Physics of Linear and Non-Linear Optical Waveguides
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Module - 02
Basic Fiber Optics
Lecture - 19
Pulse Propagation in Dispersive Medium (Contd.)

Hello student for the course of Physics of Linear and Non-Linear Optical Waveguides. Today we have lecture number 19 and in the last class we started the calculation of Pulse Broadening in a Dispersion Medium. So, today we will going to continue that calculation ok.

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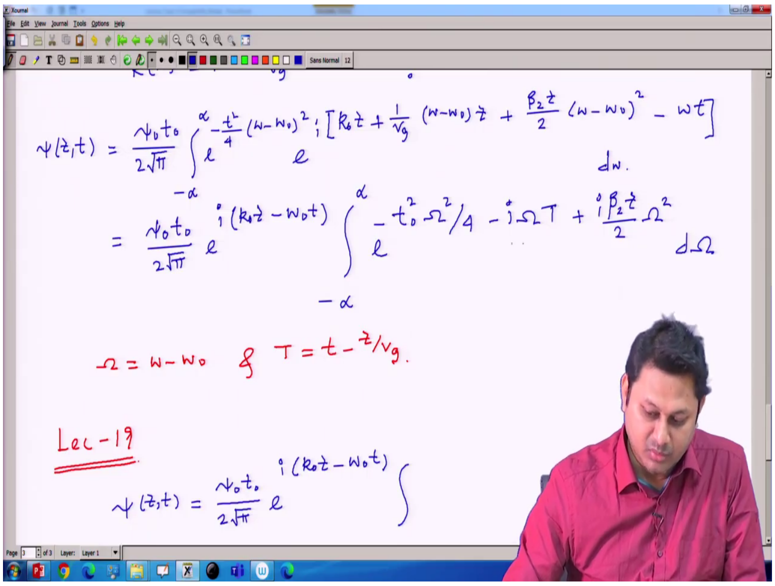

So, today we have lecture number 19 and if you remember in the earlier class the last class we derived psi t in this particular form, where omega and big omega and T was defined. Today so, today we will start from this and continue. So, I have eventually psi z, t as psi 0 t 0 divided by 2 root over of pi e to the power of i k 0 z minus omega 0 t that portion and then the next part which is inside the integration is this one. And I can simplify this a little bit because omega square and omega square are there in.

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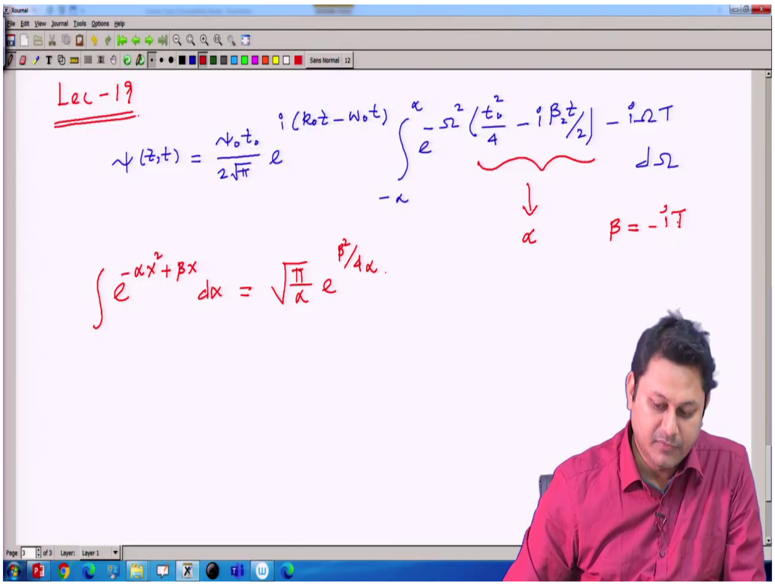

So, I will I can write this at minus infinity to infinity e to the power of minus omega square then t 0 square divided by 4 that is one term and minus i of beta 2 z divided by 2 that is another term. So, take I take minus of omega square common. So, that is why this plus sign whatever is here in front of beta 2 will be minus here.

