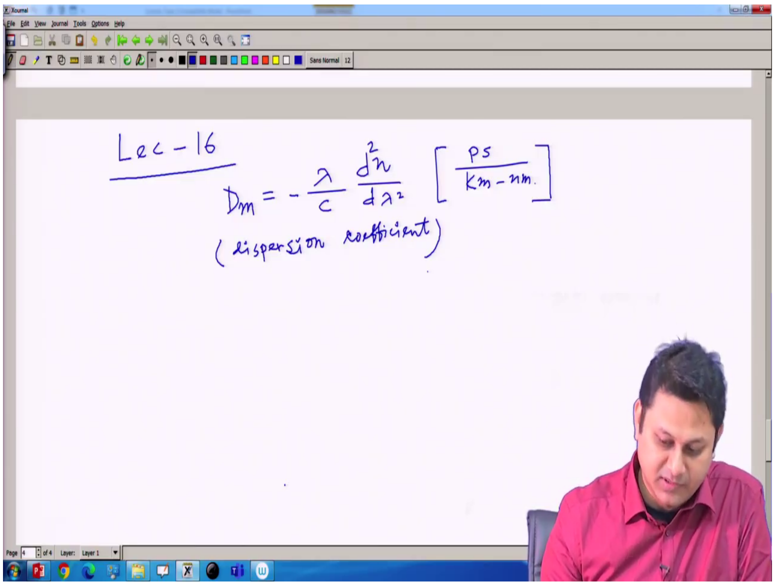
Physics of Linear and Non-Linear Optical Waveguides

Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 16
Material Dispersion (Contd.), Dispersion Coefficient

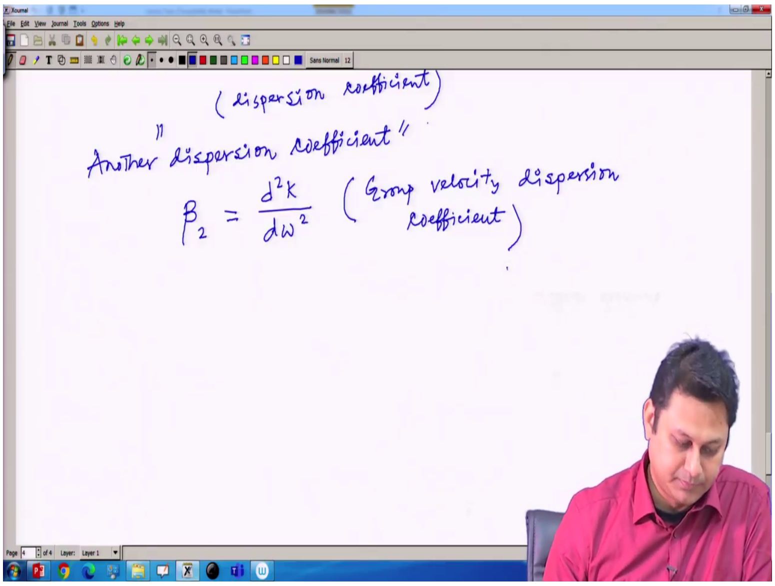
Hello student, the course of Physics of Linear and Non-Linear Optical Waveguide. So, today, we have lecture number 16 and we will going to study more about the Material Dispersion and Related Coefficients.

(Refer Slide Time: 00:28)



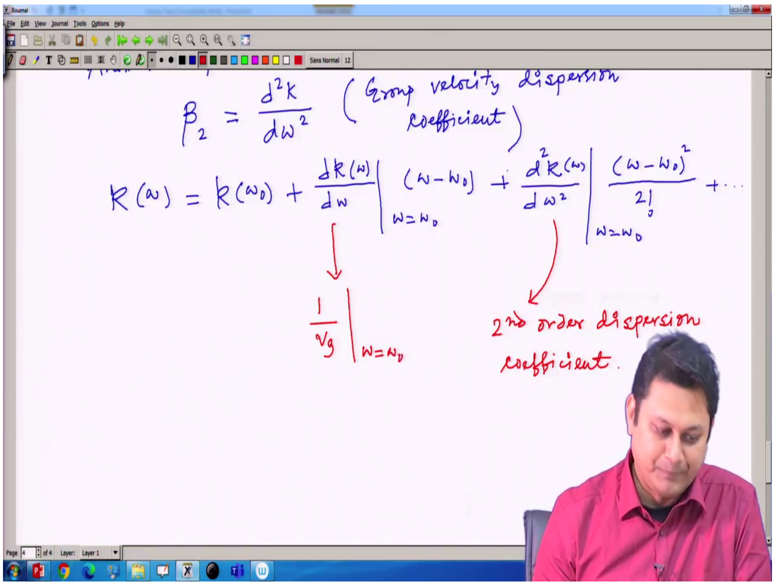
So, today lecture 16. So, in the last class we defined the dispersion coefficient we called it D m. By definition it is minus of lambda divided by C d 2 n d lambda square. And if I want to find out the unit it should be pico second per kilometer nanometer; pico second per kilometer nanometer, and we called this as dispersion coefficient. Well, apart from that another useful coefficient is also used specially in the pulse propagation problem in waveguides.

