Physics of Linear and Non-Linear Optical Waveguides
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Module - 02
Basic Fiber Optics
Lecture - 18
Pulse Propagation in Dispersive Medium

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we will going to have lecture number 18 and in this lecture we will going to study Pulse Propagation in Dispersive Medium.

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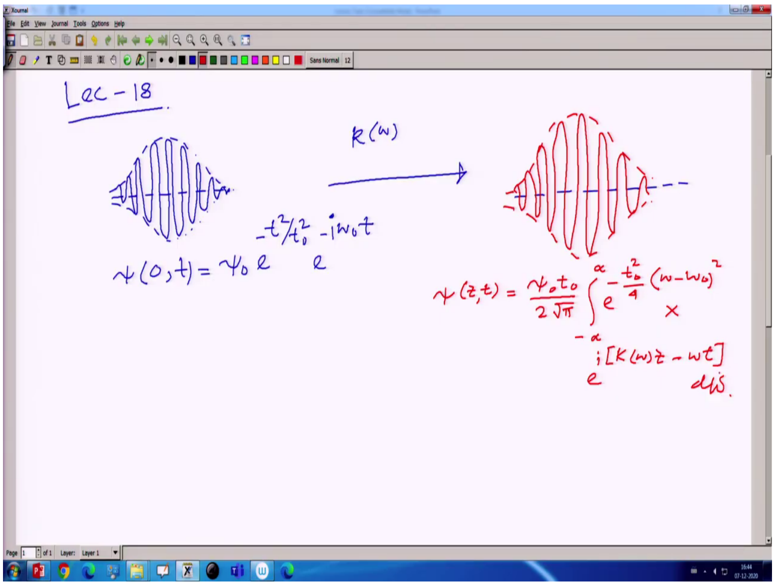

In the last class, we have done a calculation; let me draw it once again. That was the input pulse and a Gaussian input pulse and if I propagate through a medium characterized by k as a function of omega. Then at the output we also have a Gaussian pulse, but the same I can determine in this mathematical form. So, this is my output and this was at some point z t. It is psi 0 t 0 divided by 2 root over of pi.

Then integration of minus infinity to infinity then e to the power of minus t 0 square divided by 4 omega minus omega 0 square multiplied by e to the power of i k of omega z omega z minus a k of z omega function z and then omega t integrated by integrated over d omega. So, now this k omega is very important because the output is depending on the value how the k omega is defined based on that. So, case 1.

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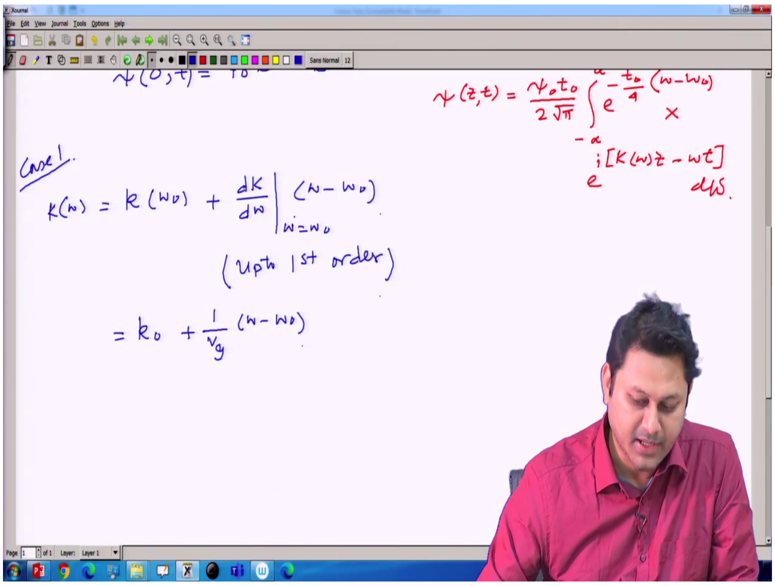

So, first we will going to find out what is happening if I expand my k omega in this way in the Taylor series. So, this is say case 1. I can expand this k omega in Taylor series and if I restrict up to the 1st order term then let us try to find out what will going to happen.

So, I just expand k in these first two term of the Taylor series and now so, this is up to 1st order. Now, this thing is nothing but k 0 plus d omega d p d d k d omega is nothing but 1 by v g omega minus omega 0. So, this is the value of the k as a function of omega I defined.

