Physics of Linear and Non-Linear Optical Waveguides
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Module - 03
Modes (Cont.)
Lecture - 21
TE and TM Modes

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number-21. And today we will going to study TE and TM modes which we already started in the last class.

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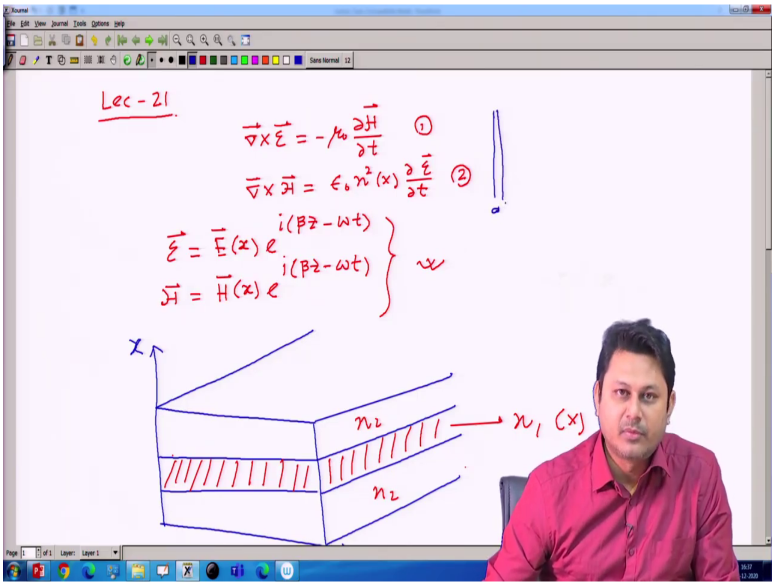

So, let me start once again what we have done so far. So, we started with an equation which is essentially the Maxwell's equation in this form. This is the this is curl H, this was one equation. And another equation was, it is not square, this is equation 2. We also mentioned that the curl E, this E vector is associated with E as this E vector which is a function of x e to the power of i beta z minus omega t. And for H we had this to form.

Now, the waveguide geometry, I loosely described in the last class that why it is so, because in our case the wave guide distribution is something like this. So, let me draw it once again. Along this direction, we have x, this is y. So, the refractive index variation is like here. So, here refractive index is n 1. And this behave as a cladding, so I can write n 2.

Since, it is a x-axis, so refractive index is essentially function of x and this is a planar waveguide. In such waveguide, we can write my field like this way; the way I have written. So, this is the way one can write the field distribution.

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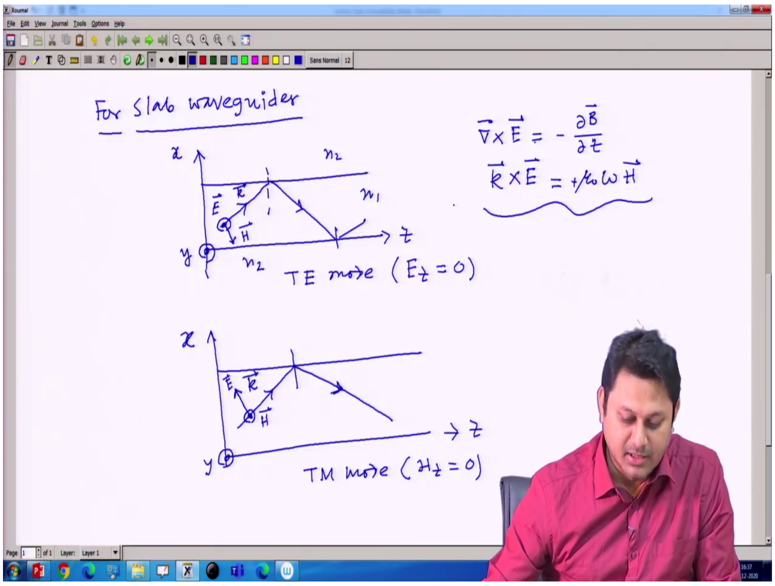

Now, I can go forward with the slab waveguide. So, let us try to understand for slab waveguide or planar waveguide whatever. So, how the TE and TM mode vectory vectorially defined or vectorially represented, let us try to do that.