And now the next term is minus of i of omega b T and d of omega. So, I need to do this integration and again this integration I will going to use the same old expression. And e to the power; we know that integration of e to the power of I already used that in earlier classes. So, here it is. I am just using that once more.

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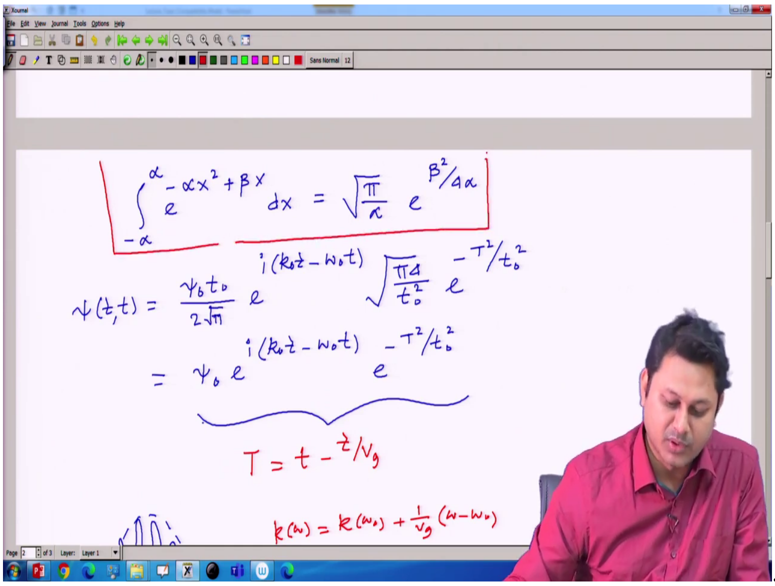

So, let me find it out where it is yeah. This one I am just for your. So, this is the integration I used in the last class. So, I am going to use this once again. So, e to the power of minus alpha x x square plus beta x d x will be root over of pi alpha e to the power of beta square divided by 4 of alpha and here again I am having my integral in this form. So, I have my omega square and which is x square and alpha I have this quantity and my beta is this one.

So, if I compare, so, this is my alpha and my beta is minus of i T as usual, but here my alpha is a little bit complicated form because it is a complex. So, no problem with that. I can I can still do the calculation. Only thing the calculation will be little bit lengthy. So, if I put this thing this integral, so, let me find out this integral first.

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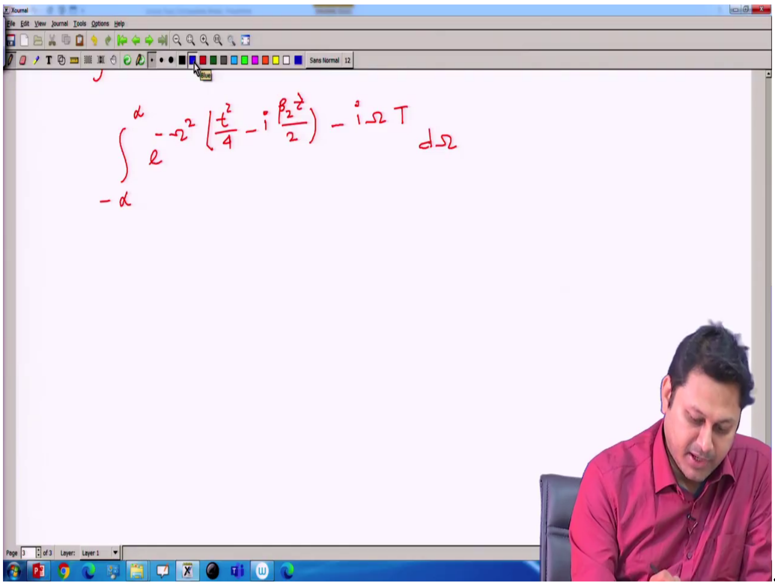

So, left hand side I want to execute this integral. So, integral minus omega square t square by 4 minus i beta 2 z divided by 2 minus of i omega T d omega. Only thing is to evaluate this integral that is all and we know the recipe as well. So, I will going to use this recipe and find it out what is the final value.

