why v is a constant. So, this equation is nothing but the equation of the circle in the eta xi plane.

So, in the eta xi plane, it form it forms a circle. So, I have two equation now – one is circle equation 2 gives me a circle and equation 1 is tan xi tan xi. So, I will going to plot this together.

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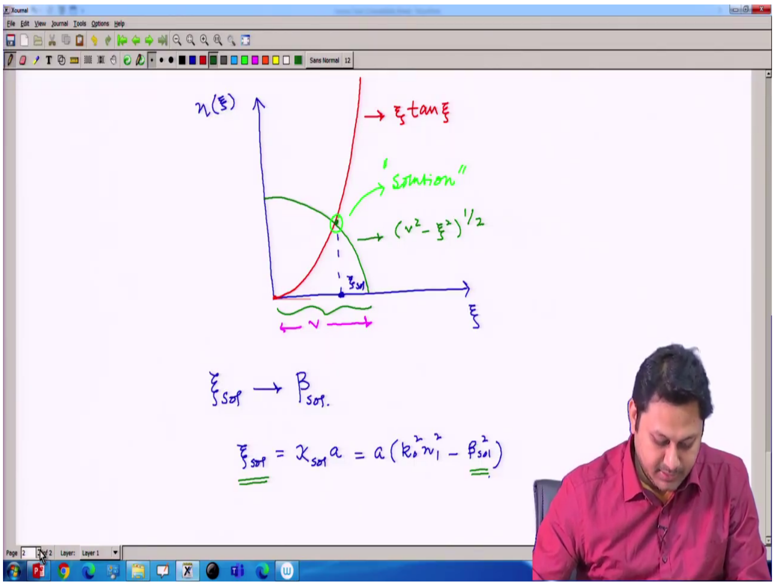

So, let me plot it first. So, along this axis I plot eta which is a function of xi and this is xi. So, when I plot equation 2, then I have a circle. So, let me draw this circle first because I do not want to plot all the quadrant. This is sufficient here and also is restrict the value because otherwise I will do not have any physical value for that. And, then if I plot xi tan xi, so, it we know what is the how the tan xi or tan x is will going to change with respect to x.

Here, if I now plot that it should be something like this. If I increase the value of xi, so, please note that for this equation so, let me write it here only. So, this is for xi tan xi note that when xi is equal to 0, I have 0 here.

When xi tends to pi by 2 then this will goes to infinity because tan pi by 2 goes to infinity and this is circle. So, this equation tells me it is v square minus xi square whole to the power half and this is the value of the radius of this is v.

So, from here to here this value, so, this value is v. So, radius of this circle is v. So, here we can see that we had a cutting point; we should have a cutting point. So, let me make it a different color, say this one. We have a cutting point here. So, this is the solution. So, this is my solution. So, that means, I should have a solution here and xi sol is my solution. So, xi solution is my solution which is the cutting point.

Well, if I know the xi solution then xi solution if I know then from that I can find out what is the beta propagation constant of that mode. So, beta solution I can have because there is a straight relationship between xi and beta. What is xi? Xi solution is I know what is xi. It is kappa multiplied by a, we already defined. Here it is kappa multiplied by a. So, kappa is a variable here. So, I should have some solution in terms of kappa.

So, I write kappa sol into a, that quantity I can write in terms of beta. So, a multiplied by k 0 square n 1 square minus beta sol square, ok. So, if I know from this transcendental equation what is the solution here at this point what is the solution, this is my solution xi sol and I can find that just putting this equation graphically.

After putting this equation graphically I can find out what is the value of the propagation constant for this mode. And, now it is interesting to note one thing that this is for a fixed value of v. So, if I increase the value what happened?

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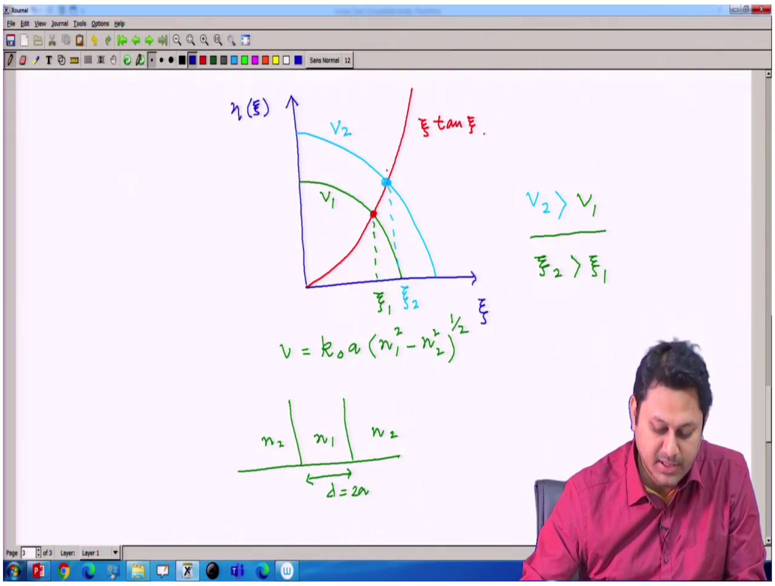

So, let me draw it once again. This is my eta as a function of xi and this is xi I am plotting. I have a circle here and then I have xi tan xi. This is the solution. This is xi tan xi and this is for some value say v 1, some value v 1. And, for v 1 I have a solution say eta 1. So, eta 1 is a solution for a v 1. What is v? v is k 0 a multiplied by n 1 square minus n 2 square.