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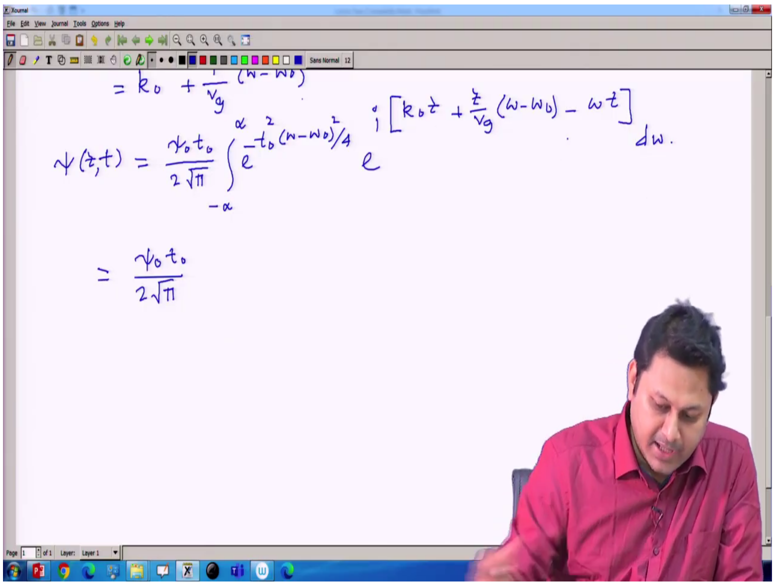

Then I can calculate the my goal is to calculate psi z, t. What is the psi? Function at output. So, psi z, t we already defined. So, it should be psi 0 k t 0 root over of 2 pi integration of this quantity 4 then e to the power of i; now I am going to put this value of k. If I put k z it should be k 0 z plus z by v g omega minus omega 0 minus omega t and then it is over d omega. So, I need to execute this integral then we will get the results.

So, in order to execute this integral I will do in this way. Sorry, it is not square. So, the first term will be as usual. Then here from the from this integral I can take this e to the power i k 0 z out because this is not a function of omega anymore and I will also put a value like this.

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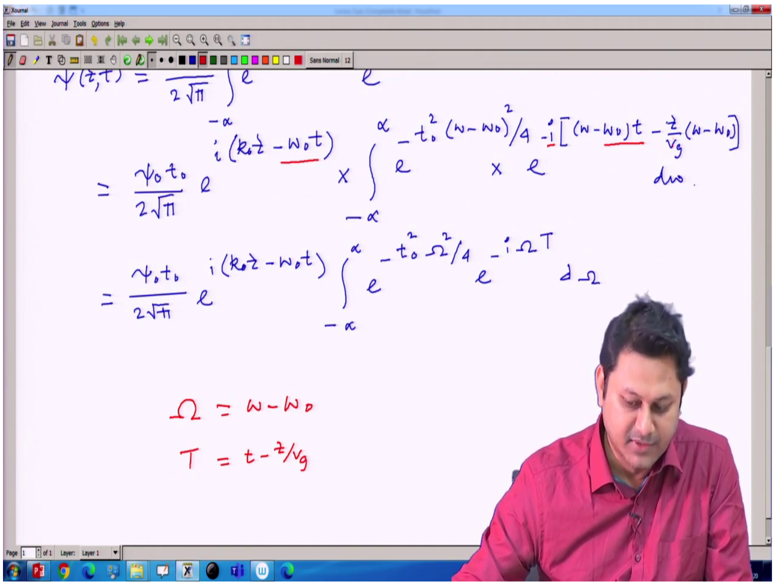

So, it should be e to the power of i k 0 z and I put an additional term omega 0 t, this term was not there, but I put it additionally. So, since I put this term I need to compensate this and I will compensate inside the integral then it should be a compact form. So, let me do that. So, it should be e to the power of minus t 0 square omega minus omega 0 4 into this into term is important; e to the power of i. Then I can write this omega minus of omega minus omega 0 t and then minus of z by v g into omega minus omega 0 and d omega.

So, you can see that e to the power of i omega 0 t is compensated by this term. So, this term, which I take at the additional term is compensated here. Well, then everything in the integral is in the form of omega minus omega 0 that is useful. So, next I will write this as omega 0 t 0. I am going to write term by term what we have and then integration minus infinity to infinity e to the power of minus t 0 square.

And then I write it as big omega square divided by 4 and then e to the power of say minus of i omega and big T d of omega. Mind it, I use two new term here. One is big omega and this big omega is nothing but omega minus omega 0. And another term I introduce this is big T which is simply t minus z by v g that I can find it here. If I take omega minus omega 0 common then it should be t minus z by v g.

