Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module – 01
Basic Optics
Lecture – 02
EM Wave in Vacuum, Poynting Vector, Maxwell's Equation in Free Space

Welcome student to the course Physics of Linear and Non-Linear Optical Waveguides. In the last class we started the wave equation concept of wave equation and then the solution and then the Maxwell's equation. And, today we like to understand the wave propagation in Vacuum and its solution and if possible the Poynting Vector and Maxwell's Equation in dielectric medium.

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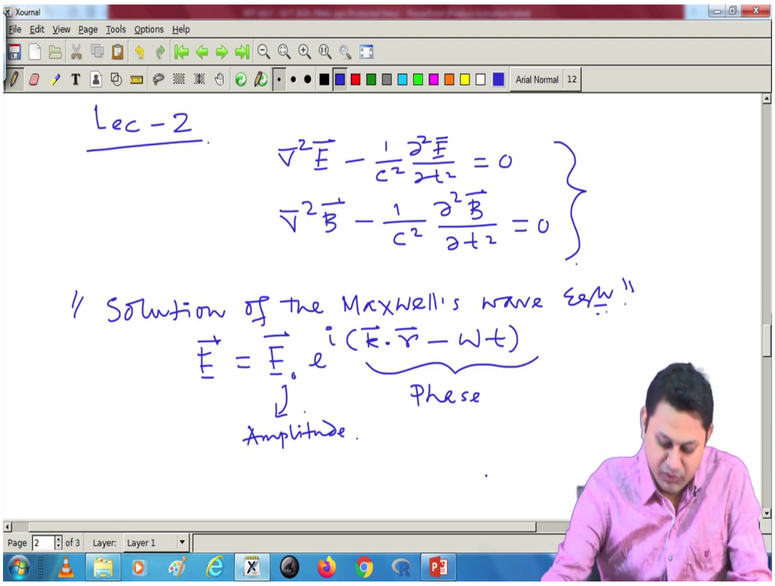

So, let us start quickly. So, today we have lecture 2. So, we have two equation that was the first equation we have the wave equation for electric field. And, also we have the wave equation for magnetic field, these two equation we have. So, today we will try to find out the solution and the solution we will find this way; so, the solution of the Maxwell's wave equation.

So, already in the first class we know that if we have a wave equation then the solution has a typical form. And, let us say this typical form let us write in this typical form as this. This is a very very special form, I have an electric field E and the right hand side I have the electric amplitude E 0. This is the amplitude part and this entire part is a phase part, inside the phase I have a space variable and also the time variable. And, then demand that this would be a solution of the Maxwell's equation.

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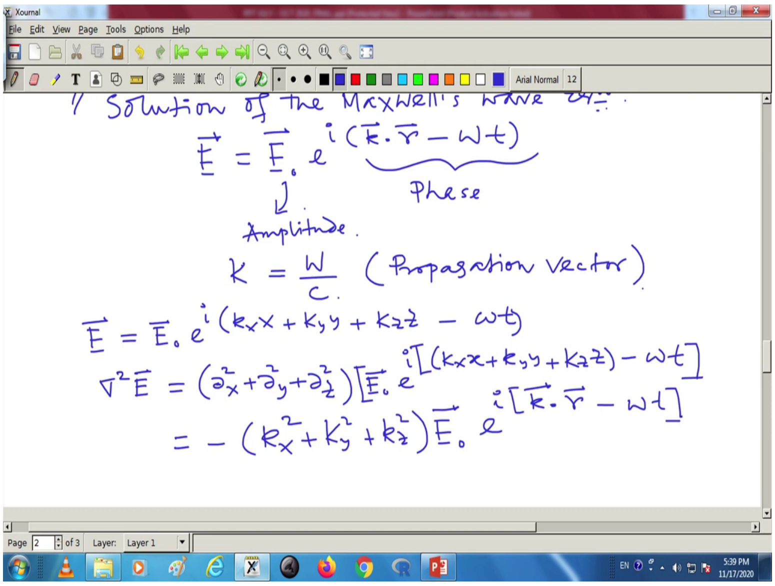

And, we can find the solution with the condition that k is equal to omega divided by c, that is the condition we will finally, find in order to have these as a solution. And, this k is called the propagation constant or the propagation vector, this is the propagation vector. Well, let us check whether really this is a solution or not. So, I have my E as E 0 e to the power of i k dot r, I can divide this k dot r as k x x plus k y y plus k z z, k x k y k z are the three components, minus of omega t.