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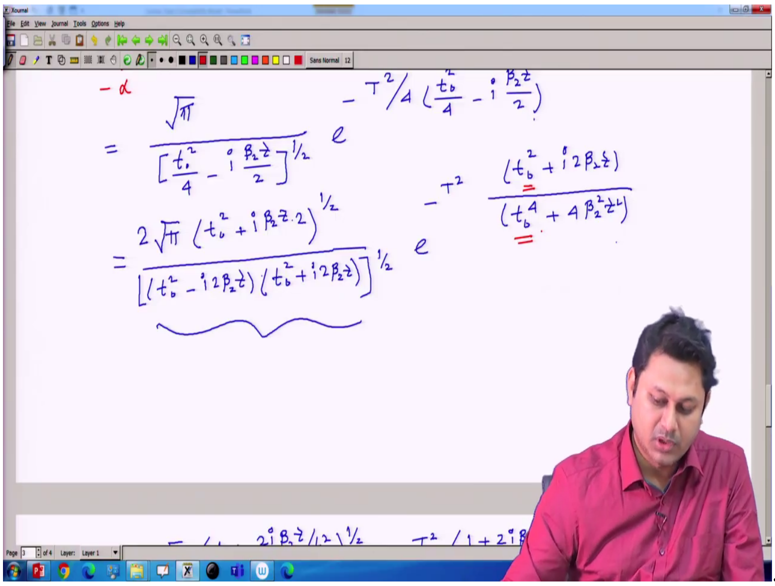

So, I should write minus root over of pi divided by root over of alpha. So, alpha here is t 0 square divided by 4 minus i beta 2 z by 2 whole to the power half, complex term and then we have a root over of that. That is why things become little bit clumsy here, but here I have 4 and then t 0 square divided by 4 minus i beta 2 z divided by 2.

So, that is the value of the integral. So, I already got the value of the integral. Only thing that we need to do next is to evaluate this a plus ib form. So, that everything can be written in a amplitude and phase form then I can extract the information out of this expression whatever the expression I have. So, in order to do this the first thing we do is to multiply this stuff with this. So, I take 4 common and then if I take 4 common from the denominator then I can have a 2 term here.

So, I will do that first. And then I can have a term like t 0 square and then if I multiply with the complex conjugate I will have 2 z multiplied by 2 whole to the power half. The denominator I have this term as usual because I have already taken 2 common. So, this additional 2 will be here and the complex conjugate. This things will be over half ok. Over the exponential I can have a similar thing.

So, I can multiply this as. So, this 4 will be absorbed. So, I should not write this 4 anymore. So, let me write it once again. So, I have only t 0 term here and then I have t 0 square plus i of 2 beta 2 z divided by the complex conjugate if I multiply it should be simply t 0 to the power 4 plus 4 beta 2 square z square. Here also I will going to get the same term by the way.

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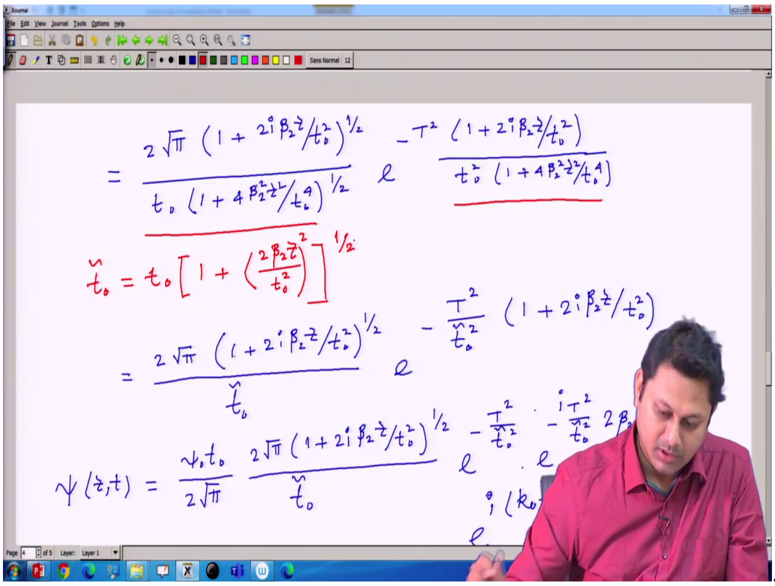

