Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 22
TE and TM Modes (Cont.)

Hello student to the course of Physics of Linear and Nonlinear Optical Waveguide. Today we have lecture number 22, where we will going to continue with the calculation of TE and TM Modes.

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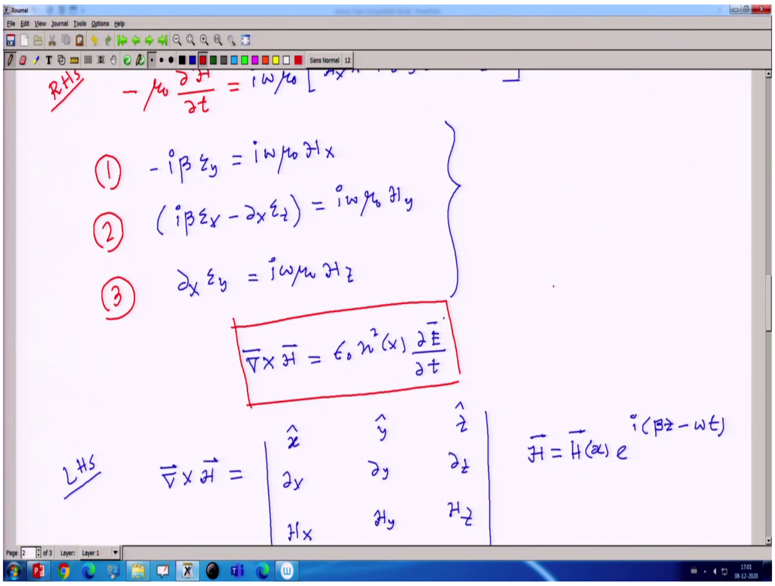

So, in the last class, if you remember we had 6 equations, these are the equations. Equation 1, 2, 3 is now shown here in the monitor.

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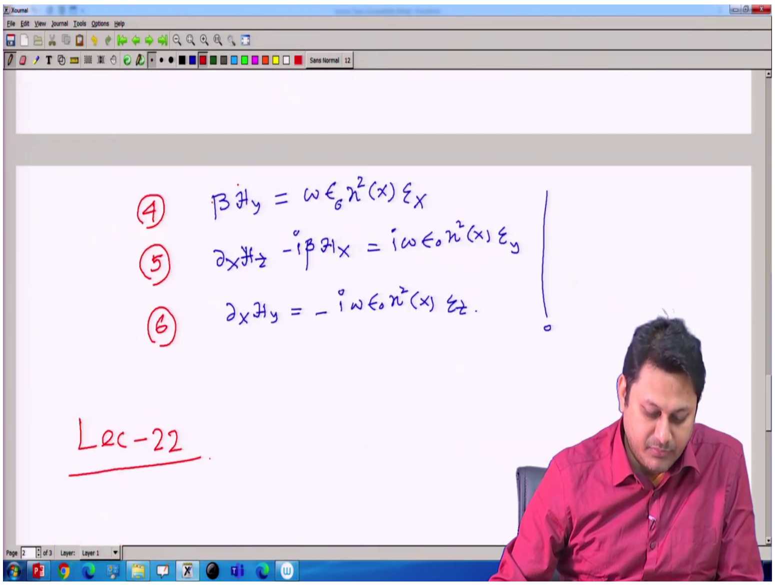

And equation 4, 5, 6 also there. And we find this 6 equation by just writing down the Maxwell's equations – two Maxwell's equation which is related to the curl of E and curl of H. Now, today what we will do, we will just extract first for TE mode the E y component I will going to extract. So, let us try to find out where we have the E y components.

If you look carefully in the previous equation, then you may find that this is one equation where we have E y; in the 2nd equation, I do not have any E y, but in the 3rd equation I have E y. In the 4th equation, I do not have any E i E y, but in the 5th equation I have E y.

So, here I have E y in the 4th equation, 5th equation; in the 3rd equation, here we have E y; and in the 1st equation, here we have E y. So, what I will do? I will just extract all the E y components and then we will try to find something out of that.