And, then after having that let me put it in the equation; I need to calculate this quantity grad square E. So, this is del 2 del x s dels x square del 2 y square, in the shorthand notation this is the way one can write. This will go to operate over this term, whatever the term I have which is E 0 e to the power of i, then k x x plus k y y plus k z z minus omega t.

Now, if I operate this thing you can readily understand that when we operate the first term over that then it should operate only on the x component of this term and here the x is present here k x x. So, when I operate these things. So, I will have i k x outside and then the rest of the term will remain same. In the similar way when you operate once again because, this is a second order derivative once again then what we have?

We have another i k x term and so on. So, due to 2 i k x, this i will be i squared. So, I will have a negative sign and I have k x square plus k y square plus k z square these thing and whatever we have in the right hand side will remain same. Because this is a eigenvalue like equation; so, i k vector dot r minus omega t.

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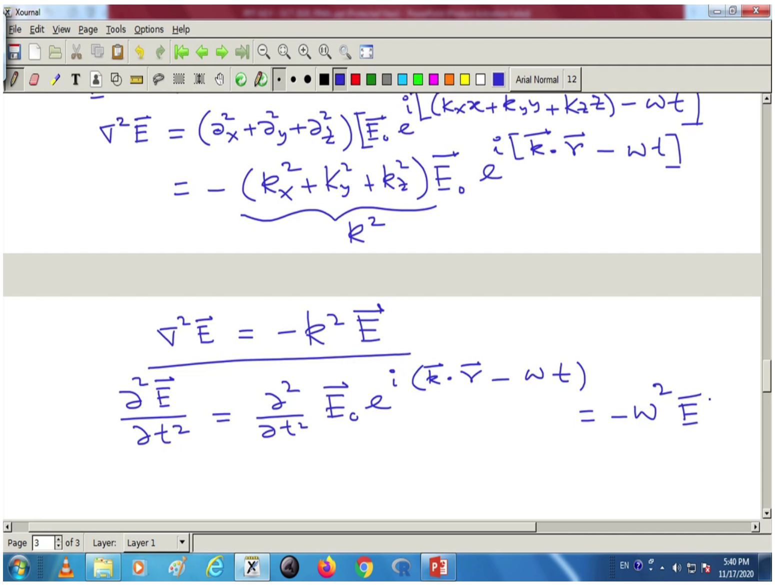

So, k x square k y square k z square whatever we have here is nothing, but k square is the amplitude of the k. So, I have grad square E is equal to minus of k square E because E 0 e to the power i k r minus omega t is nothing, but my E. So, I have this thing. The next thing is to calculate this quantity.

What is this? Again these things will going to operate whatever we have, that is E 0 vector e to the power of i k dot r, in the phase we have two terms. And, when I operate this thing, this double derivative with respect to time will simply give me minus of omega square and then the vector E.

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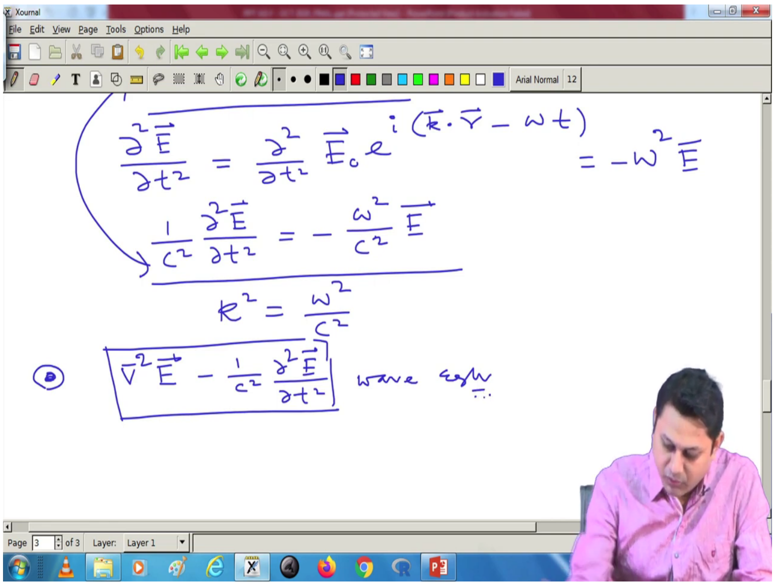

If I calculate 1 by c square because, in the equation I have 1 by c square; it should be minus of omega square divided by c square and then E. Now, these things will satisfy these things whatever we have here and these things, these two will be equal only when k square is equal to omega square by c square; then it should be solution.