So, this is the slab waveguide. So, if I make a cross section, it will be this. So, this is my x-axis. So, refractive index is varying along x-axis, y-axis is perpendicular to the plane and this is my z-axis and the slab is sitting somewhere here.

Now, we know curl cross E is minus of del B del t. And from that we also know that this equation in terms of K can be represented as plus mu 0 omega H. So, this basically gives the relative direction between the K E H etcetera.

So, for TE mode, we know that E z is 0. So, if a ray is falling like this, then I can write the E vector. So, this is the K vector. The E vector should not have any kind of z-component, so that means, the E vector should be in xy plane, so it is perpendicular to the plane. So, it is my E vector.

And according to these things K cross E has to be in the direction of H, so that means, my H will be along this direction. So, this is the vectorial representation of a TE mode, that if a wave is propagating in this slab wave guide where the refractive index here is n 1, here n 2 and n 2 – outside. Then the ray can propagate and the E vector and H vector, so this is my H vector is distributed in this way where K is a propagation along the propagation direction.

Well, for similarly for TM modes, I have H z equal to 0 transverse magnetic mode. So, along this direction, I have y; along this direction, I have my x and this is z-direction. A ray is falling like this, facing a total internal reflection, so come back to the medium. So, this is my K vector. Now, here the H z-component is 0. Since the H z-component is 0, it has to be in x y plane, that means, it should be perpendicular to this plane. So, it is H.

And if this is H, so E will be perpendicular. So, let me write H in this way. So, this is H which is perpendicular to the plane. And E vector will be along this direction. Such that the condition K cross E equal to mu 0 omega H should satisfy. So, K cross E, if I do K cross E it should be along the direction of H. So, this is the vectorial representation of the mode. It is important that how it is important to know that how this TE and TM mode vectorially defined inside a wave guide in planar waveguide.

Now, we directly calculate the equations these two equations, we directly calculate, and try to find out the six component and then try to extract the information out of that that is our goal here. So, first we calculate.

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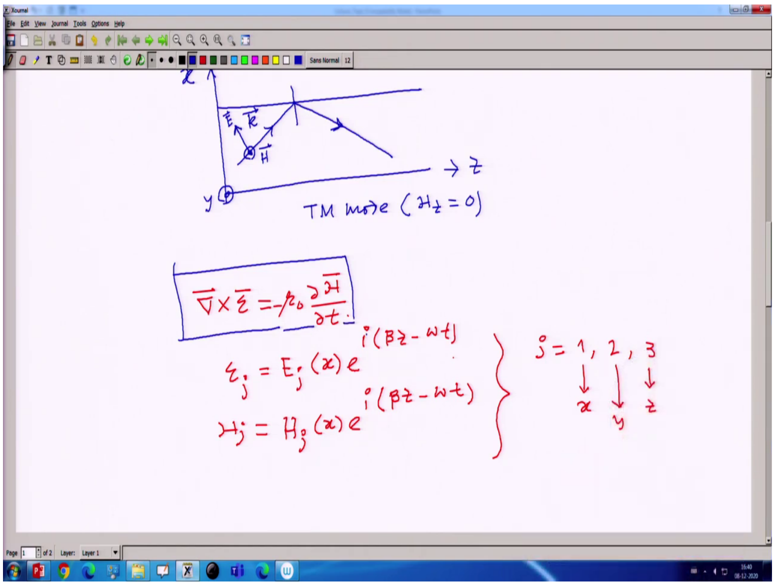

So, what are the equations? So, let me write down the equation. The first equation is curl cross E is. So, let me write once again curl cross E, first I want to execute this is mu 0 minus of mu 0 del H del t ok. And E j, the jth component of the E vector we define that in the last class, it is defined in this way. This is defined in this way.

In the similar way, H component say H j component can be defined as H j function of x e to the power of i beta z minus omega t. So, this is the way two components the components 6 components are defined. Mind it j is 1, 2, 3; 1 basically corresponds to x-component, 2 corresponds to y-component and 3 corresponds to z-component.

