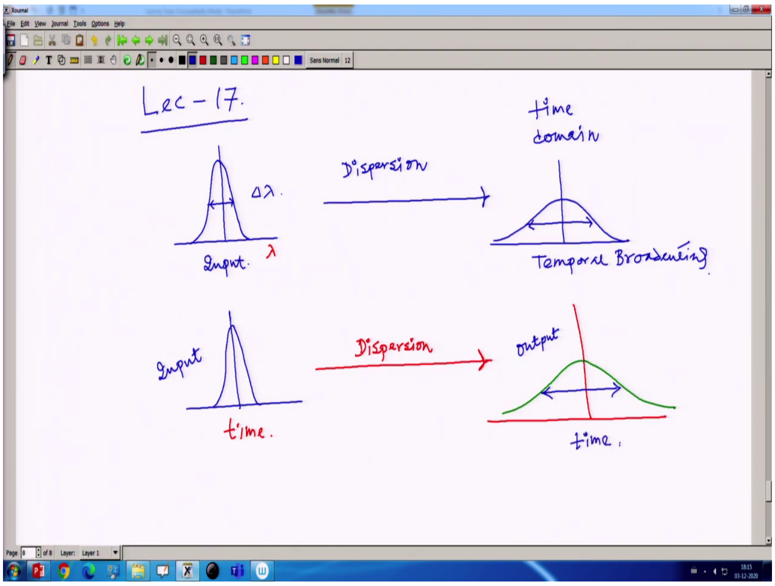
Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 17
Pulse Broadening

Hello student to the course of Physics of linear and non-linear optical waveguides. So, today we have lecture number 17. And, today we are going to discuss about the Pulse Broadening, which is related to the material dispersion.

(Refer Slide Time: 00:31)

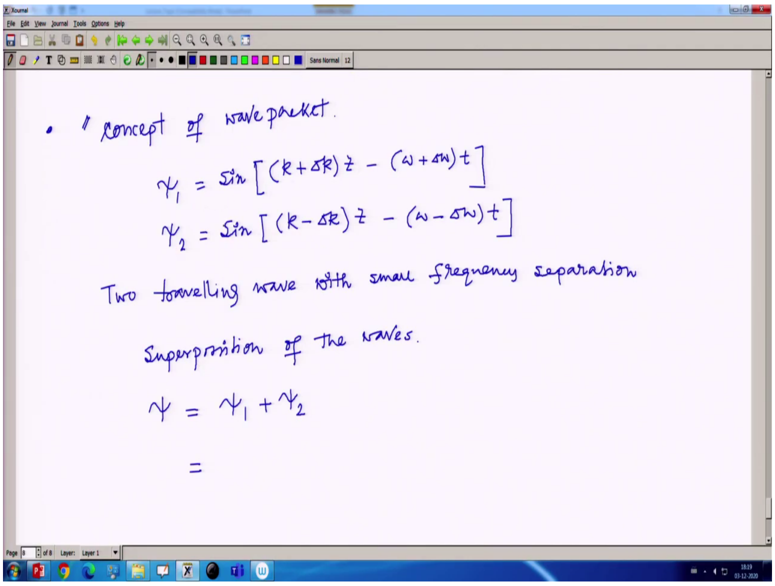


So, in the last class we understand that in the dispersion due to the dis in the dispersion what happened. Then, when an optical pulse is propagating in a dispersion having a spectral width in optical pulse having, it is having a broadening spectral broadening spectral width.

So, delta lambda is a spectral width for this input. When it is passing through a dispersive medium so, due to the dispersion what happened in time domain, this is in time domain. We have a broadening, which we call temporal broadening. So, if I just draw the time picture, this is optical pulse in time domain mind it this is time. When I draw this this x axis it is frequency lambda.

So, in dispersion we have a temporal broadening here. This is input, this is output. And, we have v like this in time. So, we are going to understand these things in in today's class and maybe in the next class as well.