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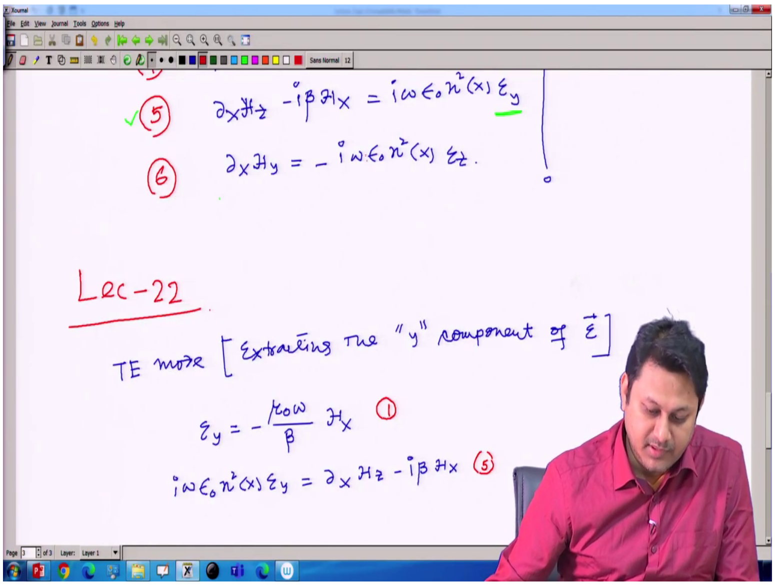

So, for TE mode what we will do? Extracting the y component of the vector E. So, I can have from equation 1, if I look carefully equation 1, I can have E y as this. So, E y will be equal to mu 0 w divided by beta and then H of x this is my equation 1. Let me go back.

So, H of x, i, i, will going to cancel out; one negative sign will be there. So, beta will go down. So, it should be omega divided by beta mu 0 H y. So, this is my equation 1. So, I have this equation, rewrite this from equation 1.

From equation 5, I can have this one i omega epsilon 0 n square as a function of x E y is equal to del x H of z minus i beta H of x that is my equation 2 that is from equation 5 whatever the equation I write. So, I should not rename this equation, because this is the same equation I am writing. So, this is equation 5. So, this is the equation actually I am writing E y i omega. So, I just write this equation, just rewrite this equation rather.

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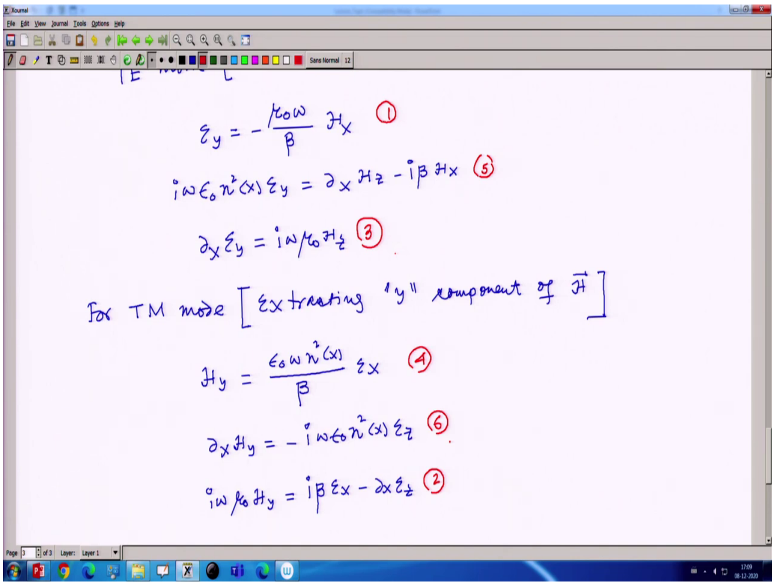

Then I have sorry then I have d x E y is equal to i of omega mu 0 H of z. This is coming from equation 3; directly coming from equation 3. So, I have 3 equations – 1, 5, and 3, out of this 6 equation; where in all this equation only E y components are there. And E y component, I can extract from this 6 equation to write down this 1, 3, 5 a new set of equation just rearranging the equation.