And, this solution is called the plane wave. So, why it is called plane wave? We are going to learn, but one thing we find that ok, I have a differential I have a wave equation like this. So, let me summarize what we have, this is my wave equation; for this wave equation I have this is the wave equation.

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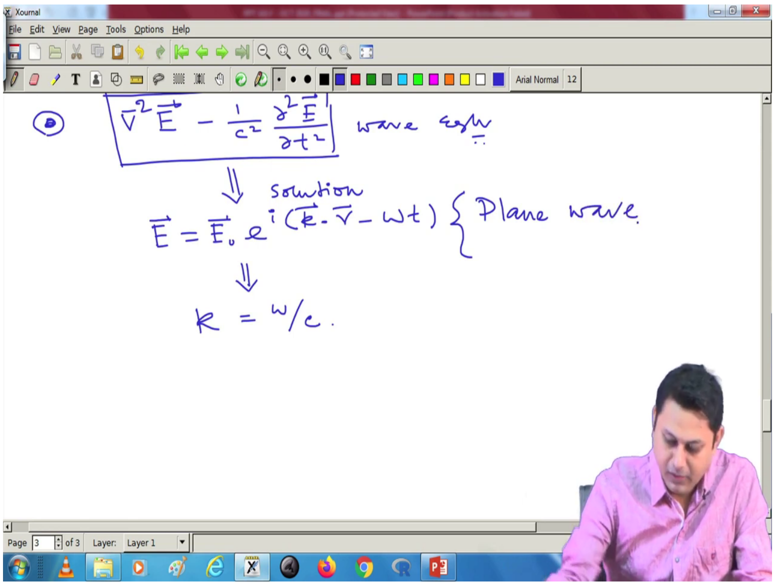

And, for this wave equation I have a solution and the solution is of the form E vector is equal to E 0 e to the power of i k dot r minus omega t. And, when this is solution then there is a relationship between k omega and c and this relation is k is equal to omega divided by c, where k is called the propagation vector. Why it is called plane wave? Why this kind of solution are called plane wave? This is called plane wave by the way, these kind of solutions are called plane wave.

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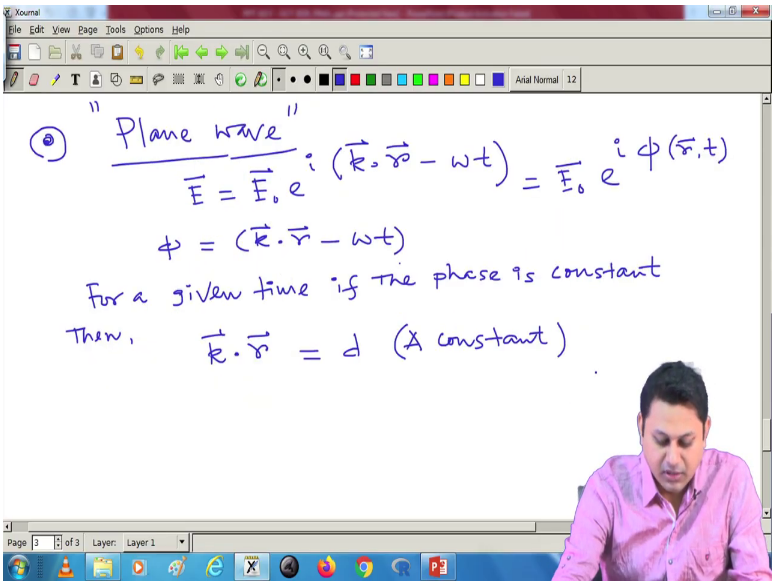

So, quickly we understand what is the meaning of plane wave here. So, I have a solution in this form. So, this is a complex kind of term. So, I can always write this as E 0 vector e to the power of i phi, this thing is scalar quantity and phi is a scalar which is a function of r and t by the way; which is a function of position r and time t.

So; that means, phase is changing. So, phi which is a phase is k dot r minus omega t. Now, if it is a plane wave then for a given time what happened? For a given time if the phase is constant, then we can write; what we can write?

That k dot r for a given time this; that means, t is a constant and the phase whatever the phase I write phi is a constant. So, k dot r is eventually A constant term say d, A constant. So, I have a equation actually I have a equation in my hand, r is a space coordinate and then with the space coordinate I have a equation in my hand.