(Refer Slide Time: 03:13)

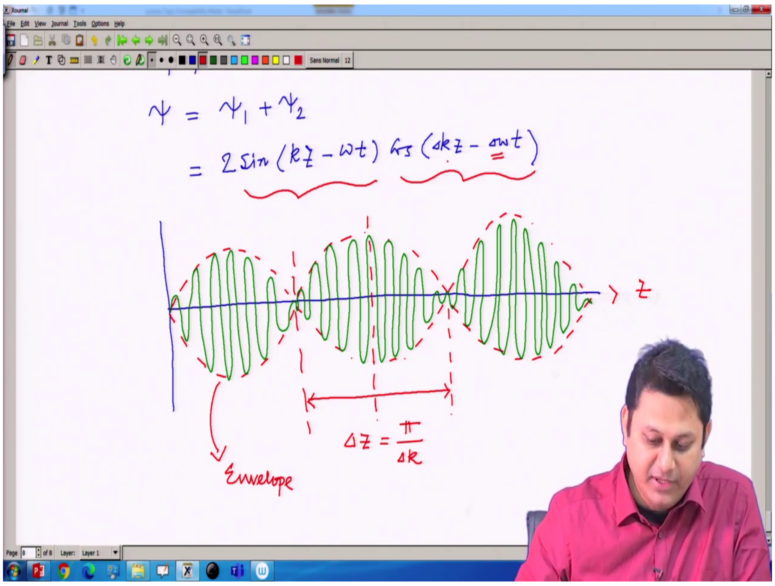


So, before that we should understand the concept of wave packet. Suppose, we have two waves propagating waves psi 1 and psi 2, having this form. So, this is sin k plus delta k z minus omega plus delta omega t. And, psi 2 is another wave propagating wave sin k minus delta k, z minus omega minus delta omega t. Now, this is two travelling wave with small frequency separation.

So, we have two traveling waves psi 1 and psi 2. And, there is a small frequency mismatch; one is omega for one case the frequency is omega plus delta omega and another case it is omega minus delta omega. And, accordingly the wave vector is also different. Now, if I superpose this wave I have super position. So, psi is psi 1 plus psi 2, because I am making a super position. So, my total is psi 1 psi is psi 1 plus psi is 2.

So, if I now add these two psi 1 is sin k delta k k plus delta k z minus omega plus delta omega t, and psi 2 is sin k minus delta k z minus omega minus delta omega t.

(Refer Slide Time: 06:42)



So, if I superimpose. So, I should have this. If, I just simply add these two wave, it should be 2 of sin k x minus omega t. And, cos of delta k x minus delta omega t this interesting expression oh sorry this is z so, I should write it as.

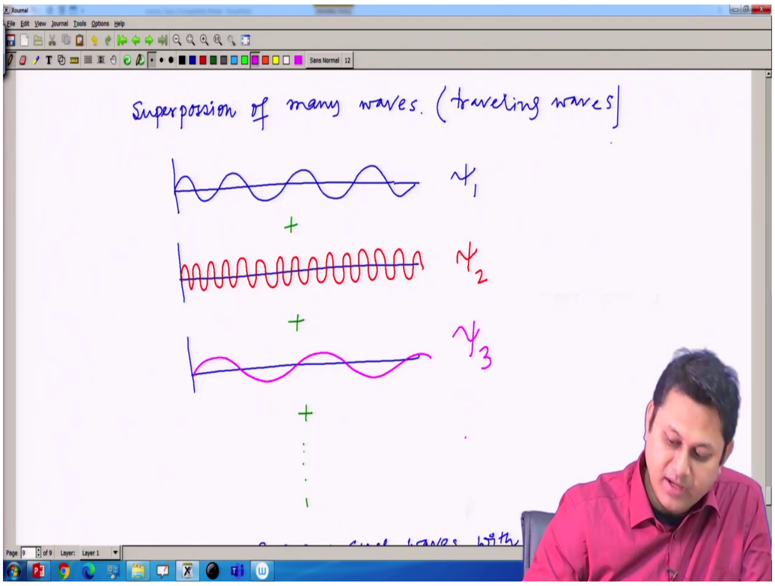
So, I have one wave with very large frequency and another wave, which we call the envelope is having a very small wavelength or frequency. So, if I draw these things, if I draw this stuff, I will have an envelope part like this. And inside the envelope I have a variation sinusoidal variation like this and so on.

So, this is called the bit formation. This is not a very new thing to most of the students and this is that, this is the distance. So, this is this is the 1 by 1. So, this is the envelope this is called the envelope, which is varying with a relatively small frequency. On the other hand inside the envelope we have the distribution of a pulse, which is varying with the frequency, larger frequency, this larger frequency is omega.