In the similar way, for this is for, by the way this is for TE mode, for TM mode, I will do the same thing. For TM mode, extracting y component of H give us something. So, again go back to, so where we have H y? So, it is easier.

So, let me put in different color here, we have H y, equation 2. In equation 4, we have H y and in equation 6, I have H y. So, I will going to write down all the equation which contain H y. So, H y will be simply epsilon 0 omega n square x divided by beta E x, this is from equation 4.

Next, del x H y is equal to minus of i omega epsilon 0 n square x epsilon z which is from equation 6. What was equation 6? In equation 6, this is. So, I am just rewriting whatever we have here i omega epsilon 0 n square E z i omega epsilon 0 n square E z. And I have another equation which is equation 2 and which is i of omega mu 0 H of y is equal to i of beta E x minus del x E z, this is equation 2.

So, I just rewrite all these 6 equation according to my convenience. And in one set of equation only E y are there; in other set of equation only H y are there.

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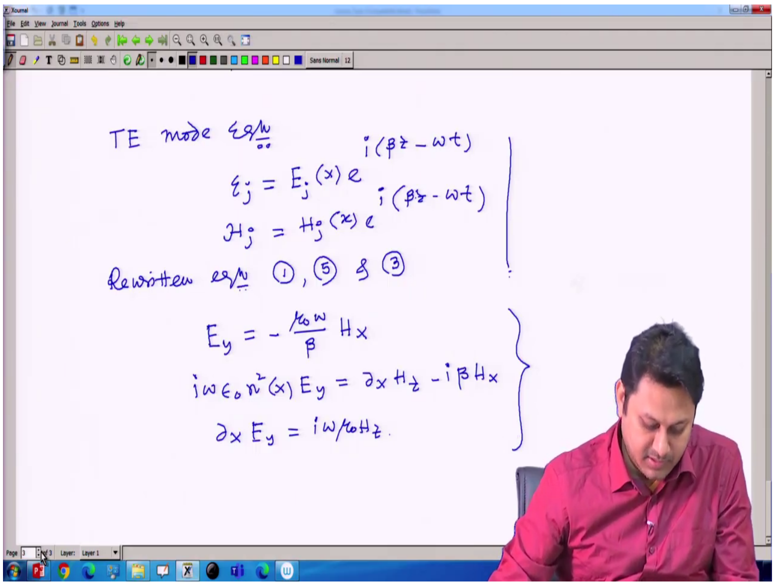

Now, I will derive my TE mode equation. So, TE, so now, we write TE mode equation. So, E y E j, so let me write it once again is equal to E of j function of x e to the power of i beta z minus omega t. And curly H j is equal to H of j x-component e to the power of i beta z minus omega t. So, this is from the very beginning we consider E and H in this form.

Now, if you look carefully to this 6 equation, I have curly E y here and curly H x in the right hand side. So, if I now write the full form, then e to the power i beta z minus omega t can be cancel out from both the side. So, this equation, this curly thing, I can simply replace by this state E and state y. So, rewritten equation 1, 5, and 3, I can have.

So, rewritten equation 1, 5, and 3 we have now we remove this curly thing and write only E y because E curly E y is E y E to the power i beta z minus omega t. In the right hand side also, I should have, for example, here curly H x, so curly H x is nothing but H x E to the power i beta z minus omega t. So, essentially E to the power i beta z minus omega t will going to cancel out.

So, these things will be minus of mu 0 omega divided by beta, then H x i of omega epsilon 0 n square function of x. Just rewriting whatever we have wrote in the previous page delta x H z minus i beta H x and del x E y is equal to i of omega mu 0 H of z. So, I am just rewriting all this one equation, 1, 5, 3. Now, this is the important thing.