So, the next step I should write it as the integral should be simply 2 root pi which we already have here and then I manipulate this by taking the t square common then it should be something like 2 i beta 2 z divided by t 0 square whole to the power half whole divided by t 0 1 plus 4 beta 2 square z square divided by t 0 to the power 4 and then half.

We have another term e to the power of minus T square 1 plus 2 i beta 2 z divided by t 0 square whole divided t 0 square 1 plus 4 beta 2 square z square divided by t 0 to the power 4. So, what I have done is simply take the t 0 square common here and from this from this term and from here I took the t 0th term common and here t 0 to the power 4 term common. So, 1 by t 0 term will be here. In a similar way here also I took t 0s term t 0 square term common, it is half. So, one t 0 will come out and it will be cancel out by the t 0 square that term that I took from both these two terms.

So, one t 0 will be still here. So, now, I will define my t 0 tilde as whatever we have here. So, t 0 tilde is t 0 then 1 plus 2 beta 2 2 beta 2 divided by t 0 square whole square of that and this. So, I define my t tilde in this way.

So, that I can write this term and this term because here you can see here we have this term and also here we have a term like this which basically characterize the width. And you can if you look carefully by the time you understand that width is not same that we have in the previous case. It is now changed, it is increased. So, that we will going to explain, but let us first find out the expression in terms of tilde.

So, this is 2 root pi whatever we have. So, it is 1 plus 2 i beta 2 z divided by t 0 square whole to the power half and the denominator in the denominator I have t 0 tilde square. No, only t 0 tilde here because this square term is not here and in the exponential term we have minus of T square divided by t 0 tilde square then I have something 1 plus 2 i beta 2 z divided by t 0 square. So, that is the integral part I have calculated so far. So, this is the integral part I have calculated.

So, if I now write the total function, what is the total function? Psi z, t this is my output. If I now write my output it will be psi 0 t 0 divided by root of 2 pi then I have the integral part.

So, let me write it. This 1 plus 2 i beta 2 z divided by t 0 square whole to the power half then divided by t 0 tilde, some term like this.

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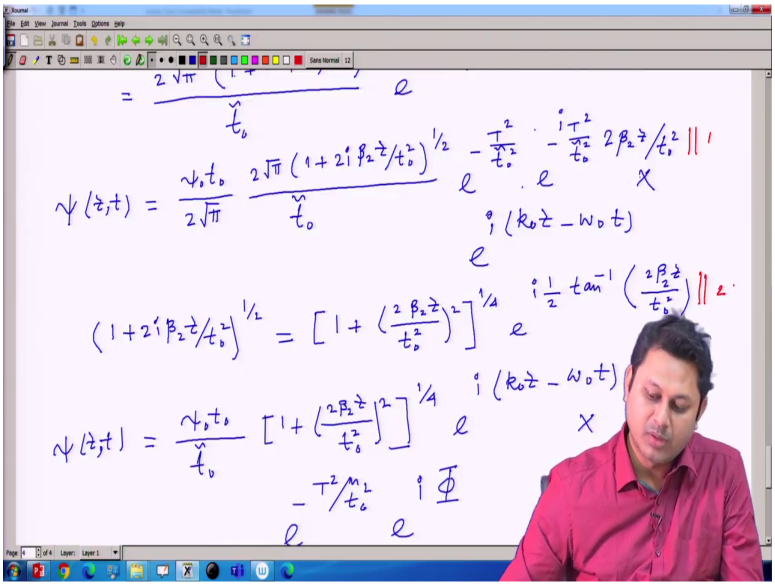

And one term like T square by t 0 tilde square into e to the power of I just dividing the two part. This is i of is the phase part actually this and then 2 beta 2 z divided by t 0 square. I just divide this term here which is written and over that I have an exponential propagating term which is always there. So, this is the overall expression of psi. This is the overall expression of psi.