Now, I can calculate and find that these the width from here to here. If, this is location interesting Z, it should be pi divided by delta k. So, easily one can find out from this. Well, this is the superposition of mainly two waves and we find that we are having a concept of envelope. And, inside the envelope I have a varying distribution like this.

Now, instead of having two waves, if I have a large number of wave with different frequencies and superimposed to them, then we should have something called the wave packet. So, that is that is the thing we wanted to find out.

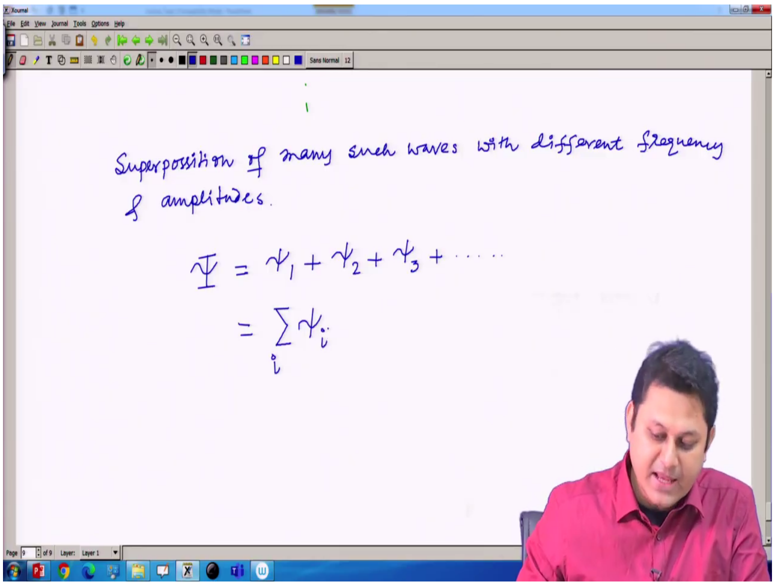
(Refer Slide Time: 11:52)



Suppose, so, I should write it here that I want to superimpose. So, this is superposition of many waves and all the waves are travelling waves.

So, we have many travelling waves. So, now, I want to super impose that so, 1 by 1. So, suppose I have one wave like this. I have another wave like this having a larger frequency. I have another wave like this and so on and all the waves I add and so on.

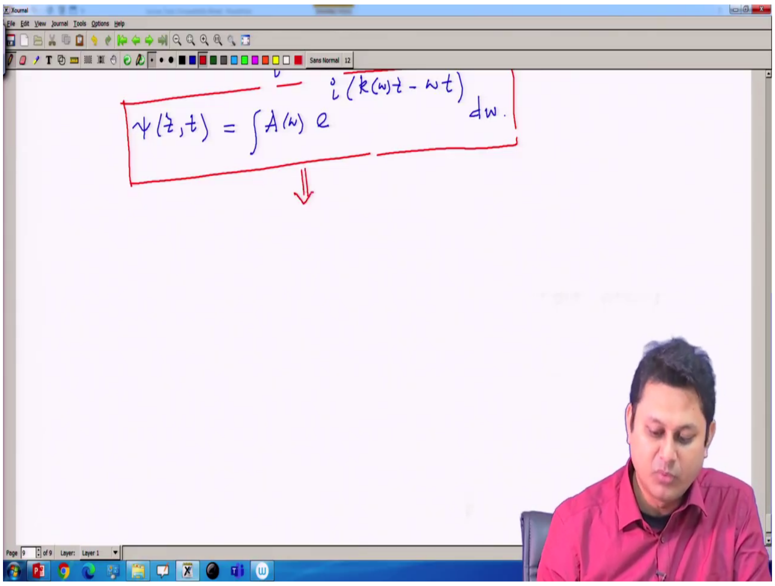
(Refer Slide Time: 13:32)



So, superposition of many such waves with different frequency and amplitude, what will be the general form of that so, I want to superimpose all the wave. So, it should be something like summation of. So, if this is psi 1, if this is psi 2, if this is psi 3 and so on.

So, in principle I want to superimpose everything. So, my big psi is psi 1, plus psi 2 plus, psi 3 and so on. If, I want to use the summation sign it should be summation of i psi i.