Now, I need to eliminate this H x and H x and H z if possible. So, let us try to do that. So, really whether it can be done or not, let us see.

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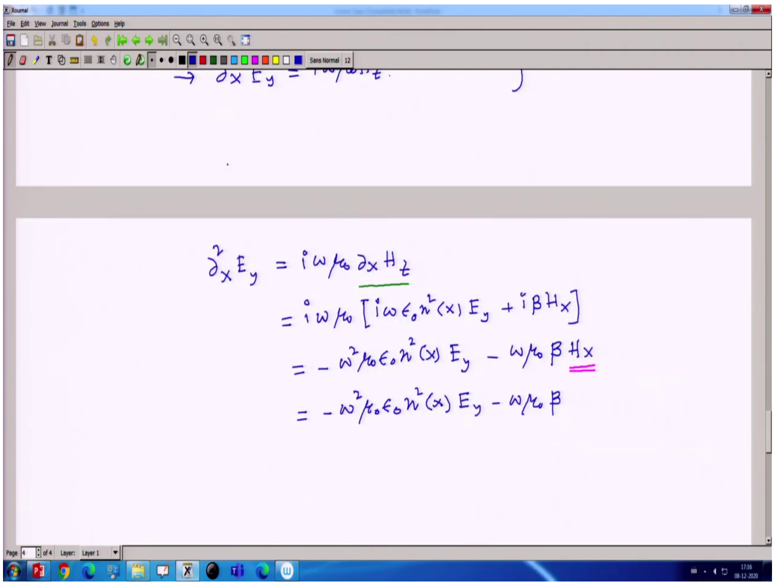

If I make a double derivative with respect to from this equation; from this equation, if I make a double derivative with respect to x derivative with respect to x both the side rather, then I have this is equal to i omega mu 0 del x H z. Now, you can see del x H z is sitting in this equation. So, I will going to replace these things here.

Please note here I have this quantity and I am having this quantity here. So, in the next line, I should write it as i of omega mu 0. And I will replace this as i of omega epsilon 0 n square function of x then E y plus i beta H of x. Well, I can write it as minus of omega square mu 0 epsilon 0 n square function of x E y minus omega mu 0 then beta H x.

Now, from here I have E y, E y, but the term H y is present here. So, I need to eliminate that as well. Now, this H y term is present here. So, I can eliminate this H y from this equation. If I do, then I will going to have I will going to find this, omega square mu 0 epsilon 0 n square x E y minus of omega mu 0 beta. H y I will going to replace what is my H x sorry it is E y then multiplied by beta divided by mu 0 omega.

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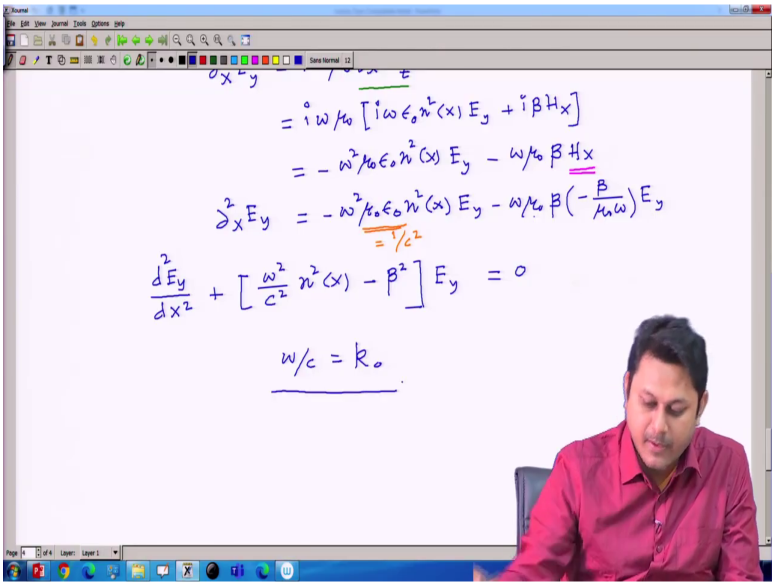

So, I should write here a plus sign ok, let me write in this way. So, it is beta divided by mu 0 omega then E y. So, you can see that in this equation, this portion is double derivative with respect to x of the field E y. So, I can have. So, now, let me write clearly d 2 E d y d x square plus let me write it as this equation as omega square divided by c square, because mu 0 multiplied by epsilon this quantity is 1 by c square.