Now, I can write this term 1 plus 2 i beta 2 z divided by t 0 square whole to the power half as this is a plus b a plus i v whole to the power half. So, we know how to write this. We know how to write this. And I can write it as amplitude and phase form. And if I write the amplitude and phase form it should be simply like this. 1 plus 2 beta 2 z divided by t 0 square

square of that whole to the power 1 by 4 and then the phase part e to the power of i half tan inverse of this quantity because it is tan inverse of y by x.

So, and half term is coming because of this to the power half. So, it should be like this, ok. So, I execute this a plus i b whole to the power half in this way; real and imaginary part in phase and amplitude dividing into phase and amplitude part. So, after doing that I can have a final form in this way.

So, my psi z, t can now be written as psi 0 t 0 divided by t 0 tilde square sorry, it is not square because t 0 tilde was just sitting i remember that. And then this is the part and this 2 pi 2 pi only cancel out. So, I just now need to write this in this way. So, I write it as 1 plus 2 beta 2 z divided by t square whole square whole to the power one by 4 that is fine and e to the power of i. I just write the propagation term in this way. After that I need to write this term which is envelope and phase overall phase.

So, it should be multiplied by the envelope term which is very important minus t square divided by t 0 tilde square and then e to the power of i a full phase term. In this full phase term I can I can write this this. So, in the full phase term I have this one. One term is this one and another term is this one. So, these are the two phase term. This is 1 and this is 2. This two term will be inside this phase term. So, let me write this phase term carefully.

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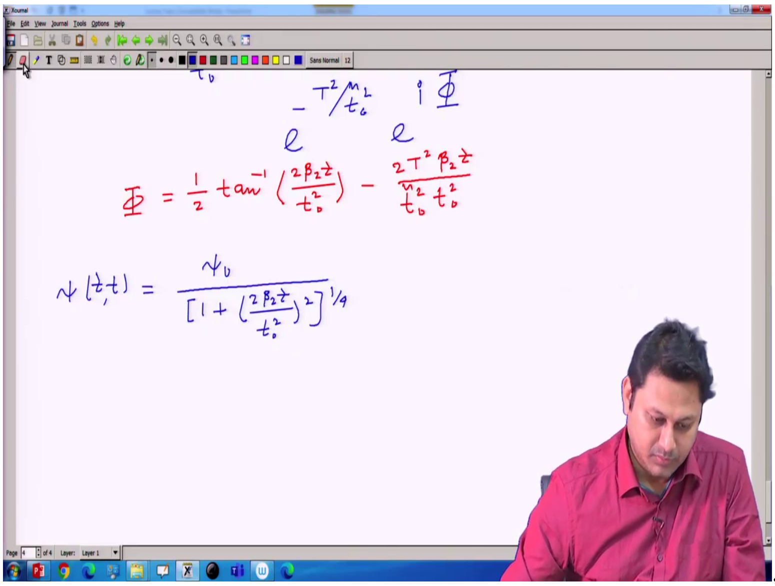

So, my phase term big phase phi will be half of as I mentioned tan inverse of 2 beta 2 z divided by t 0 square this is one term and another term will be minus of 2 t square beta 2 z divided by t 0 tilde multiplied by t 0 t 0 square I think yeah because I am writing this one.

So, 2 this is 2 of t square beta 2 z 2 of big t square beta 2 z divided by t 0 tilde square multiplied by t 0 t 0 tilde square. So, one t 0 tilde square should be here yeah. Then dimensionally also it is correct. So, I have my phi in this form. So, I now I have a total idea what is going on here.

So, I have amplitude term, I have a phase term and I execute this amplitude and phase both. So, now, I write this amplitude and phase in more general way. So, more compact way. So, psi z, t is now psi 0 divided by now I write this 1 plus 2 beta 2 z divided by t 0 square square whole to the power 1 by 4.