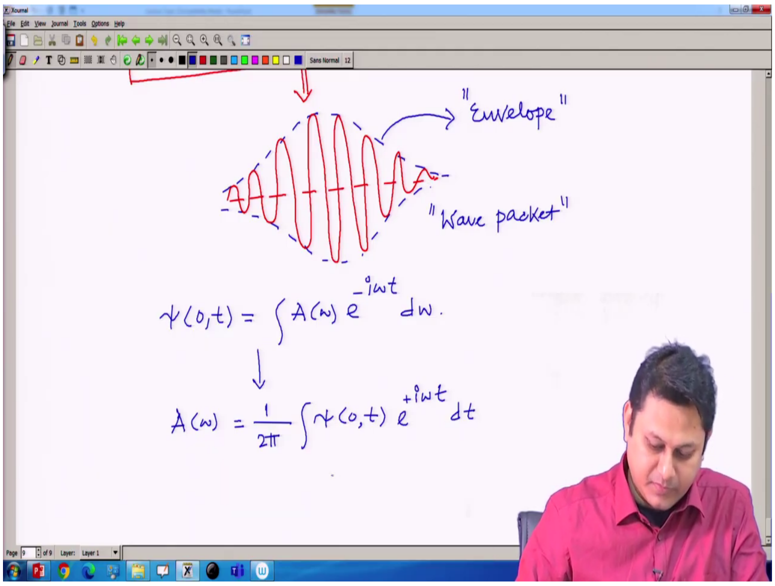
(Refer Slide Time: 15:28)



But, if the variation is continuous, almost continuous the variation of amplitude and this variation of frequency is continuous. I can write in a more compact form and it is something like this. So, this is the representation of many waves that is superimposed. And, when the many waves are superimposed, then what happened we will have a structure like this, which we call the wave packet we can have a structure , because, I am now superimposing more and more and more and waves.

So, instead of having bits in this case only two waves are there So, I have this kind of bit formation. If, I now adding more and more waves only one portion of this wave will be amplified or sustained the other portion will go away.

(Refer Slide Time: 16:44)

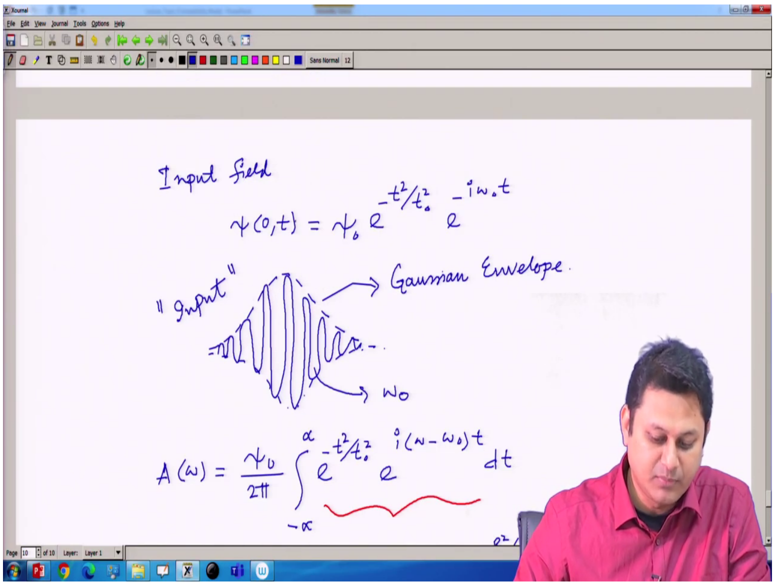


So, eventually I have a structure like this, which is my wave packet. This is the envelope I am drawing and inside the envelope, we have something this this is the envelope part the blue dotted line, represented by blue dotted line and this is called the wave packet.

Now, here it is interesting to note that, if I want to find out the wave at z equal to 0; that means, that input it should be simply A omega e to the power of minus of i omega t d omega. Now, this is this relation is a well-known relation, because if I want to this is related to the Fourier relationship.

So, if I want to find out A omega, then it should be simply 1 divided by 2 pi integration of the input wave input wave like this. So, once we know the value of psi 0 t; that means, the input wave in time domain, I can calculate the A omega. And, normally the input wave is of the form of Gaussian.