And then n square function of x that quantity and minus of omega mu 0, omega mu 0 is going to cancel out. So, I have beta square which is a propagation constant of the field is equal to 0. Now, omega divided by c is k 0 the propagation constant in free space.

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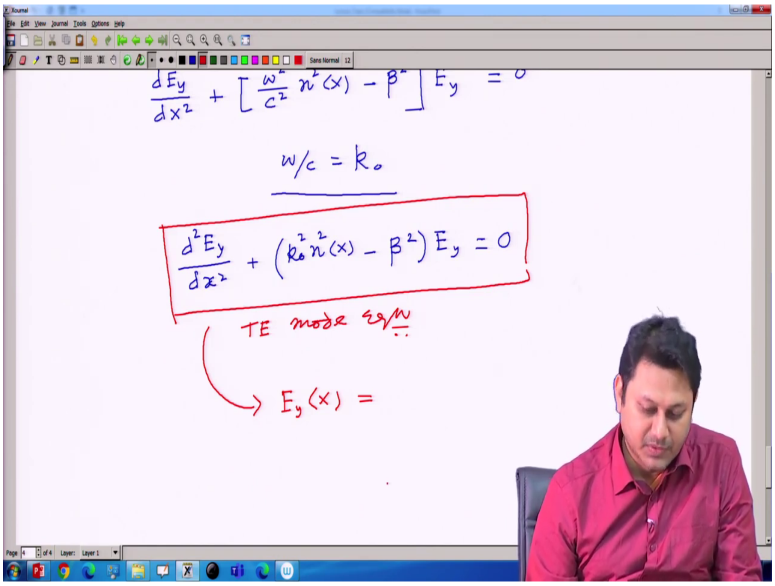

So, eventually I have a differential equation in this form d 2 E y d x square plus k 0 square n square x minus beta square E y is equal to 0. So, this equation is my mode equation for TE mode equation. So, this is my TE mode equation. In TE mode equation, what we have? I have beta which is a propagation constant and then we have the refractive index.

So, if the refractive index profile is given, then I can solve this differential equation. And when I solve this differential equation, I should have the solution E y x this is my solution.

So, I should have something in this side. So, I can have the distribution of the field; I can have the distribution of the field. So, we will use this TE mode equation to find out the mode this TE mode distribution in the planar waveguide. So, planar or slab waveguide. So, let us define the problem first.

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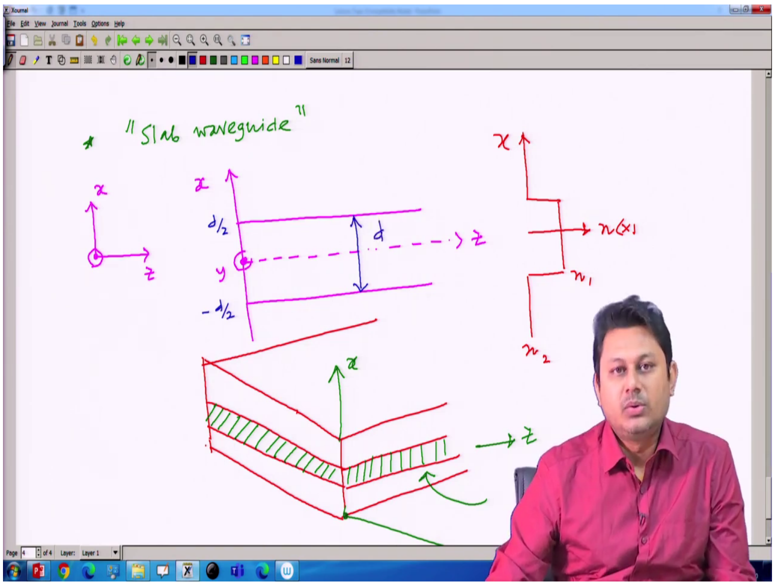