(Refer Slide Time: 01:35)



So, another dispersion coefficient ok, which we defined as beta 2. This is standard notation to define a dispersion coefficient beta 2, which is by definition d 2 k d omega square and is called the group velocity dispersion coefficient.

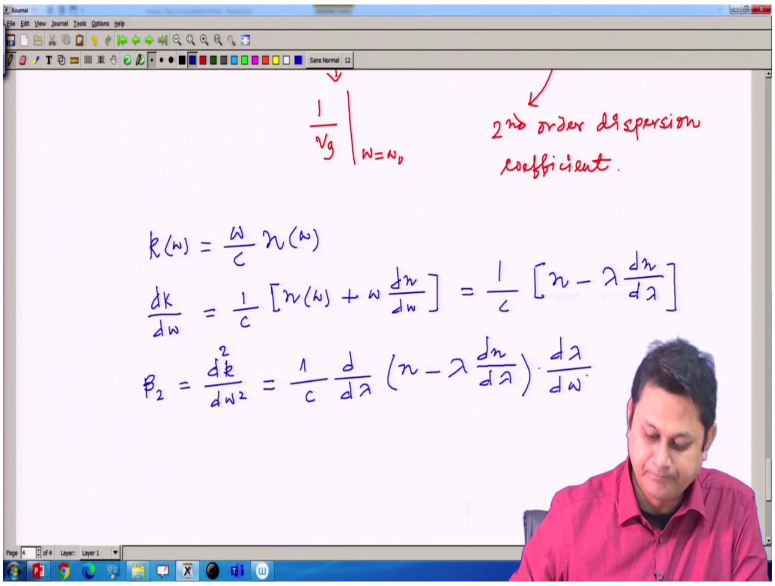
(Refer Slide Time: 02:52)



Now, one can realize this coefficient by expanding the propagation constant k as a Taylor series, which is a function of omega. See if I expand around the wavelength around the frequency omega 0 this is the operating frequency. So, it should be this evaluated at omega equal to omega 0 and so on.

This quantity is called the 2nd order because there are higher order also dispersion coefficient. This term if we look carefully it is 1 by V g evaluated at omega equal to. So, at operating wavelength what is the group velocity can be defined by this term.

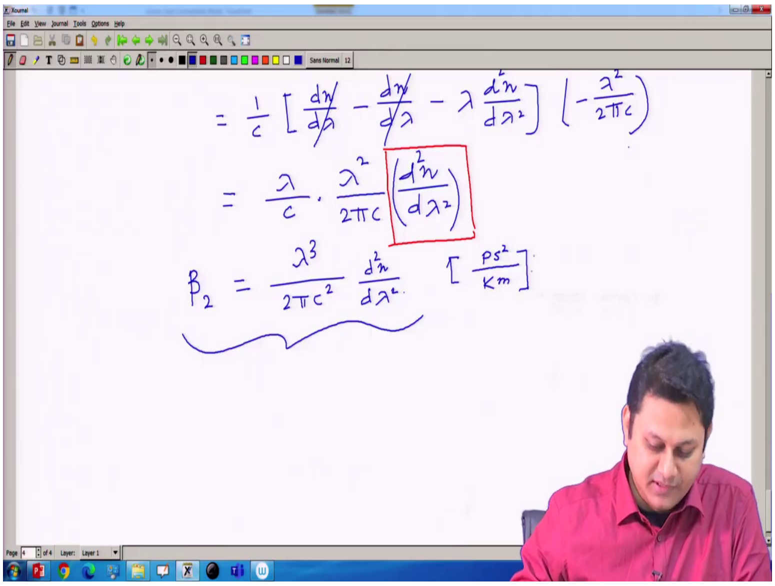
(Refer Slide Time: 04:54)



Now, if I want to find out the value of beta 2, we can do that because I know what is the expression of k, because k omega is omega divided by C n of omega. d k d omega I can calculate which we have already calculated. This is 1 by C n of omega plus omega d n d omega in terms of omega. In terms of lambda it is 1 by C n minus lambda d n d lambda in last class we have already calculated that.