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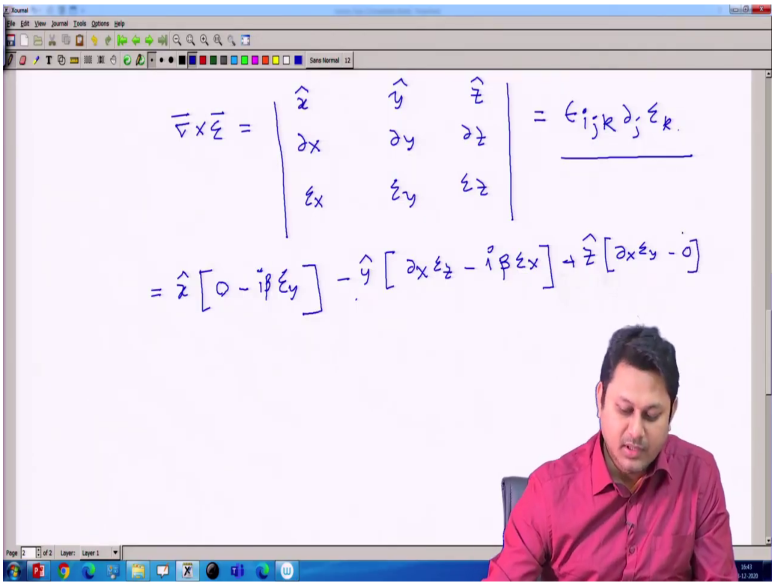

After having that, now we will going to find out what is the value of this curl, curl of E, first I am going to execute. Because in this equation, this is my equation and I divide this equation into component wise that is the first thing we will do. So, what is curl cross E? So, I have x unit vector, y unit vector, z unit vector and then del x, del y, del z and then E x, E y, E z, and let me see with symbols it should be simply this.

Well, I can now know what is my x-component, y-component, z-component, because it is already defined in the previous page. So, I will going to use this to find it out what is the value of this cross product, this curl of these things.

So, it should be x, then E z-component does not contain any kind of y, so it should be simply 0, then minus of i beta E y ok. Let me write the beta in the nicer way. This is the second component because when I derive the with respect to z, then i beta will come out and because beta is associated with z.

So, in after making this derivative, these things will come out with a negative sign, then minus of y unit vector del x E z. Because I do not know what is the x dependency and this x derivative are there, so I need to put it like this way because the explicit form is not known yet minus of i beta E x. The second term after making the derivative with respect to z i beta term again will going to come out. And finally, z unit vector del x E y minus 0. So, these are the three.

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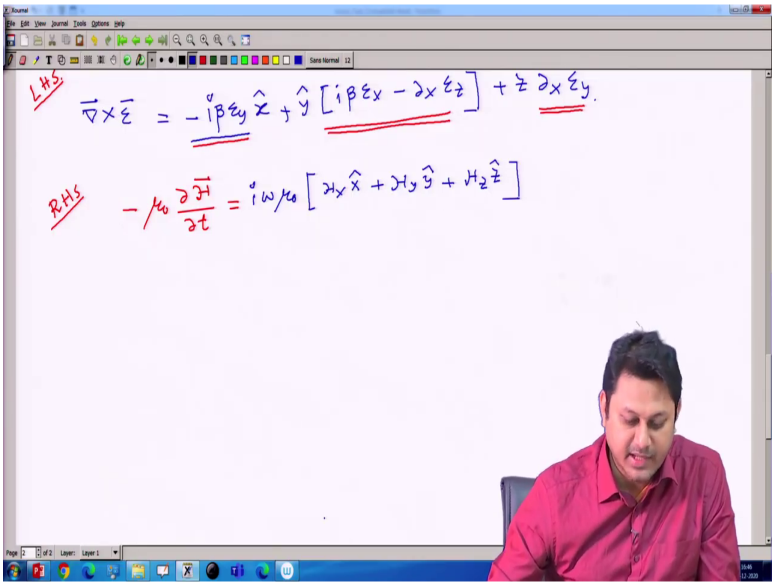

So, if I now write again curl cross E is simply minus of i beta E y x plus y unit vector then i beta E x minus del x E z – second term and third term just writing rewriting the term we have in the previous line this. So, this is my x-component, this is my y-component and this is my z-component that is important because I need to tally with the x, y, z component with the right hand side.