This one because this is 1 by 4 and t 0 tilde if i now write it is also there with a t 0. So, this 1 by 4 and to the power half this term we will going to cancel out and we have in a denominator like 1 by to the power 4 because yeah. So, this term will be half. So, not to the power 4.

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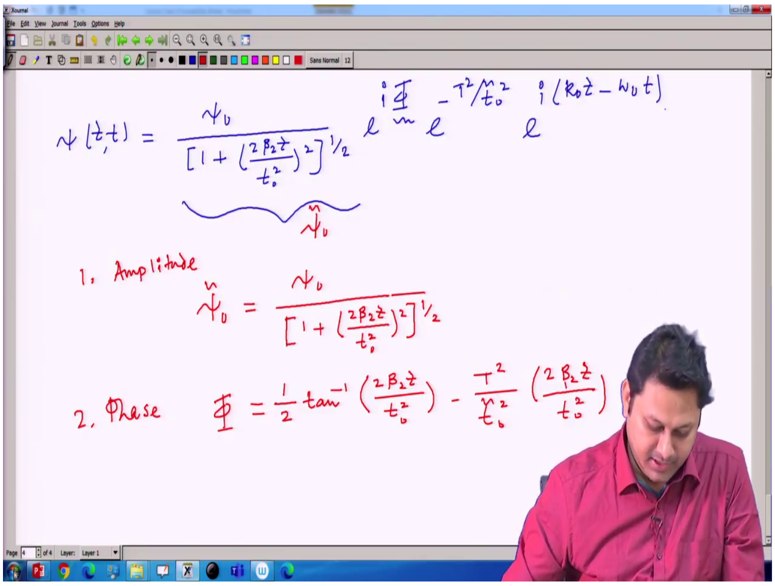

But I think this is half. This is one term. I just write my t 0 tilde at t 0 this quantity and then I have to the power 4. So, I just execute and I have this one and now after that I have the phase term.

So, I write e to the power of i full phase and the envelope term which is minus of T square divided by t 0 tilde square and e to the power of i k 0 z minus omega 0 t the propagation term.

So, I have a amplitude term, I have a phase term and I have an envelope term. So, this is my amplitude. So, let me write it one by one. So, what is my amplitude, what is my phase? So, 1, amplitude term.

So, if I write this as my psi 0 tilde. So, amplitude psi 0 tilde after propagating distance z is equal to psi 0 divided by 1 plus 2 of beta 2 z t 0 square whole square whole to the power half. Next is my phase term. Phase term will be already we write it; tan inverse of 2 beta 2 z divided by t 0 square this one minus t square divided by t 0 tilde square then 2 beta 2 z divided by t 0 square. This is my overall phase and finally, I have the width term that is the most important term here.

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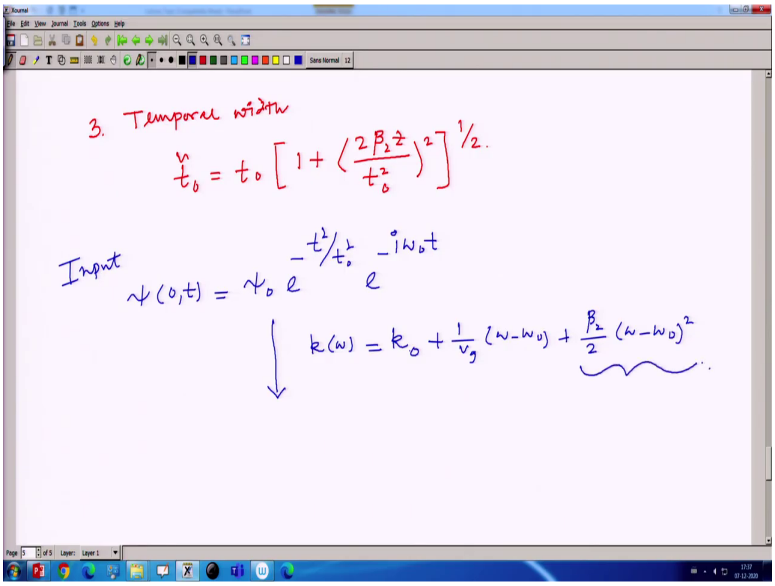

So, 3, we have temporal width. So, the temporal width term which is t 0 tilde that I can write is as t 0 1 plus 2 beta 2 z divided by t 0 square whole square whole to the power half. So, all the terms I execute one by one and then t 0 is let me check ok. Here I am making a mistake.