So, for a given planar waveguide I can have two parameter, this a. So, this is the structure of the planar waveguide, this is n 1, this is n 2, this is n 2 and this length is d which is equal to 2a according to our notation. And I define v in k 0 multiplied by a n 1 square minus n 2 square they should be to the power half here.

So, I can change the value of v by changing either n 1 either n 2 or the value of a. These are the three geometric parameter associated with inside the v and through which I can change the. So, if I want to increase the value of v so, what I simply can do? I can increase this value of a or the value of d.

If I increase this value then what happened? This now this curve will increase and I will have another circle with a bigger radius. So, then I have a new solution here. Say this solution is xi 2 and this is for another value v 2. So, obviously, v 2 is greater than v 1. If that is the case, then I have xi 2 which is greater than xi 1.

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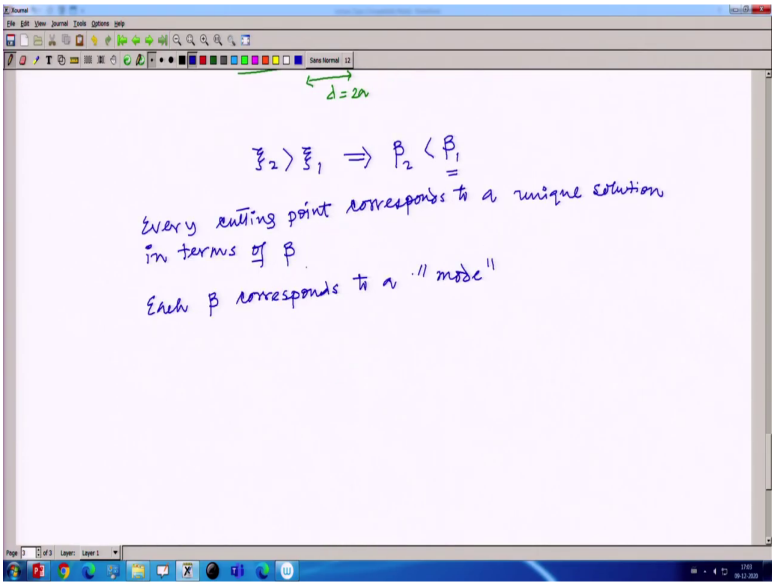

And xi 2 is greater than xi 1 means from here I can say, so, here let me write once again. So, I have a condition when xi 2 is greater than xi 1. These two are solution for two different values of v. In first case it is v 1 and it is v 2. So, I just increase the value of v the; that means, I am increasing the v parameter by increasing suppose by increasing the value of a.

Then what happened? I should have two solution such that xi 2 is greater than xi 1. If xi 2 is greater than xi 1, then that follows that beta solution for 2 is less than beta solution for 1. So, beta 1 will be greater than beta 2. So, the second case I should have another beta, but that beta is lower than the previous one.

So, let me whatever the findings let me write. Every cutting point; so, every cutting point corresponds to a unique solution in terms of beta. So, I have one cutting point. So, a v parameter is given to me for a slab waveguide, I then use this transcendental equation xi tan xi is equal to v square whatever the transcendental equation I figure out here equation this. So, I can so, this equation I am basically solving.

And, every time I am getting a cutting point for a given value v, I have a single cutting point here. I will show there would be other cutting points as well, so, how it will come then this thing will give me different values of beta depending on the value of v. So, for 1 value of v I will have only one cutting point and mind it this is for a symmetric mode, this is for a symmetric mode.

Now, each from here I can write each beta corresponds to a mode. So, beta is unique for a given mode. So, if I know what is the mode then I can find it what is beta.

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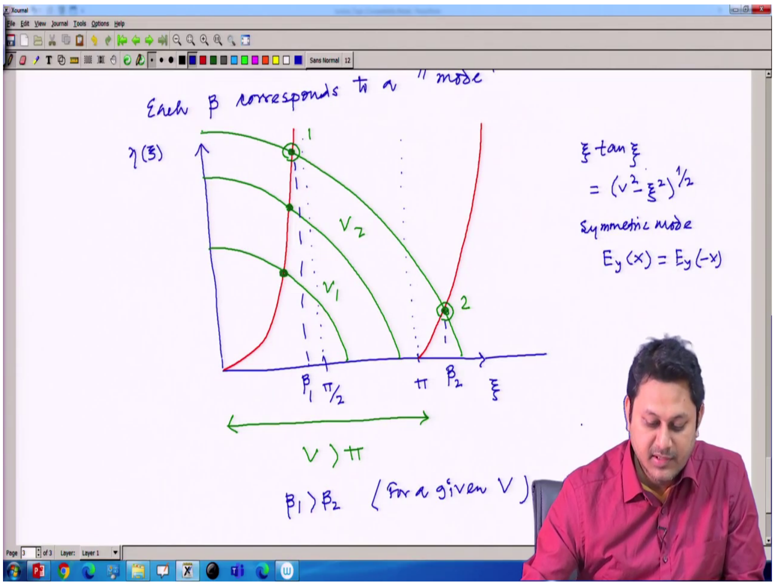