So, I have done this and then I reduce this omega minus omega 0 in terms of big omega and t minus z by v g is big T. So, this is a well known term because we are dealing with this term when we start understanding what is the meaning of plane wave and propagating wave and wave equation. So, that is why t minus z by v g is a well known term. This is a transformation in t when the wave is moving with a group velocity v g. So, after doing that I can have my integration in that form. So, now, I can solve this integration and in order to solve this integration I know the identity integral.

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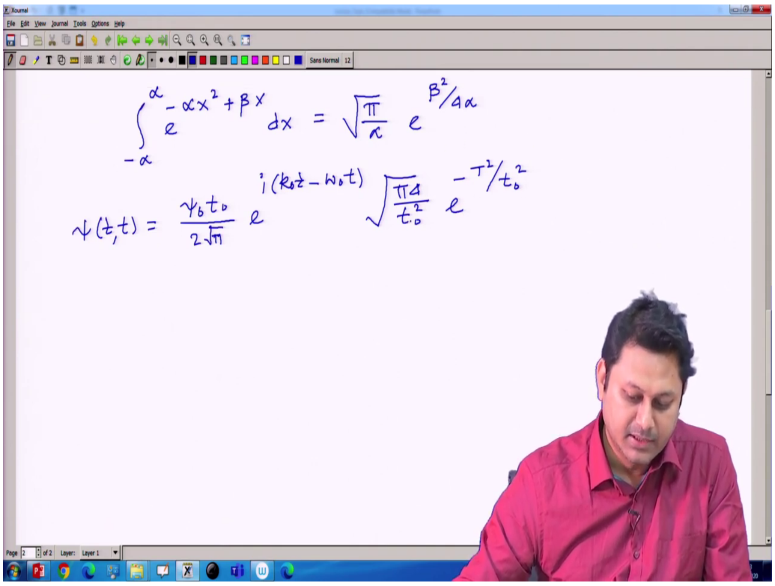

So, this integration we define earlier that if we have this then the result will be root over of pi by alpha e to the power of beta square divided by 4 alpha. This is a standard integral based on the gamma function.

And if we do that if we apply that to this integral then I can execute this value and this value will be. So, my psi z, t will be psi 0 t 0 then 2 root pi then e to the power of i k 0 z minus omega 0 t as usual and from this integration I have pi divided by alpha. So, it should be 4 alpha should be 4 1 by alpha should be 4 t 0 square and it should be e to the power of minus T square divided by t 0 square.

Just use alpha as 4 by t 0 square and beta is minus of i T. So, if I put this value here in this identity then I will get this result.

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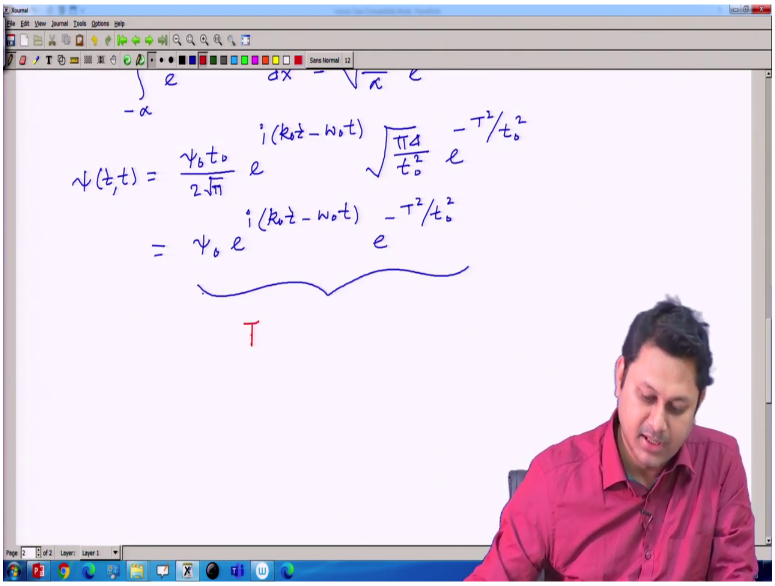

So, I can simplify it. So, you can see this 4 will be 2 because it is under root over. So, this 2 and 2 and this pi will going to cancel out. So, eventually I have a simplified form and this is this one. So, if I look carefully this expression, it is nothing but a Gaussian envelope. So, it is propagating the Gaussian pulse is launched and it is propagating. Only difference is coming through this value of T.