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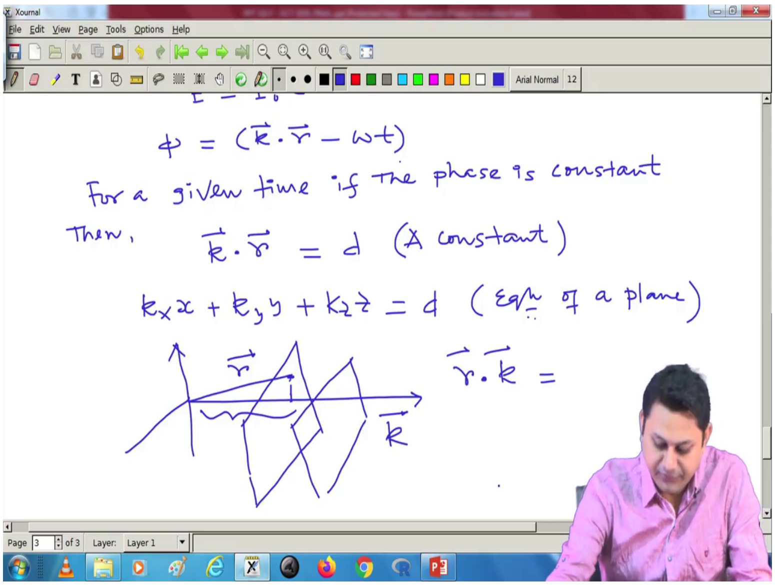

And, what is the equation? The equation is k x x plus k y y plus k z z is equal to d. Now, we know that this equation is the equation of a plane. This is the equation of a plane so; that means, whenever I have k dot r constant then it should be a plane. So, we can understand in this way that suppose I have a coordinate system like this and in this coordinate system, this is the plane I have and I have a point over this plane which is r vector and this is my k vector.

Along this direction the entire plane is moving like these, the next point is like these and so on. So, these planes are moving. So, every time when I calculate k dot r for this particular position k dot r or r dot k, this quantity is basically the projection of r over this point. So, if I make a projection; so, this is the point I am calculating every time. So, these is always constant if I have a plane.

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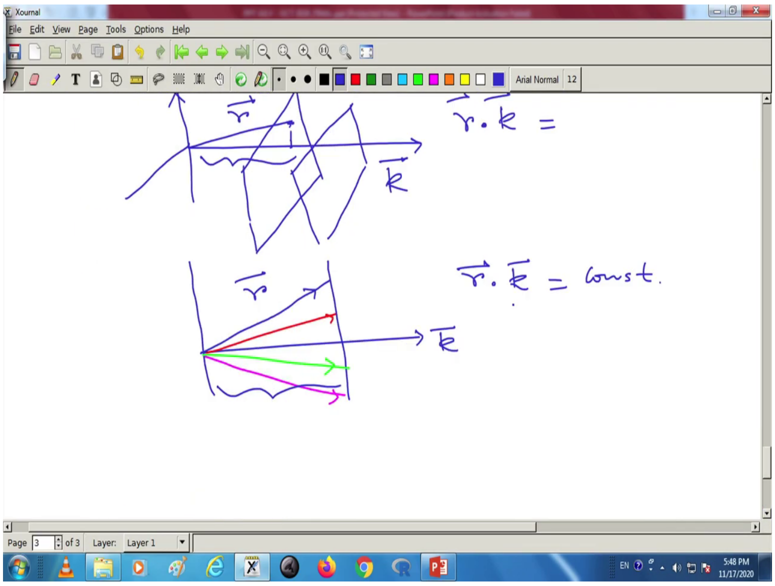

So, if I look this thing in this way; so, this is the plane I have and this is my r vector and this is my k. So, when I have k dot r; so, this is one r, I can have more r. So, this is another r, another point, this is another point, this is another point. So, I have a several different points over the entire surface and each time when I calculate r dot k each time these will remain a constant.

So, this is a constant. So, it happens only when we have a structure of plane. And, whatever the solution we have? Is the form of a plane? So, that is why it is called the plane wave solution. This is a very very important thing that the solution of Maxwell's equation is a plane wave, if we assume is a plane wave solution then we can extract few other things as well from that.