Now, beta 2 is the 2nd derivative of k with respect to omega. And if I do from this side it should be 1 by C d of d lambda, because this is a function of lambda. So, eventually I like I want to derive this with respect to lambda and using the chain rule I should have d lambda d omega. Well, d lambda d omega and all these things is known.

(Refer Slide Time: 06:52)



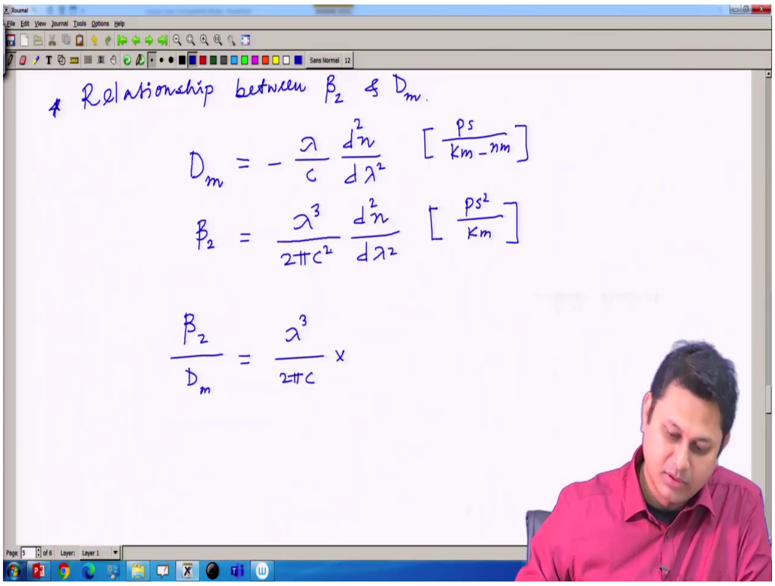
So, I will write here as 1 by C d n d lambda minus d n d lambda minus lambda d 2 n d lambda square and this quantity also we know. It is minus of lambda square divided by 2 pi C bracket close.

Now, this quantity will going to cancel out. So, we have lambda divided by C multiplied by lambda square 2 pi C and d 2 n d lambda square. So, important thing is when you calculate the d n as a dispersion coefficient, we have an important term here this one, which basically tells how the dispersion coefficient will going to change with respect to lambda. And d 2 n d lambda square is a factor that is sitting here. Also in calculation of beta 2 I have a same factor sitting here. So, the calculation of d 2 and d lambda square is very important in calculating the dispersion values.

So, this is essentially lambda cube divided by 2 e, 2 pi of C square and then d 2 n d lambda square. This is my beta 2 another dispersion coefficient and in units this is slightly different. This unit is pico second square per kilometer. The unit become pico second square per kilometer.

So, for D m and beta m both the cases the units are different, but both the things are measuring the property which is dispersion and very important property. So, once we know the derivative with respect to d and d lambda one can find the value of beta 2 also the value of D m. So, there should be some kind of relationship very easily one can find out the relationship between these two parameters because in many cases it is required.

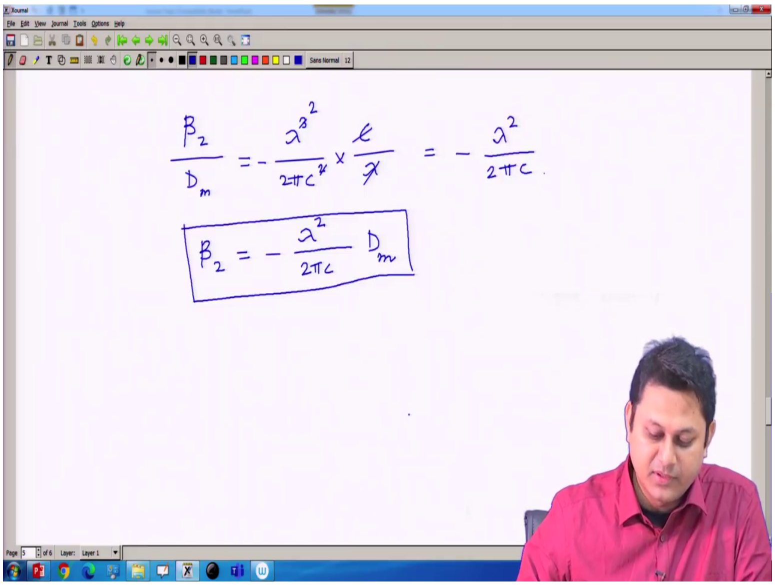
(Refer Slide Time: 10:06)