What is in the right hand side? In the right hand side, so this is in the left hand side of the equation. What is in the right hand side of the equation? In the right hand side, I have minus of mu 0 del H del t – this is the right hand side, better to write in a blue. This quantity is simply because H what is my H? H is H j x e to the power i beta z minus omega t. So, t is associated with omega and I am making a derivative with respect to t.

So, what we will going to we what we have is essentially the i omega mu 0 and then the H vector. And H vector if I write in component wise it should be H x x unit vector plus H y y unit vector plus H z z unit vector. This is the; this is the three components these are the three components of this right hand side.

Now, we this left hand side and right hand side are equal. So, now, we will try to find out what is in the x-component, what is in the y-component, and what is in the z-component. If two vectors are same, then x-component, y-component, z-component should be equal. So, the first equation that I have these are the very important set of equations.

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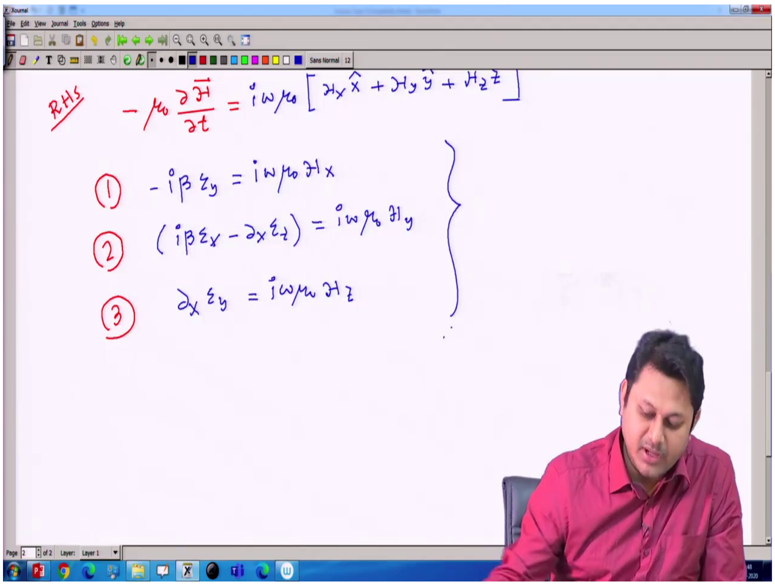

So, let me write equation 1. And if I tally the x-component, it should be simply because here I have the x-component i beta. So, I will just going to write that. So, minus of i beta E y because this is my x-component here that is equal to; that is equal to i of omega of mu 0 H x.

What is equation 2, telling the y-components? It is i beta E x minus del x E z equal to i omega mu 0 H y.

And what is my equation 3? Sorry, I need to write it in a red ink. So, for equation 3, I have del x E y because if you remember it is del x E y and here it is i omega mu 0 H z should be equal to i omega mu 0 H z.

So, I have three components. And three components are related to x-component, y-component, and z-component. So, I get this three equation by just equating the x, y and z-component of this equation. Now, I have another equation in my hand, so this is important. So, this set of equation I have. So, I should have another set of equation with three equations there.