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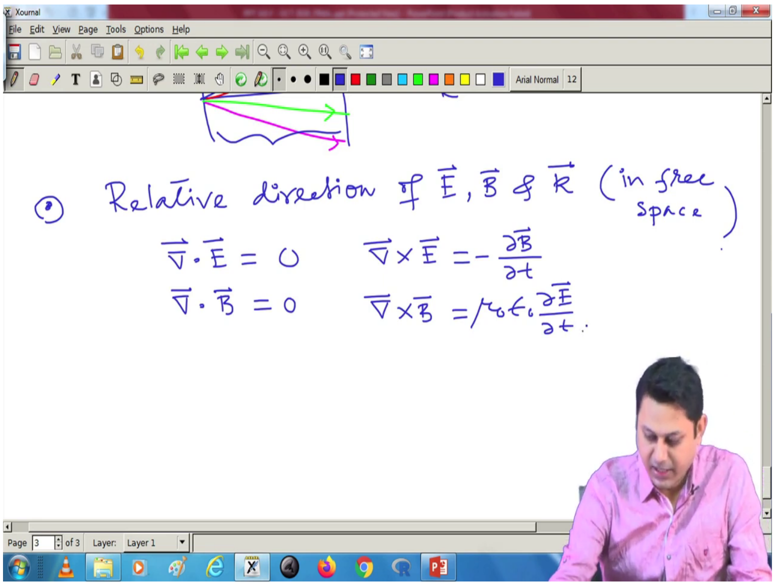

Now, after having the plane wave solution, the next thing important thing quickly we are going to understand is the relative direction of E, B and k. So, three vector quantity we have; so, we are going to find out the relative direction of in free space.

So, again in free space we have three equations grad dot E, four equation four Maxwell's equation rather grad dot B, these are the two equations and another two equation is the curl equations. The curl of E is minus del B del t and then curl of B is mu 0 epsilon 0 del E del t. We have four equations and also on top of that we have the solutions for that.

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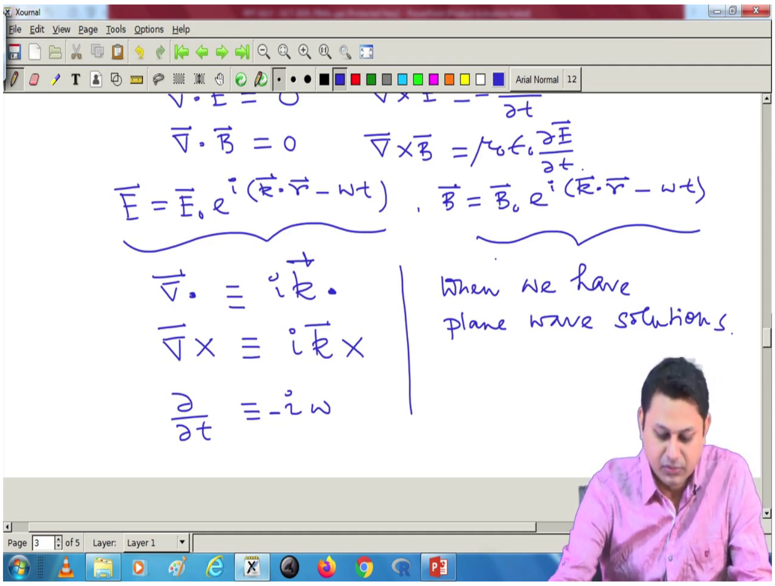

And the solution is E vector is equal to E 0 e to the power of i k dot r minus omega t and similarly I have a solution for B as well. So, B will be equal to B 0 e to the power of i k dot r minus omega t. So, two plane wave solution we have; one is E and another is B. And, now one very important identity, one very important one very important expression that one can extract from the solution is that.

So, if I have these kind of solutions; so, every time when I operate over this, it like it operates over E or B whatever. And, every time if I operate then this del will be replaced by the ik for this particular form, whatever the solution we have exponential. So, it is like this; so, these things will be equivalence. So, if I operate the gradient I if I operate the divergence then it simply ik, you can verify that; you just operate over this.

And, you will find that in the right hand side you have ik dot. In a similar way when we have a curl operator, I will have in the right hand side as this. You can verify quick very quickly that this is really happening, if I have a plane wave solution.

And, also another thing is this operator del del t operator, if I operate del del t then it is equivalent to multiply like minus of i omega. And, these three things can happen when we have plane wave solutions; when we have plane wave solutions then these things will be valid.

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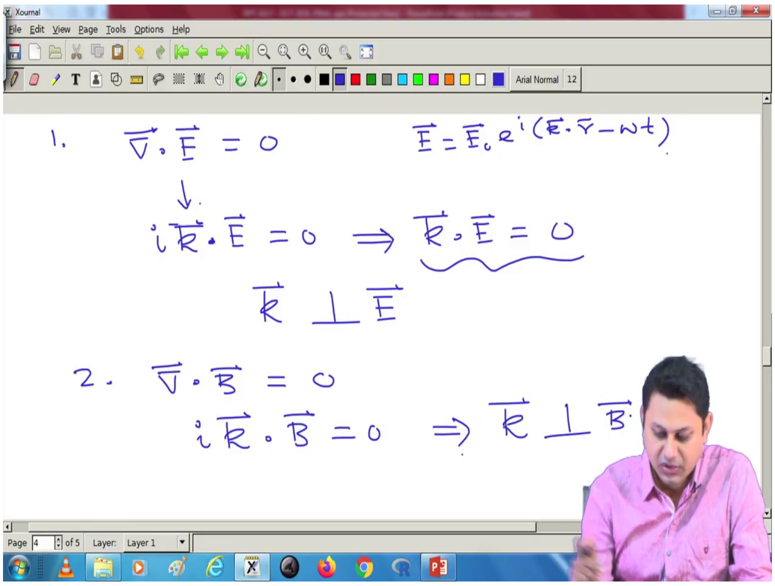