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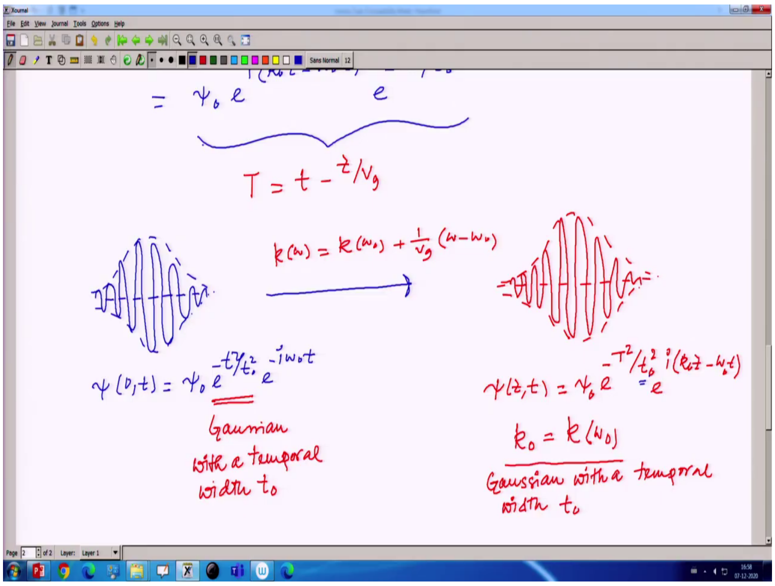

So, please note that my T here is in moving frame and in the moving frame we know it is defined in this way. So, physically what is the meaning of that? So, let me write once again. I have an input with a Gaussian envelope having a frequency. So, this is the shape of the wave packet. So, this is minus by my psi 0, t that was simply psi 0 e to the power of minus t square t 0 square e to the power of minus i omega 0 t that was my initial pulse shape.

And now it is moving in a medium, where the k value is defined in this way; k as a function of omega and I mention that this function of omega is this one. Then in the output I am having a pulse like this. And it should be if I write in terms of t it should be something like psi 0 e to the power of minus T square t 0 square e to the power of i k 0 z minus omega 0 t, where k 0 is nothing but k at omega 0 that I define.

So that means, this pulse is moving without changing any kind of it shape. Initially, it was a Gaussian. It is a Gaussian with a temporal width t 0. In the output also we have a Gaussian with a temporal width t 0. You can see the temporal width does not change here. This value does not change. Only thing is the t and that is true because the pulse is now moving with the group velocity v g and this is in the moving frame that is moving with the same group velocity v g.

So, if I move with the group velocity v g then in the moving frame I can see the pulse shape like this way at some point z. So, there is no change. The important thing is there is no change in the pulse width. If I extend my k value what is the k; where k is a function of omega and if I expand this k as a function of omega and Taylor series up to first two term there is no such change. Now, we will going to do something special here.

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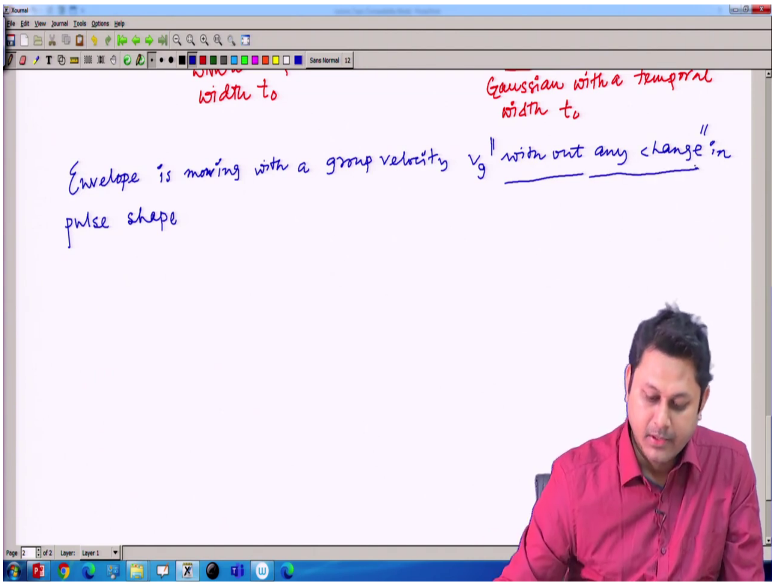

So, let me conclude what we have done here. So, envelope; so that means, envelope is moving with a group velocity v g without any change,without any change so, without any change in shape in pulse shape, now without any change. So, there is no change at all in the pulse shape.