So, the next thing is to is to the relationship between the two dispersion coefficient, beta 2 and D m; m stands for the material dispersion. So, we already know what is D m. So, let me write it once again. D m is equal to minus of lambda divided by C d 2 n d sorry d lambda square. And beta 2 is lambda cube divided by 2 pi C d 2 n d lambda square. For this the unit is pico second per kilometer nanometer and for this it is pico second square per kilometer.

Now, the relationship one can easily find. See if I just make beta 2 divided by time it should be lambda cube divided by 2 pi C and then multiplied by this is 2 pi C square by the way because beta 2 is lambda cube divided by 2 pi C square. So, I miss a C square.

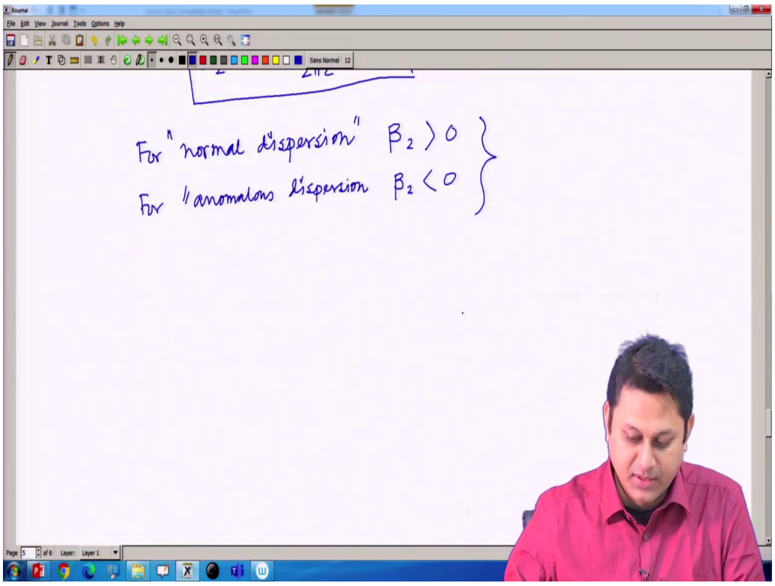
(Refer Slide Time: 12:24)



So, it should be C square and then D m is C divided by lambda. d 2 n d lambda square d 2 and d lambda square these two term is going to cancel out. So, I have eventually one C one C square cancel out one lambda this cancel out. So, I have lambda square such eventually I have one negative sign is there because

So, negative sign of lambda square divided by 2 pi and then C. So, my beta 2 if I calculate beta 2 I can convert this term to D m. If I calculate D m I can convert this term to beta 2. So, this is basically the relationship between beta 2 and D m. So, if the value of D m is given. So, you can calculate the value of beta 2 for a given lambda. And if the value of ah beta 2 is given you can calculate the D m for a given lambda; both the cases they are measuring a quantity called the dispersion.

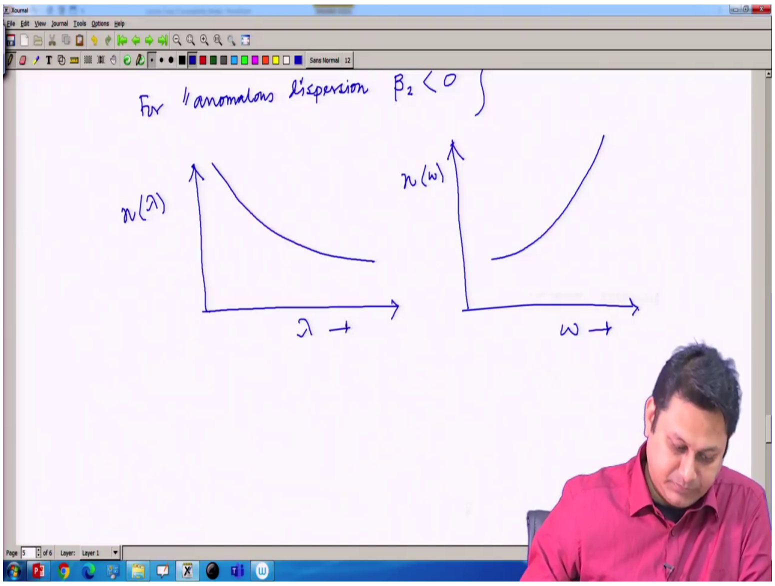
(Refer Slide Time: 13:42)



Now, for normal dispersion when the refractive index is increasing with respect to frequency that we call the normal dispersion. So, for normal dispersion we have beta 2 greater than 0 and for anomalous dispersion, this is another thing called anomalous dispersion beta 2 is less than 0.