So, we will going to make use of that to all the four equations, we have four Maxwell's equations. So, first equations I have grad dot E equal to 0. And I know the solution of that E is equal to E 0, the plane wave solution e to the power of i k dot r minus omega t, this solution we know.

As soon as I know this solution what I do? I will just replace this thing that this is nothing, but ik dot with this solution E and this is 0 or I can write that is simply k dot E is equal to 0; k dot E is equal to 0 readily gives me the information that the vector k is perpendicular to vector E.

So, the direction of the electric field and the propagation direction which is k is perpendicular to each other; this is the first information we have. In the similar way exactly in the similar way for B, I can have a same equation because gradient divergence of B equal to 0 gives me the fact that ik dot B is equal to 0. And, this tells me that k is a vector which is also perpendicular to the B.

So, k is a vector that is perpendicular to E and k is a vector that is perpendicular to B; that means, k is perpendicular to the plane containing E and B both. This information we have, just using the Maxwell's equation.

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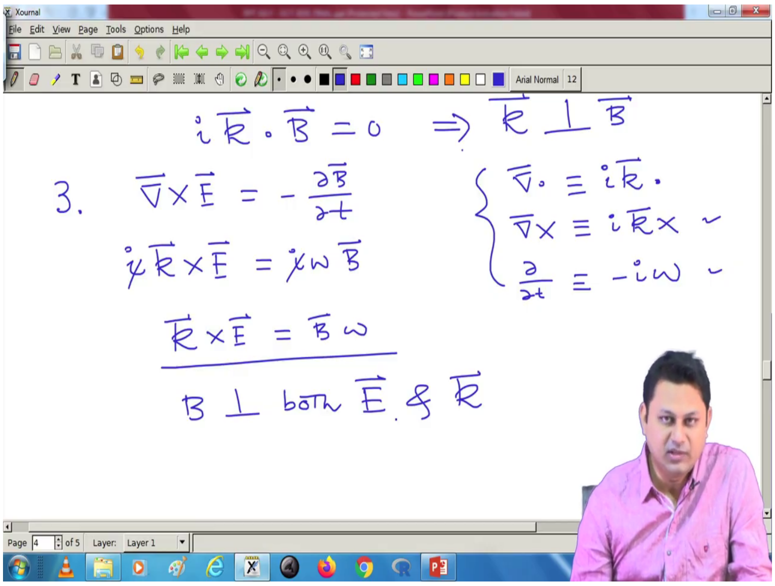

So, k is perpendicular to B and E both fine, then we will going to use my 3rd Maxwell's equation which is the relationship between E and B. So, curl of E is equal to minus of del B del t, again I can replace this curl operator because let me write it once again. So, whenever I have this, this is equivalent to i k dot, whenever I have curl is equivalent to ik curl, whenever I have del del t this is equivalent to minus of i omega.

So, I will use this 2nd and 3rd relation and I find that ik cross E is equal to i of omega of B. This is interesting equation; I can cancel out this i and i both side. So, I have k cross E is equal to B vector multiplied by the frequency omega. So, now, from this equation I can tell that B is again perpendicular to both E and k. Previously, we find k is perpendicular to E and B, now we find that B is perpendicular to both E and k.