(Refer Slide Time: 19:37)

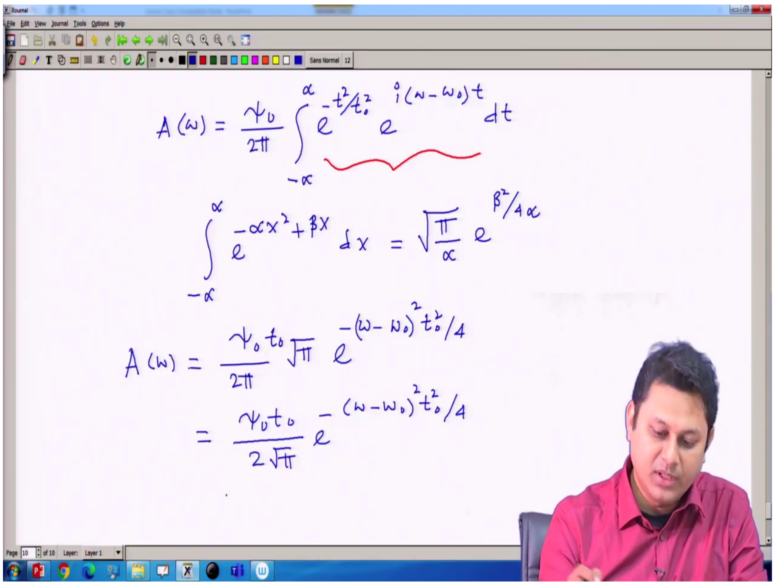


So, input field, which is psi 0 t is now given. It is something like psi 0 e to the power of minus t square this is a function of t t 0 square, e to the power of minus i omega 0 t. So, I can write this in this form.

So, this is nothing, but if I now I try to understand what is this form. So, this is nothing, but the input wave packet. And, input wave packet is having a Gaussian envelope like this. And, inside the envelope we have a frequency distribution a frequency distribution having omega 0 frequency. So, this is the Gaussian envelope and this is the frequency omega 0 fixed frequency it is vibrating.

So, once we know psi 0 t I can calculate the A omega t. So, A omega. So, A omega is now psi 0 divided by 2 pi integration of the field that is given which is Gaussian. So, I am just writing the expression. So, once the input is given I can calculate what is my A omega. And, if the A omega is there I can calculate the value of psi at some point z. So, we know one very useful expression which is this minus infinity.

(Refer Slide Time: 22:02)



So, in order to evaluate this integration, this is a well-known relation that we frequently use related to the gamma function. And, this integration is root over of pi divided by alpha e to the power of beta square divided by 4 alpha.

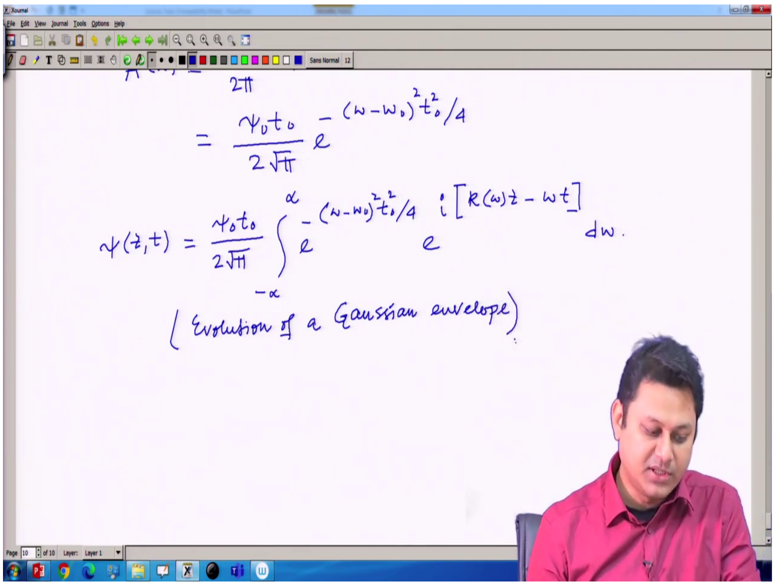
So, that is the result of this integration. And, if you look this integration here which is sitting, I can evaluate this using this expression. And, if I do then I can write it as A omega is equal to psi 0 divided by 2 pi this quantity in this quantity alpha is 1 by t 0.