So, this t 0 tilde should be 1 by half yeah because this I execute. So, t 0 to the 1 by half. So, i execute all the terms. So, now, the interesting thing is to check what is going on actually. So, my initial. So, what is my final field? So, final field let me write. So, initial field; my initial field or rather I should write my input was how much? Input psi 0 t was psi 0 e to the power of minus t square divided by t 0 square e to the power of minus i omega 0 t that was my input.

And after that what we find that k omega is a medium through which the pulse is propagating and this medium is characterized by this 1 by V g omega minus omega 0 and then the important dispersion term plus beta 2 divided by 2 omega minus omega 0 square. This is the important dispersion term. When this term was added then we find my final form in this way.

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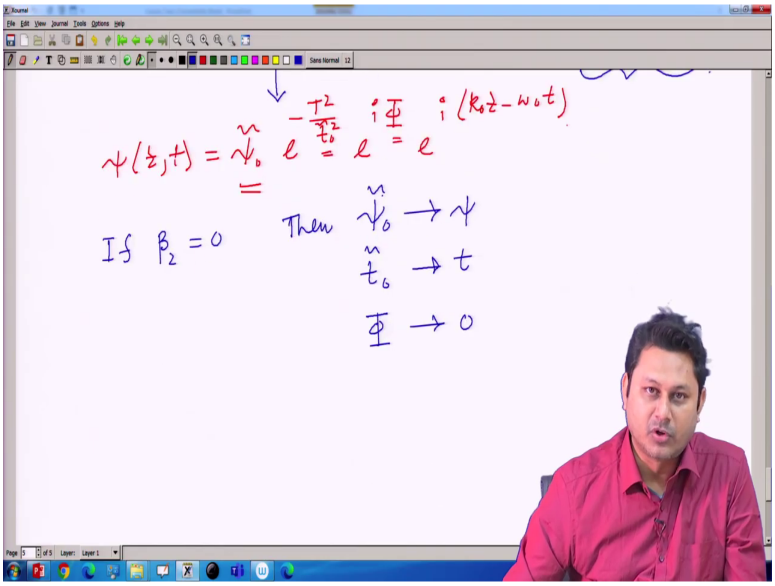

My after propagating a distance z, the pulse looks in this way. So, there is a change in amplitude. So, I put it tilde. The pulse is moving. So, I have a moving reference frame that is why t become big T divided by t 0 tilde square. So that means, the width is also changing and extra phase was there and I have the propagation term as usual like this.

So, my amplitude is modified, my width is modified, my phase is modified. So, I can have three term that is incorporated due to beta 2. And now if I look carefully if I put beta 2 equal to 0, my psi 0 becomes psi, my phi will be 0 and my t tilde will be t.

So, here interesting note: if beta 2 equal to 0 then obviously, psi 0 tilde become psi, t 0 tilde become t and the big phi will be 0 as well. Because it depends on if I look carefully the definite the value of phi it is beta 2 and beta 2 sitting here. If I put both the beta to 0 then phi will not be any more.