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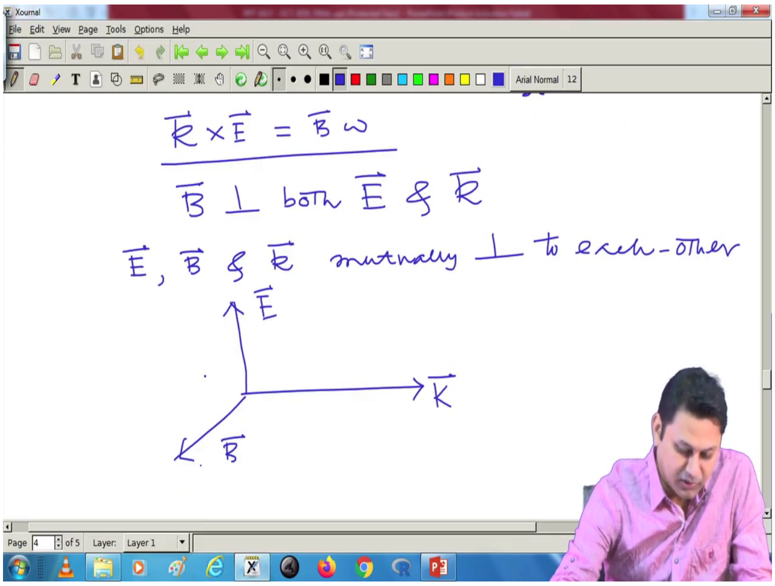

So; that means, that readily means readily tells us that E vector, B vector and k vector mutually perpendicular to each other. That means, if I plot in this direction E then B will be in this direction and k will be in this direction and that is precisely the k is for electromagnetic wave.

We know that electric field and magnetic field they are vibrating each other and their direction they are perpendicular to each other with a sinusoidal variation and they propagate in a direction which is perpendicular to the both.

So, k is the direction of propagation and E and B are placed in this way. Another important information piece of information also we can extract from that and that is the magnitude of B.

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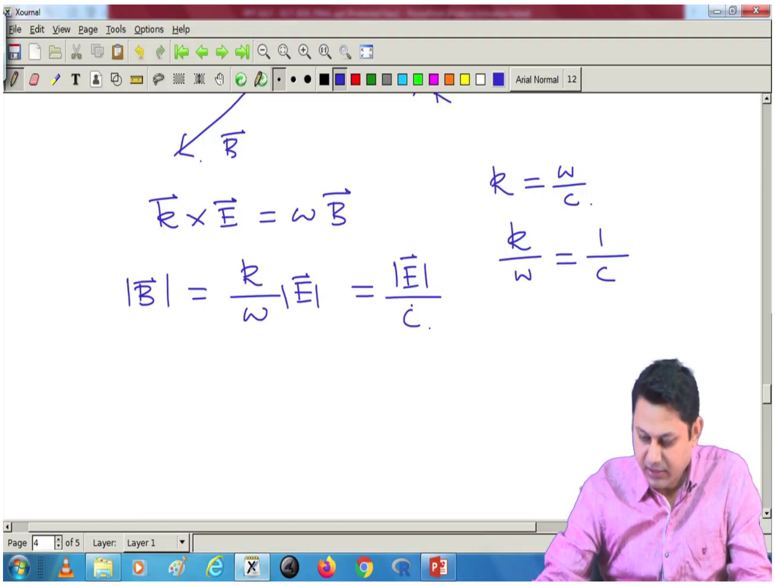

So, I have curl cross E k cross E is equal to B omega. So, I have k cross E equal to omega of B. So, if I want to find out the magnitude of B, it should be k divided by omega and then the magnitude of E and now we have a relation between k omega and c. So, k is equal to omega divided by c.

So, k by omega is simply 1 divided by c. So, it is magnitude E divided by c. So; that means, the magnitude of B and magnitude of E is correlated with the value c and you can see that it is c is sitting in the denominator. So, magnitude of B is much much smaller than the magnitude of E in the electromagnetic wave.

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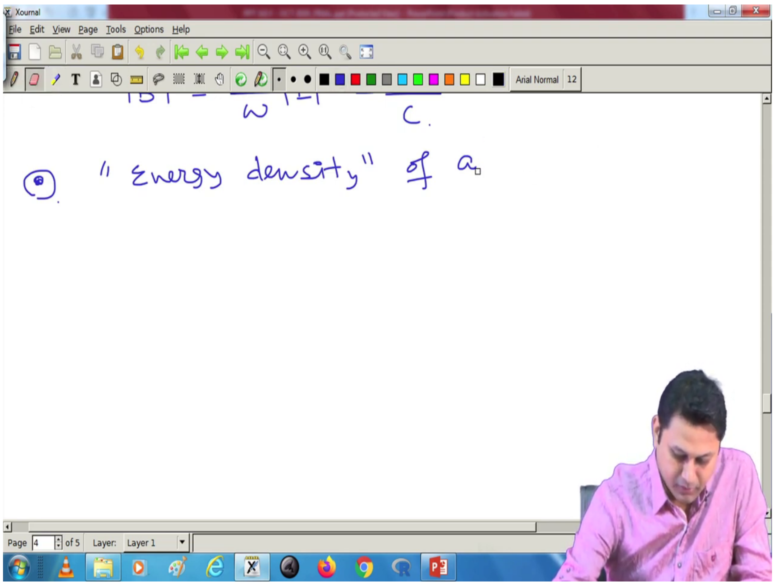

Well, after that we quickly define one thing which is energy density of electromagnetic wave of EM wave.