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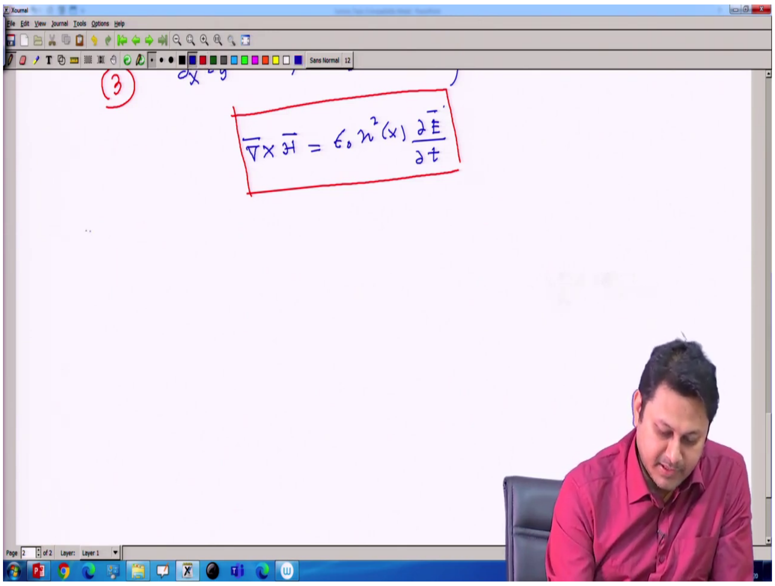

If I use the next equation. So what is my next equation? My next equation is curl cross H equal to epsilon 0 n square x d E d t. This equation also I mentioned here that I need to use these two equation. So, this is equation 2, this equation 2 I am writing. And now in the similar way I can extract these components.

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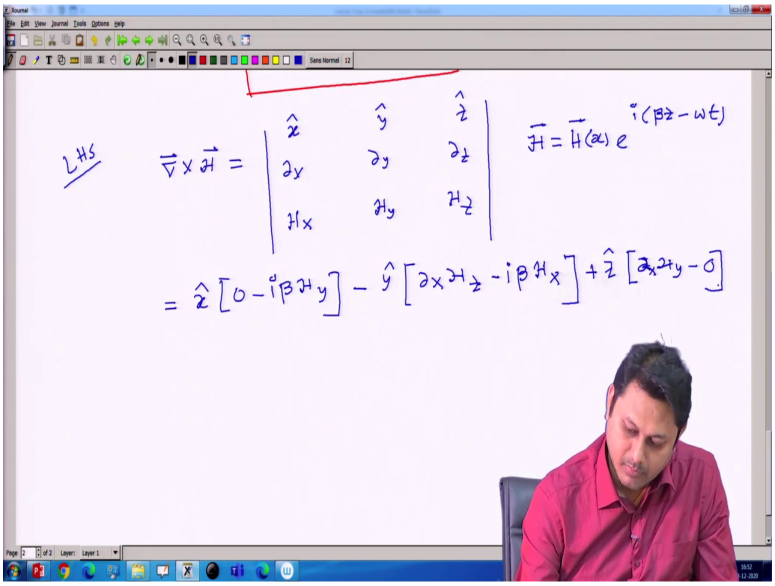

So, again the left hand side, I have curl cross curl H which is x y z del x del y del z and then H x H y H z. Where my H vector is defined in this way, it is H function of x only e to the power with a vector, e to the power of i beta z minus omega t. This is the way we define my H. So, the x-component in the similar way that we have done, so that is equal to x unit vector first term will be 0 because H z-component does not take have any y-component. So, y minus i beta of H y minus y unit vector del x H z and then minus of i beta H x.

Whenever I have a derivative with respect to z, I should have i beta because of this form, this is second term. And finally, I have plus z unit vector the first term will be y z unit vector, the first term will be sorry it should be x. So, del x H y minus 0.

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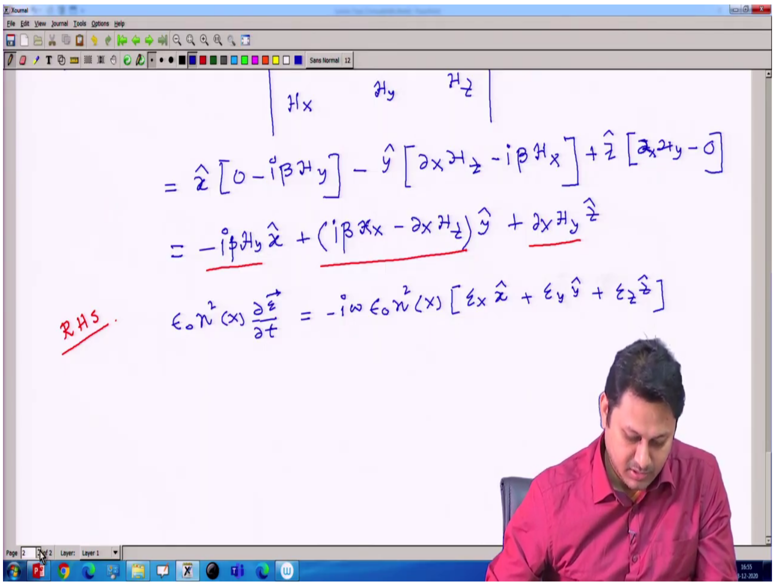