So, this is a very important concept that for normal dispersion we have beta 2 greater than 0 and for anomalous dispersion we have beta 2 less than 0. Now, in different way one can understand the normal and anomalous dispersion.

(Refer Slide Time: 14:57)

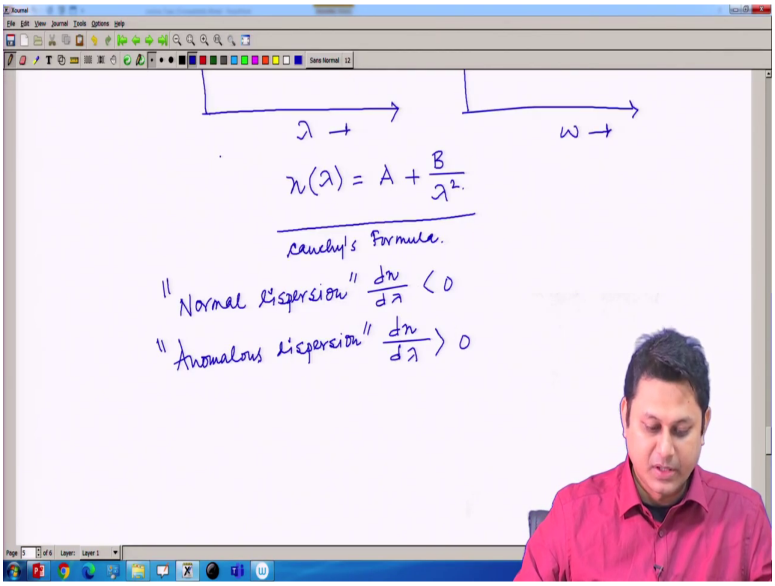


So, let me try it out. So, this is if the curve of dispersion the refractive index as a function of lambda and this curve is drawn in a normal dispersion regime when the refractive index is changing with respect to lambda and decreasing. So, when the refractive index is decreasing with respect to lambda then it is in normal dispersion and when it is increasing with respect to lambda, then it is an anomalous dispersion.

In frequency if the refractive index is increasing with respect to. So, if I draw this in a frequency domain then normally what happened? There the curve will be something like this.

It will going to increase with respect to frequency because with respect to lambda if it is decreasing it is increase. Both the cases this figure is very much true for normal dispersion.

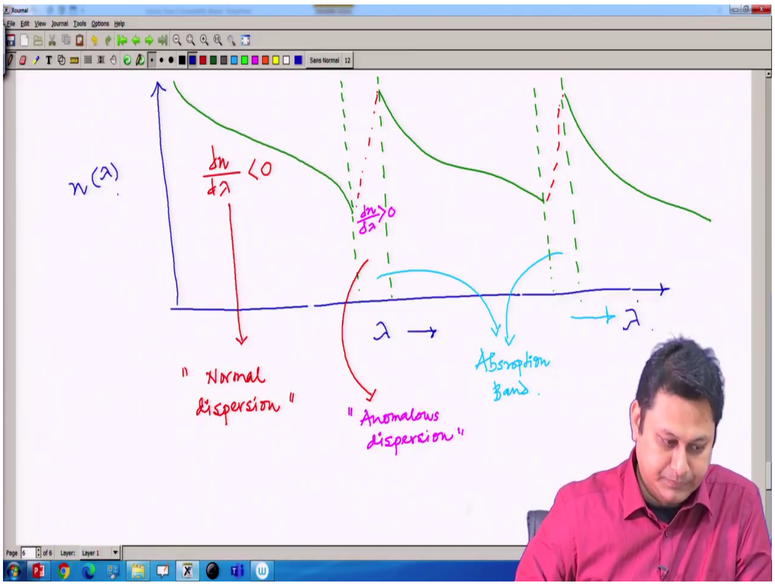
(Refer Slide Time: 16:04)



So, this equation follows the Cauchy's relation. So, if I write the Cauchy's relation side by side it should be this equation is following this. So, it is very much in normal dispersion regime, this Cauchy's formula. Now, for normal dispersion as I mentioned we have d n d lambda is less than 0 and anomalous dispersion it is d n d lambda greater than 0.