Now, the next thing I like to know that if I increase this xi, then what happened because we know that xi tan xi if I go beyond pi by 2 then what happened. So, in general so, how the solution it look like, let me draw that. So, I have eta as a function of xi. Now, I want to plot the full picture, this is xi and suppose this value is pi by 2 and this is pi.

Now, so, what happened if I plot xi tan xi? So, I am plotting xi tan xi. So, the plot should be something like this. So, it should go to infinity at pi by 2. Again, here I can have another branch like this. Again, here I have I should have another branch like this. Now, if I have a value of v given value of v, you can see that I have one cutting point here for this v, say this is my v 1.

Now, if I increase this v value then up to that I should not have only one cutting point, say I have another value v 2. Again, I have one cutting point here, but if I increase the value if according to this if the value of v is according to this plot this is the radius. So, if this is pi, so, if v is greater than pi then I can have a circle like this and now, I should have two cutting point.

This is one cutting point, this is one solution and another solution is also here. This is the second solution. So, far I am having only one solutions which is this, there is no other cutting point for symmetric mode by the way this is for symmetric mode. So, this equation this transcendental equation is for symmetric mode.

You remember, we consider, this is only for symmetric mode. In symmetric mode, what was the condition? The condition was E y x is equal to E y of minus of x. So, we interestingly find that if I increase the value of v, I am having two solution now. This solution if it is beta 1 and this solution if it is beta 2 we already show that beta 1 is greater than beta 2.

So, this is for one mode and this is for another mode for a given value of v for a given v. So, that means, when the v is greater than pi at least for this plot, then instead of having one solution I have two solution and this two solution gives me two modes.

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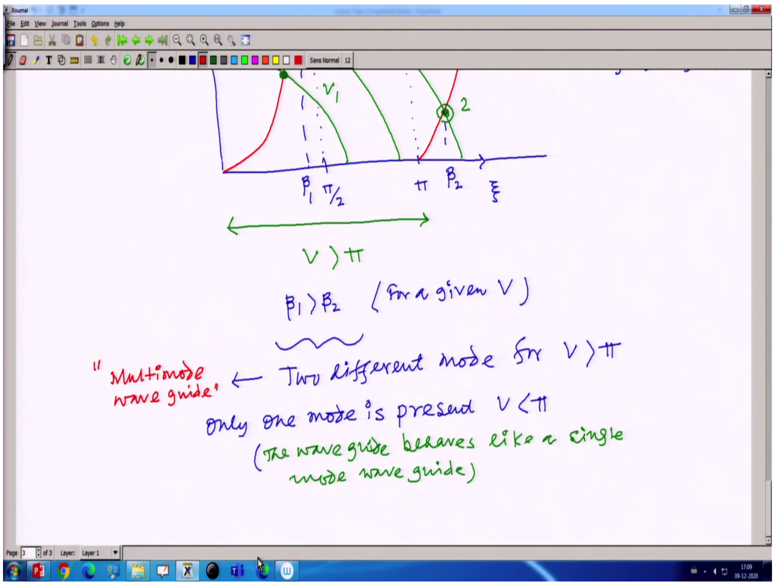

So, these corresponds to two different modes, for v greater than pi according to this plot. Only one mode is present when v is less than pi; because only one cutting points we are have we are having only one cutting points and when the value of v is greater than pi we are having two modes. So, it tells us that for this case the waveguide behave like a single mode waveguide.

So, the wave guide behaves like a single mode waveguide allowing only one mode. So, behaves like a single mode. Now, if I increase the value of v, I can also increase the value of v by just reducing the wavelength then I can have a multi mode waveguide. So, this case two different modes for V that means, this is corresponds to multimode waveguide.

So, in the next class, we will understand more on other modes. So far we are dealing with symmetric modes and for symmetric modes from this curve shown here in the picture that if I increase the value of v, then there is a possibility that at some point we have another solution. So far we are having only one cutting point and after that certain point, we now start having another solutions.

As soon as I have another solution; that means, the waveguide is now allowing not only one mode, but another mode also. But, the value of beta 1 is always greater than beta 2. So, that means, the there is a fundamental mode for which the beta is maximum and then I have a other also higher order modes by reducing the value of beta.

So, next class we will going to discuss more about that. So, till then bye and.

Thank you for your attention.