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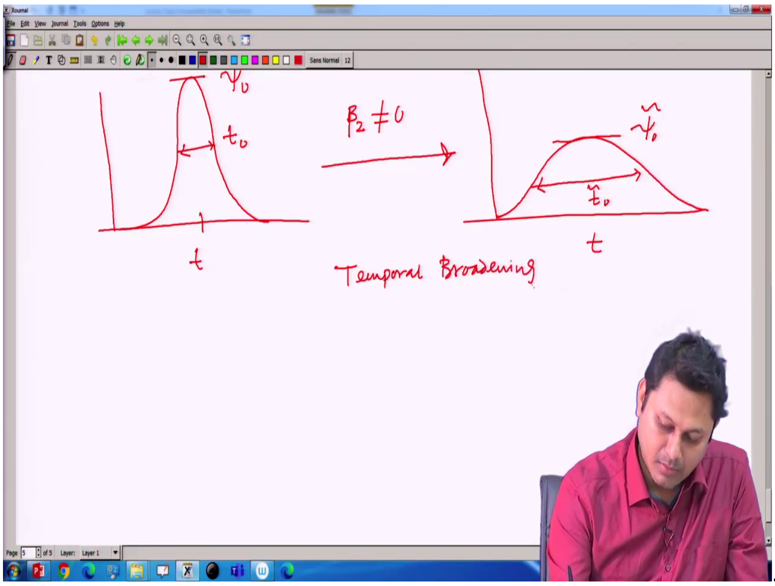

So, what is the physical interpretation of that? The physical interpretation is if I launch an envelope a gaussian envelop suppose is a Gaussian envelope and I have a amplitude peak amplitude psi 0 and width t 0 when it propagates to a dispersion medium where beta 2 is not equal to 0, what happened that there will be a broadening of the envelope. There is a broadening.

So, this value is now my psi 0 tilde and this value is my t 0 tilde both are broadening. So, this is called this is in time domain. This is in time domain and this is moving. So, this temporal broadening is happening.

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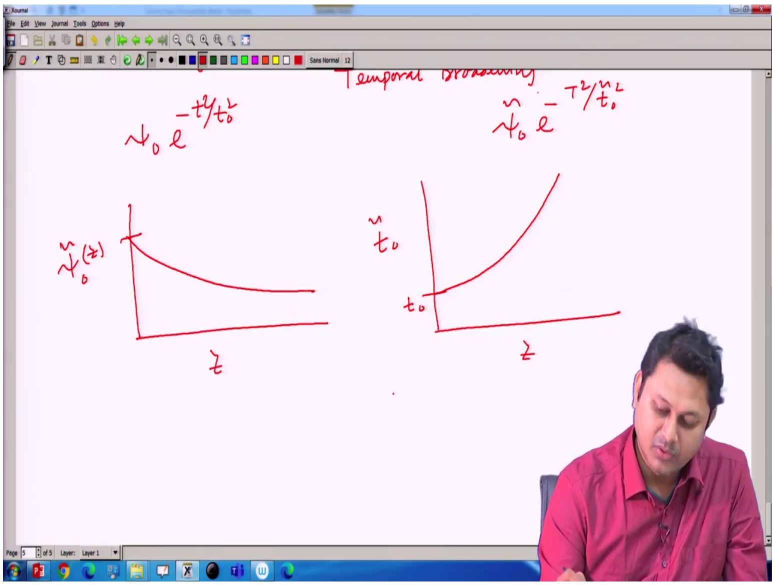

This is psi 0 e to the power of minus t square divided by t 0 square and this is psi 0 tilde e to the power of minus big t square divided by t 0 tilde square. If I plot here my psi 0 as a function of z because this expression already we have psi 0 is psi 0 divided. So, if I increase z what happen? This denominator we increase. So, there will be a decrement of the amplitude. So, it will be decreased like this.

So, this is I plot psi 0 as a function of z. In the similar way if I plot pulse width which is this we will going to increase like this. So, this value is t 0 at z equal to 0 and it is gradually increasing over the z. And how it going to increase is determined by this equation 3 that t 0 is this.

So, with this detail calculation now we understand that under the second order dispersion or group velocity dispersion how an optical pulse will going to broaden. It will not broaden in time domain, but the amplitude will also going to decay that is the first thing. And second thing there will be a mod modification in the phase as well. The phase will going to modify as well. So, with this note I like to conclude today’s class.

Thank you for your attention.