So, the regime where d n d lambda is less than 0 is the normal dispersion regime and in anomalous dispersion regime this is different. So, from this curve you can see that when the lambda is increasing the n is decreasing. So, d n d lambda is essentially less than 0. So, we have a normal dispersion regime in whatever the curve is shown here. However, if I draw the full curve including the [Refer Time: 17:45]relation then it should be something like this.

(Refer Slide Time: 17:53)



So, let me draw that. So, let me draw in a bigger. So, I have a wide range of lambda and this is n lambda and my dispersion my refractive index curve is something like this. At some point here I have a discontinuity because this falls in near to the resonance frequency and we know that at resonance frequency some sort of discontinuity one should expect.

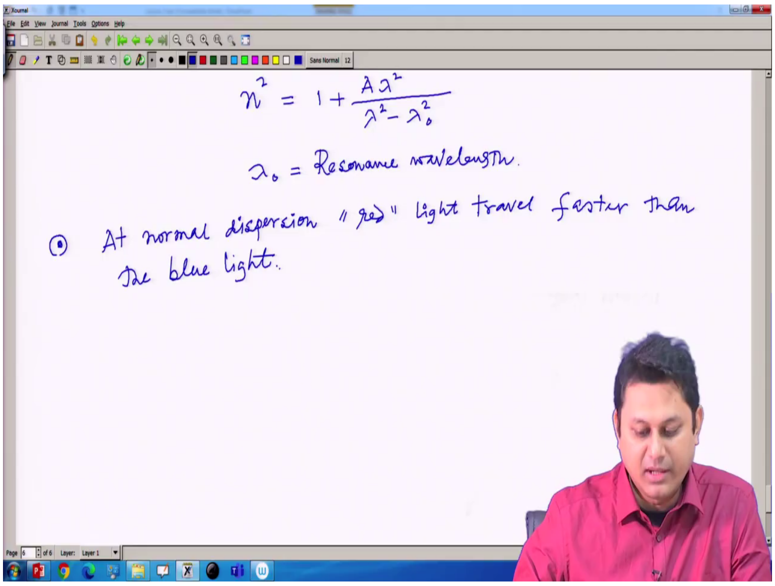
So, this is a region where we have an absorption band and then there is a change of refractive index and then suddenly again it falls like this. And we have again another resonance wavelength falling here somewhere and then this region it is refractive index is changing. It is basically increasing with respect to lambda that is why at this regime we will find the anomalous dispersion and we have something like this and so on.

So, here in this regime if I look d n d lambda is less than 0 because when the lambda is increasing d n is increasing d n is decreasing. So, this is called the normal dispersion regime.

In this gap we have d n d lambda greater than 0. So, this is anomalous dispersion regime and these bands by the way these bands are absorption bands.

So, these bands are absorption bands. If you launch a light at this frequencies or this wavelengths what happened? There will be a resonance frequency; so that means, the molecule or the, this things will going to vibrate and we should have an absorption in this region and along this we have lambda.

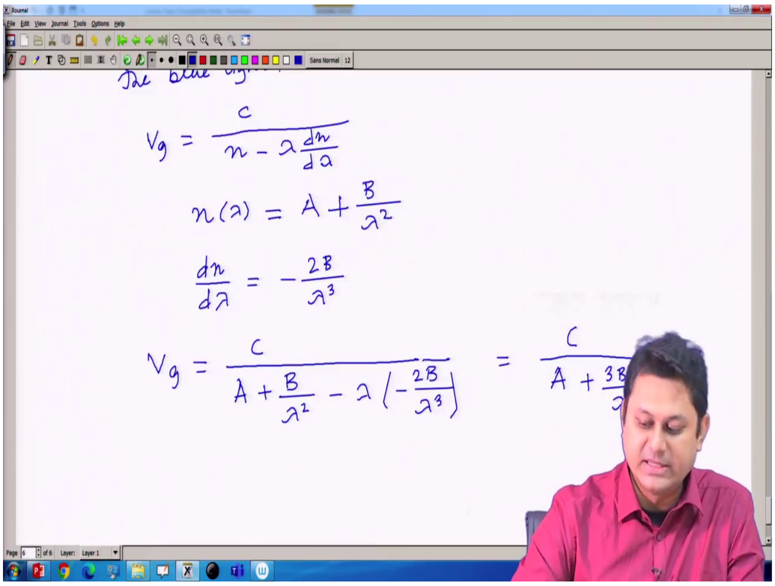
(Refer Slide Time: 21:41)



Well, this equation is valid here n square is equal to 1 plus a lambda square divided by lambda square minus lambda 0 square. Lambda 0 here is a resonance wavelength. Now, what happened to the light wave when it is propagating in normal or anomalous dispersion regime that we need to understand now.