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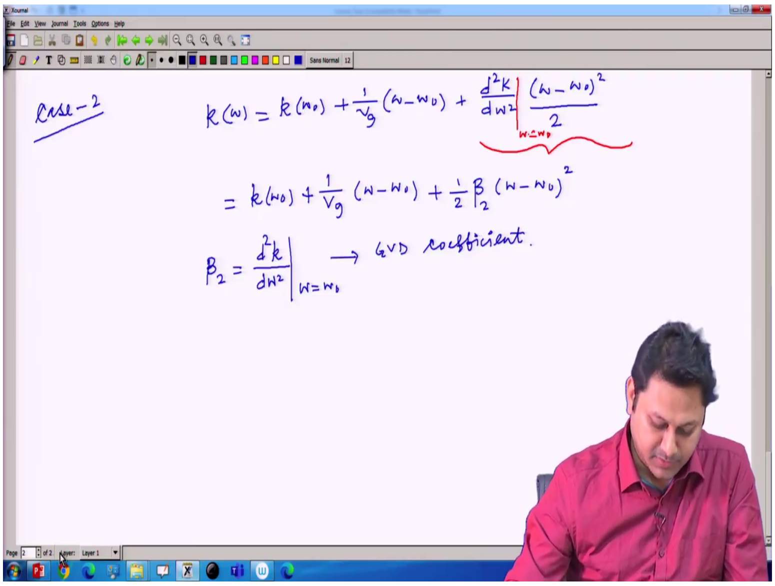

Now, we will move on and try to find out in a more general case. I called it as case 2 and in case 2 what we do that now my k omega I can expand my k omega up to 2nd order. So, I know if I expand my 1st order term is 1 by v g omega minus omega 0 plus now here I have an additional term because I am expanding into another order and this term is this. So, this quantity, so, this is evaluated at omega equal to omega 0 point. So, this term is now very very important because this is nothing but the dispersion term. Already I defined that the coefficient dispersion coefficient in earlier classes.

So, if I use this notation then this expression will look like this. So, beta 2 here is my dispersion coefficient. So, beta 2 we know which is d 2 k d omega square measurement of the dispersion omega at omega equal to omega 0; that means, at the launching frequency this is my beta 2 and that is precisely my group velocity dispersion coefficient.

So, now the pulse is subjected to moving under group velocity dispersion and when I introduce this 2nd order term then these things the calculation will be little bit lengthy. So, let us try to start the calculation. I do not know whether I can complete this calculation in a single class or not, but let us try.

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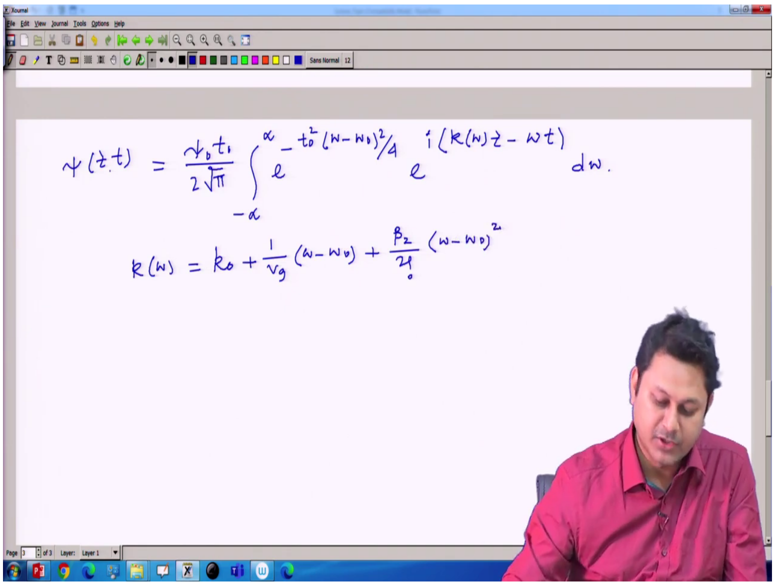

So, I have my psi z t as a general form and that is psi 0 t 0 divided by 2 root over of pi integration minus infinity to infinity e to the power of minus t 0 square omega minus omega 0 square divided by 4. And then e to the power of i k as a function of omega z z minus omega t d omega that was the general form of a wave function that is moving in a system where k this is the k omega is the dispersion relation.