So, I can also simplify or just rewrite the x-component as ok. Let us write in this way minus of i beta H y x that is the first term, second term is plus of i beta H x minus del x H z y and then finally, plus del x H y z. So, I have the components, here this one and this one. This is my left hand side. I also should have expression in the right hand side.

So, what is right hand side? In the right hand side, I simply have epsilon 0 n square function of x del E del t sorry del curl d E, you have to be careful about the curl E and straight forward

E – this one. This thing when I have a derivative with respect to t, so I know my result would be i minus of i omega.

And then the rest of the term epsilon 0 n square function of x and then the E vector; E vector I write as E x y E y y E z z unit vector. So, I can also tally the x and y-component for these two equations – left hand side and right hand side. And I am going to get three equations like we got in the previous case. So, let me go to the next page, yeah.

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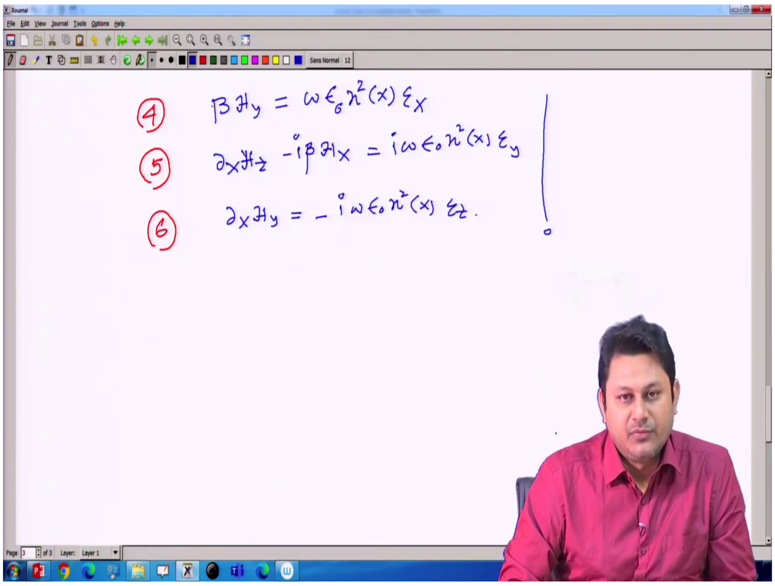

So, I have equation four as equating the x-component whatever we have in two equations here i beta and this. So, I have simply beta H y is equal to let me see what we have I have i will minus i minus a cancel out. So, it should be omega epsilon 0 n square. So, omega epsilon 0 n square as a function of x and E x that is my equation 4.

In the similar way, I have equation 5 equating the y-component change it to color. So, it should be simply del x H z minus i beta H x is equal to i of omega epsilon 0. Right hand side we always have that stuff i of omega epsilon 0 n square x E y. And finally, I have equation 6, which is which is del x H y is equal to minus of i omega epsilon 0 n square x and then E z.

So, again I have three equations in my hand I write it at 4, 5, 6. So, I have 1, 2, 3, and then 4, 5, 6, all the six components of E x, E y, and E z, and H x, H y, H z, these six components are now in my hand.

In this case, obviously, one component is missing because of the way we define the geometry of the plane waveguide that along y direction it is infinite, so that we will going to take care. And then something will be reduced and we will going to see what happened in the next class.

Today, I do not have that much of time to execute these six equations to find out what is the field equation. So, in the next class, we will do that. So, thank you for your attention. And keep looking what happen in the next class, because next class is important, I am going to derive the field equation. So, see you then in the next class.

Thank you for your attention.