So, what happened is interesting. At normal dispersion red light travel faster than the blue light. So, in normal dispersion the red light is normally traveling in a faster with blue light.

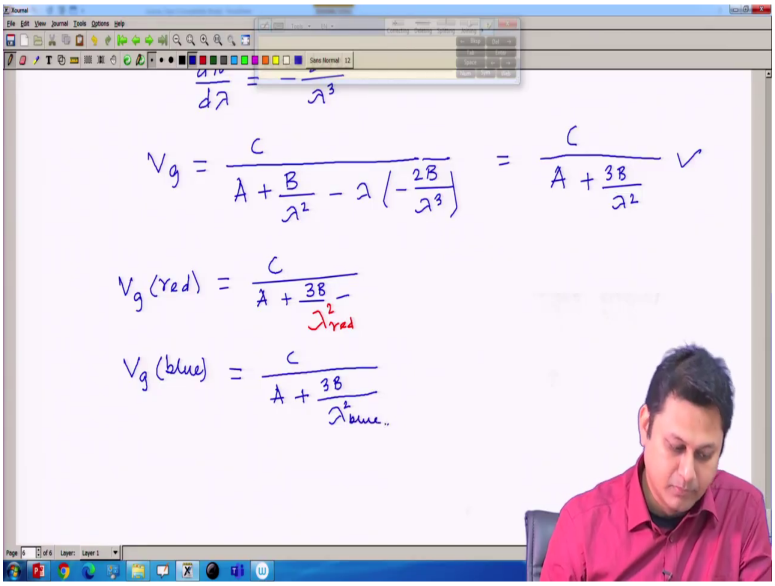
(Refer Slide Time: 23:36)



And we can see that with expression of V g. So, V g is a group velocity, which is a function of lambda and one can write as it is n minus lambda d n d lambda, this is expression of that. Now, we also know the Cauchy's relation and Cauchy's relation is valid for normal dispersion regime as I mentioned.

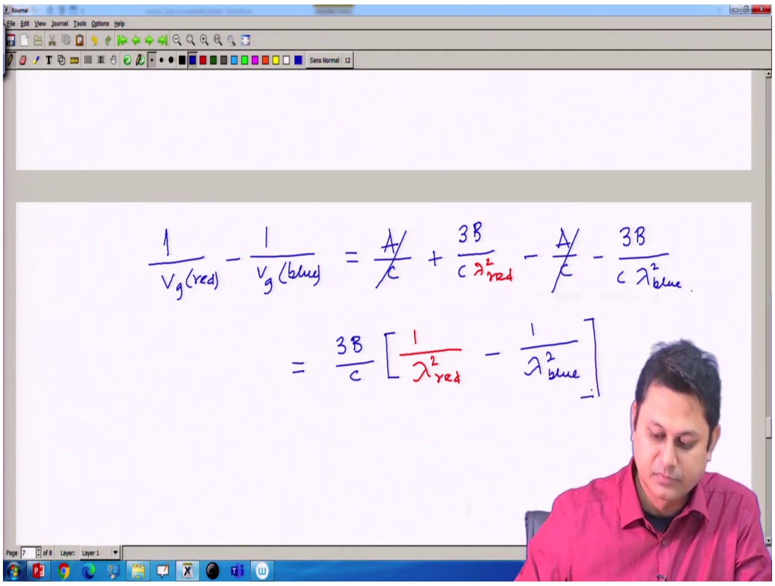
So, n function of lambda in explicit form is something like this. Now, from here I can calculate d n d lambda which is minus of 2B lambda cube, just a derivative of this expression. And V g is now simply C divided by n I write it as A plus B lambda square and then minus of lambda and then minus of 2B of lambda cube, which is essentially C divided by A plus this sign will be plus. So, it should be 3B divided by lambda square. So, that is the expression we have for V g using the Cauchy's relation.

(Refer Slide Time: 25:40)



Now, if I calculate for red so, V g for red will be C divided by A plus 3B divided by lambda red square and maybe I can write this in a. So, this lambda square because since this is a red. So, I can write it in this way. And V g blue is C divided by A plus 3B divided by lambda square blue.