Now, I put the dispersion relation that we derive. So, after having this expression now I will going to put these k omegas. So, my now k omega let me write it once again is now k 0 plus 1 by v g omega omega 0 plus beta 2 divided by 2 omega minus omega 0 square, factorial 2 and 2 are the same thing, well.

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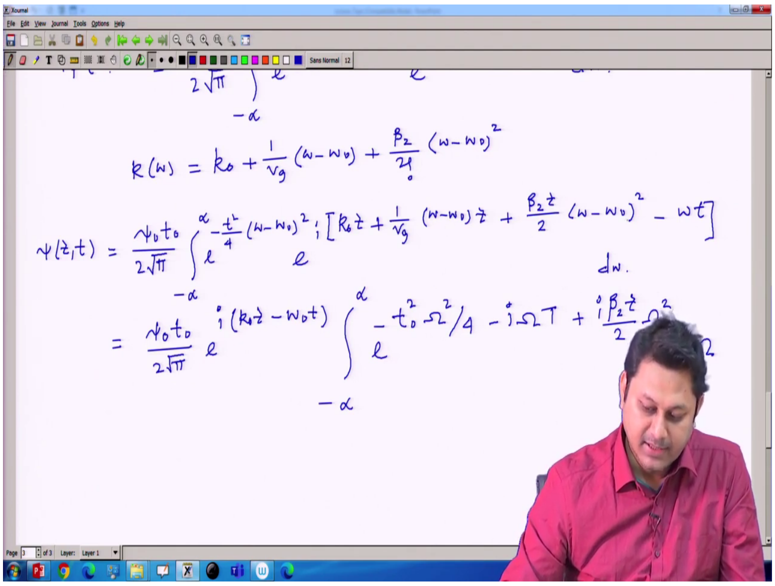

So, my psi of z, t will be psi 0 t 0 divided by 2 pi integration minus infinity to infinity e to the power of minus t 0 square by 4 omega minus omega 0 square then e to the power of i, I expand the k. So, now, I put this k. So, k 0 z plus 1 by v g omega minus omega 0 z and then beta 2 z divided by 2 omega minus omega square and then minus of omega t that term was there already there and d omega.

Well, now, I need to execute these things. The process is same that we have used in the earlier cases in case 1. So, it should be this and I will take this e to the power i k 0 z outside with an additional term omega 0 t for our convenient.

And then I integrate it over minus infinity to infinity e to the power of minus t 0 square omega square divided by 4 minus i of omega T big omega T plus i to z by 2 big omega square d omega. All the omega minus omega 0 value now I replace to big omega and then I combine this t minus z by v g term as big T and the rest of the term beta 2 z with omega minus omega 0 square I write it here.

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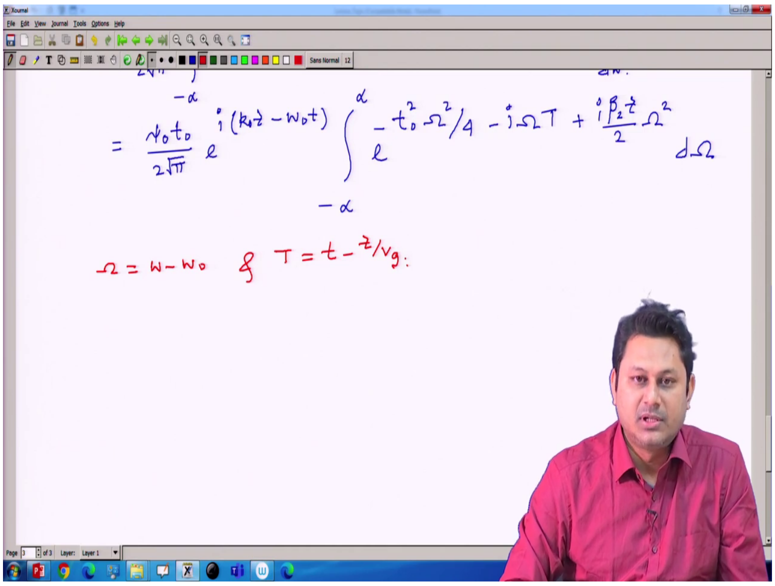

So, my omega here again I should define. My big omega here is omega minus omega 0 and my big T is t minus z by vg. So, today I do not have that much of time to complete this integration. So, in the next class, we will complete this integration in the similar way that we have done in the earlier classes. So, please try to do this integral by your own hand in as a homework you can take as a homework, but anyway in the next class I will do that with all the steps. So, with this note let me conclude.

Thank you for your attention.