So, root over of alpha pi alpha. So, it should be simply t 0 multiplied by root over of pi. And, e to the power of beta square here beta is i omega minus. So, it should be minus of omega minus omega 0 square, then t 0 square. Because, it is divided by 4 of alpha is 1 by t 0. So, it should be t 0 square divided by 4, which is psi 0 t 0 divided by 2 root pi e to the power of minus omega, minus omega 0 square, t 0 square divided by 4.

This is also a Gaussian in frequency domain you can see, in time domain we launch a Gaussian pulse in the input, this is the input which is a Gaussian pulse in time domain. And, in order to find out A omega we just make the Fourier transform of this input Gaussian pulse and eventually we will going to get A value the expression of A omega, which should be a Gaussian in frequency domain and we are getting that. So, far it is fine.

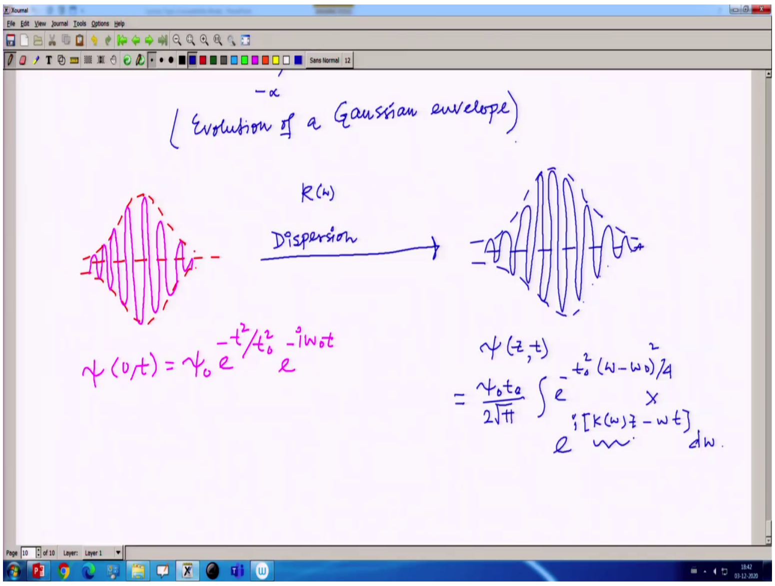
Now, once we know A omega, then the wave packet that we defined here is known. So, simply I just put this here and I can find out the wave packet.

(Refer Slide Time: 25:18)



So, my wave packet at some point z, some point t, that should be equal to now I put the value of A omega here. So, it should be psi 0, t 0 divided by 2 root pi integration minus infinity to infinity e to the power of minus omega minus omega 0 square, t 0 square divided by 4. And, then this propagation path k of omega z, minus omega t d omega. So, this is nothing, but the evaluation of a Gaussian envelope. This is the evolution of a Gaussian envelope. If a Gaussian envelope is launched at the input.

(Refer Slide Time: 26:49)



Then so, physically what is going on here, let me quickly understand. Let me make you understand that, this is the Gaussian pulse we are launching. Gaussian envelope with some this is psi 0 t, this is given. This is psi 0 e to the power of minus t square 0 divided by t 0 square e to the power of minus i omega 0 t.

So, this is my input. Now, it is allowed to pass through a dispersive medium. So, I have dispersion here; that means, k is a function of omega. If k is a function of omega something will happen here in the output. And, the expression of the output we already find so, this is my output; output also I have a structure like this.

And, the mathematical form of the output this is my psi at point z t, which is psi 0 t 0 that we already evaluated. Divided by 2 of root of pi integration of e to the power of minus t 0 square omega minus, omega 0 square by 4 multiplied by e to the power of i, k omega z minus omega

t. So, this should be the form at output, but the point is we need to find out what is k omega? If the dispersion medium is there then this k omega has to be has to be evaluated.

So, in the next class we are going to expand this k omega and we put certain condition that if it is dispersion medium how the k omega is expanded. Mainly it will going to expanded in a Taylor series form. And, when we expand in the Taylor series form then we will going to find that if I take only the first few terms, then how the pulse shape will going to modify. So, with this note I like to conclude today's class.

Thank you for your attention.