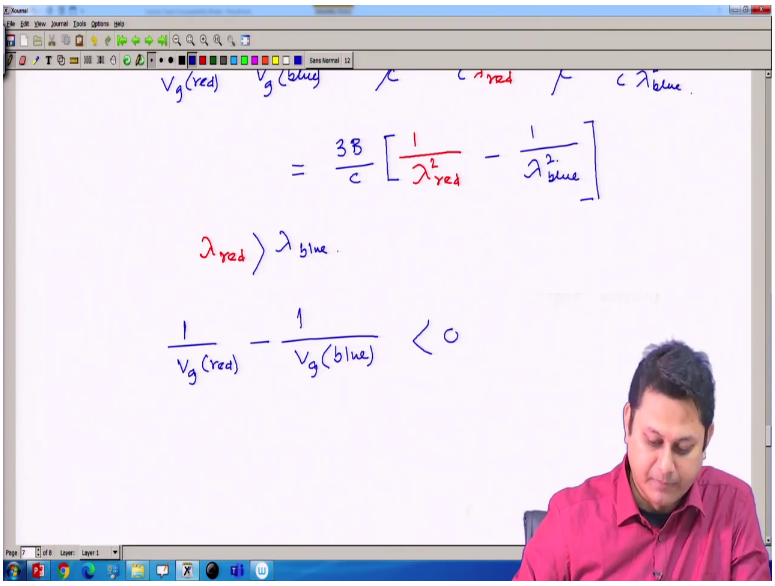
(Refer Slide Time: 26:57)



If I try to calculate this term 1 divided by V g red minus 1 divided by V g blue, then it should be A divided by C plus 3B, making the reciprocal and then I just try to find out what is 1 by V g minus 1 by V g red and 1 by V g blue.

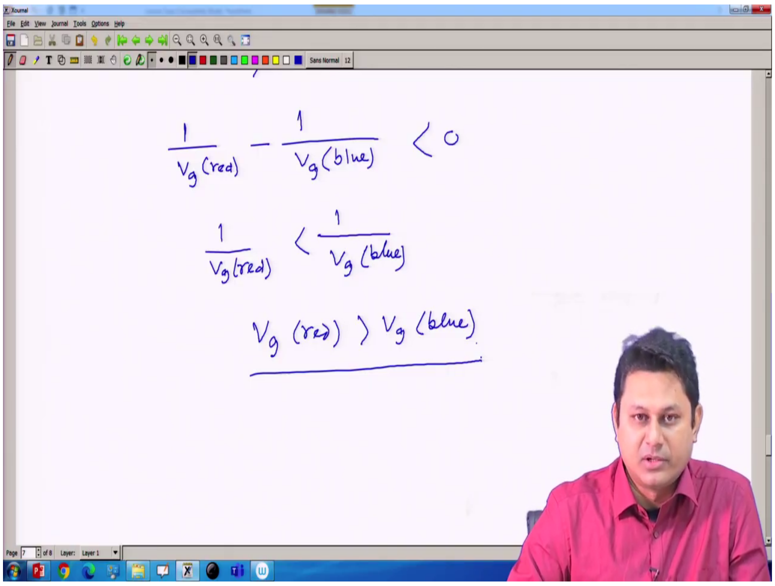
So, A divided by C 3B divided by C into lambda red. So, I just put it as red. So, lambda square red minus A by C minus 3B divided by 3 lambda square blue. A by C A by C is going to cancel out. So, eventually I have 3B. So, this these things will be c. I am making a mistake here. I write 3 here. So, it is C. C I have the term 1 divided by lambda square red minus 1 divided by lambda square blue that is all.

(Refer Slide Time: 29:10)



Now, from this expression I can I can readily write that as lambda red wavelength of the red light is greater than lambda blue. So, this quantity from this quantity I can write 1 divided by V g red minus 1 divided of V g blue is simply less than 0, because if lambda is greater than this quantity. So, this minus this should be less than 0, because this is great this is lesser compared to this one.

(Refer Slide Time: 30:03)



So, 1 by V g of red should be less than of 1 by V g of blue and from that we can conclude that V g of red should be greater than V g of blue. So, this is essentially the thing we try to find out using the concept of dispersion. How the red light is propagating in normal dispersion regime, how the red light is propagating as a faster rate compared to the blue light that we tried to prove and this is basically a proof of that using the basic Cauchy's relationship of refractive index as a function of lambda.

So, with this note I like to conclude here. So, in the next class we start we try to understand that how the pulse will going to broaden in a more rigorous way. So, with this note let me conclude.

Thank you for your attention.