So, so, we will try to calculate the field distribution or the mode distribution in a slab waveguide. The slab waveguide already we define; this is nothing new. So, I have a refractive index n 1 sandwiched by 2 refractive index refractive index n 2. So, this is the direction of z, y is perpendicular to this plane and along this direction we have x. So, this is the coordinate system. The coordinate system is this.

And now if I draw the refractive index profile along this, we have n x and along this it is x. So, we have n 1 and n 2. So, it is like a step index fiber, the profile is like a step index fiber, but this is a slab wave guide we are considering, where the refractive index is restricted in x-direction; z is a propagation direction, and y-direction is extended to a very large distance in principle infinity.

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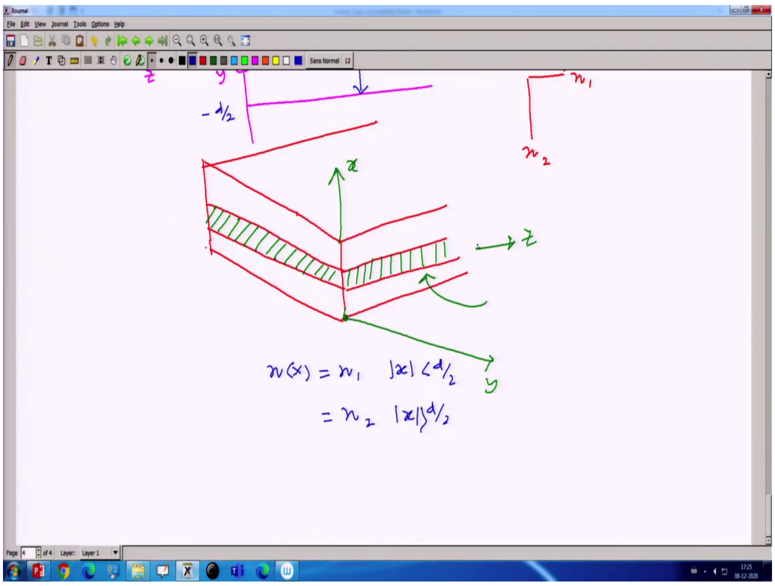

So, with this structure, so let me once again draw that, so that, so it should be like this. In 3D already we draw this figure; so I am drawing once again. So, this is the region where the refractive index is changing. So, along this axis, we have x; y is along this axis, this is y; and along this axis, we have z.

So, if I take a cross section of this side, if I just try to find out what is happening for this side which is presented here with this pink lines. So, this side is presented here with a pink line. We have y-axis here; x-axis along this direction; y-axis perpendicular to the plane; and z-axis along this propagation direction.

So, this is the geometry of the system. This is the geometry of the wave guide. For this geometry, I need to find out the distributions, I need to basically solve this TE mode equation for that distribution that is my goal here. So, here the refractive index n x is equal to n 1 for x less than d by 2 and equal to n 2 for x greater than d by 2 where d by 2 is this. So, from here to here, this length is d by 2 and this is minus d by 2 if this point is the origin. And this length is d.

So, this is the geometry I will just introduce today. So, I will not going to solve this today, because it will going to take some time. And also I do not have that much of time today. So, in the next class, we will start from this point and try to find out what should be the field distribution. I just solved this differential equation this T E mode differential equation, and simply find out because n as a function of x is given for this structure.

And I will going to find out what is the value I mean what is the solutions what are the solutions rather. So, with this note, I like to conclude my class today.

Thank you for your attention. So, see you in the next class.