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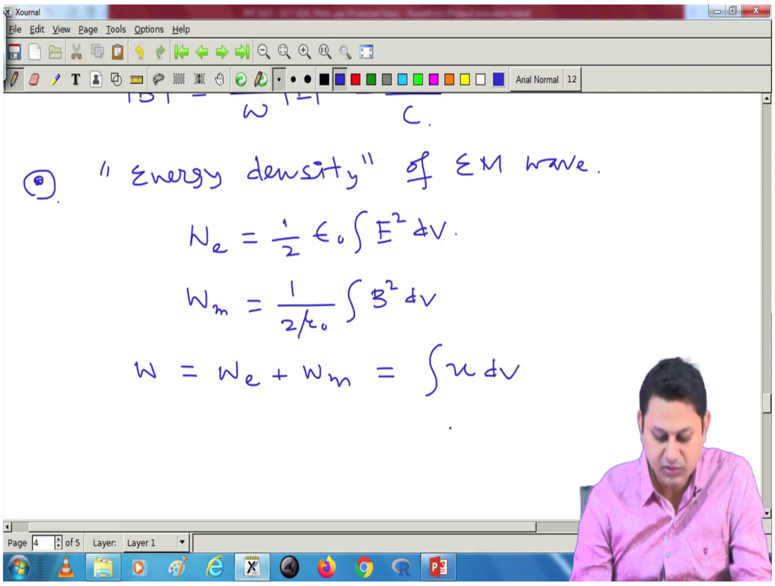

So, we know that the energy in electro electric field we know that the energy can be represented as this. And, for magnetic field also we have an expression similar like this that we know from our basic electrostatic electromagnetic theory. So, the total energy is this plus this and I can write this total energy as the energy density multiplied by the volume, this is integrated over the volume small volume dv.

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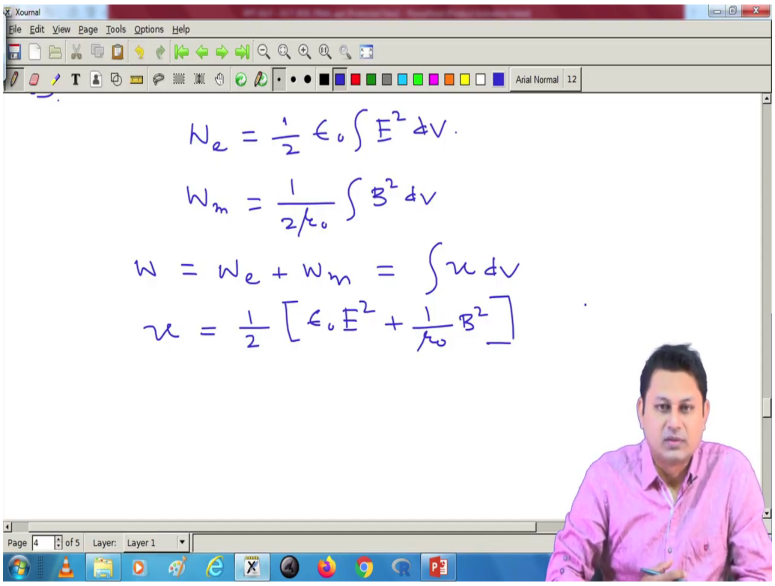

So, from this three equation you can really find out the energy density is simply half of, the energy density means the energy stored in a EM wave, energy stored per unit volume. So, this quantity is simply half of epsilon 0 E square plus 1 by mu 0 B square this.

So, I like to stop here because I do not have much time for this class. So, till now we understand that what is electromagnetic wave, starting from the Maxwell's equation we understand ok; the Maxwell's equation can be written in a wave equation form.

When we have a wave equation then we find that the solution can be represented in a specific form which we call the plane wave; we understand what is plane wave. And, then when the electromagnetic wave is there finally, we just write down what is the amount of energy that it will going to store. So, when the electric field is moving; so, some sort of energy is associated with that.

And, if I calculate the energy density, it will be related to the amplitude of the electric and magnetic field like this way and this is the amount of energy. So, in the next class we will continue with this concept and try to understand what is the meaning of Poynting vector and few other things. With that note let me conclude.

Thank you for your attention.