Physics of Linear and Non-Linear Optical Waveguides
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Module - 02
Basic Fiber Optics
Lecture - 15
Material Dispersion (Contd.)

Hello student, to the course of Physics of Linear and Non-linear Optical Waveguides. So, today we have lecture number 15. In the last lecture we started the concept of Material Dispersion. So, we will going to continue with that concept.

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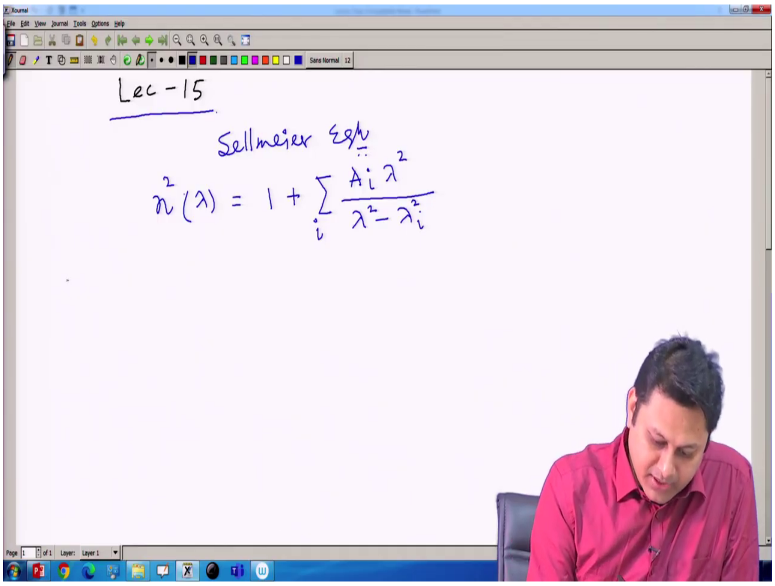

So, we started with Sellmeier equation. We derive this equation, this is the equation we derived in the last class that how the refractive index should vary with respect to lambda.

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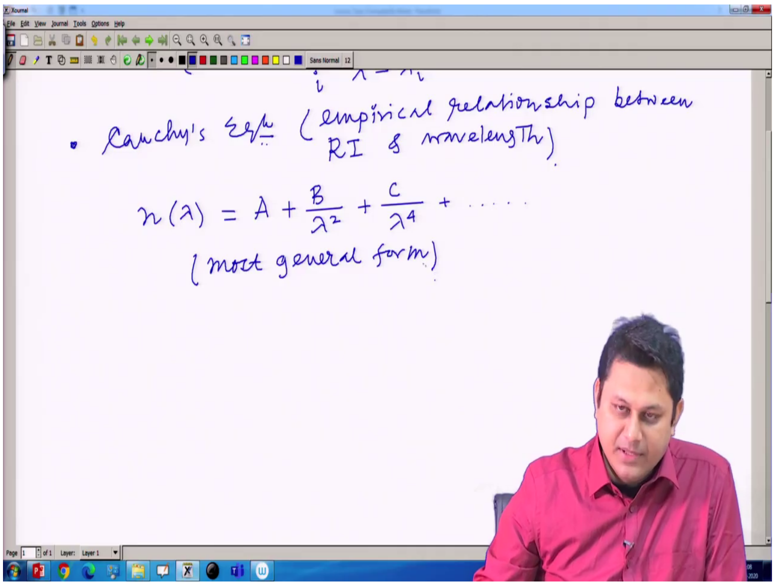

Now, today we will introduce the well known Cauchy’s equation, which is a empirical relationship between refractive index and wavelength. So, if I write n as a function of lambda, then one can have a empirical relationship an explicit form like this. This is the most general form and A B C these are the Cauchy’s constant. In a few experiments we determine this A B C this Cauchy’s constant and then we know, how the refractive index is varying with respect to lambda, we have the explicit form.

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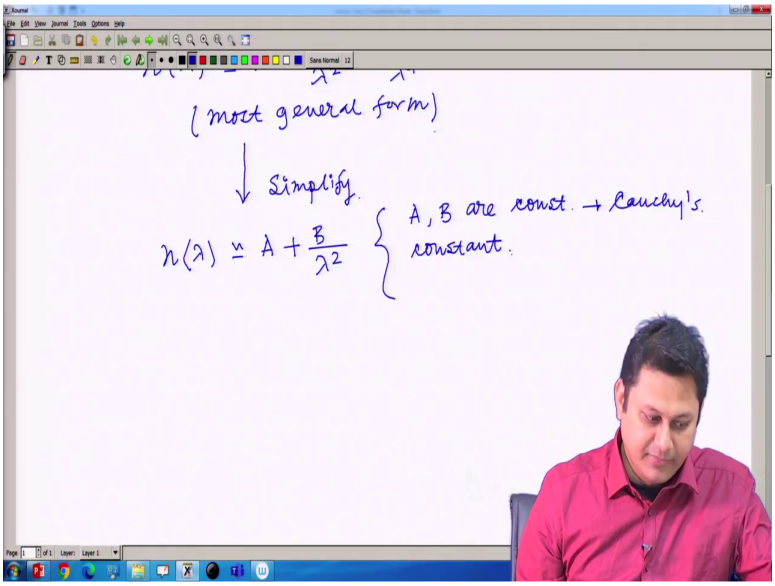

Now, I can simplify that, simplify to the first order and then, I can have the relationship like this. As I mentioned A, B are constant it is called Cauchy's constant. Well, one can directly derive the Sellmeier equation the this Cauchy's equation from the Sellmeier equation.

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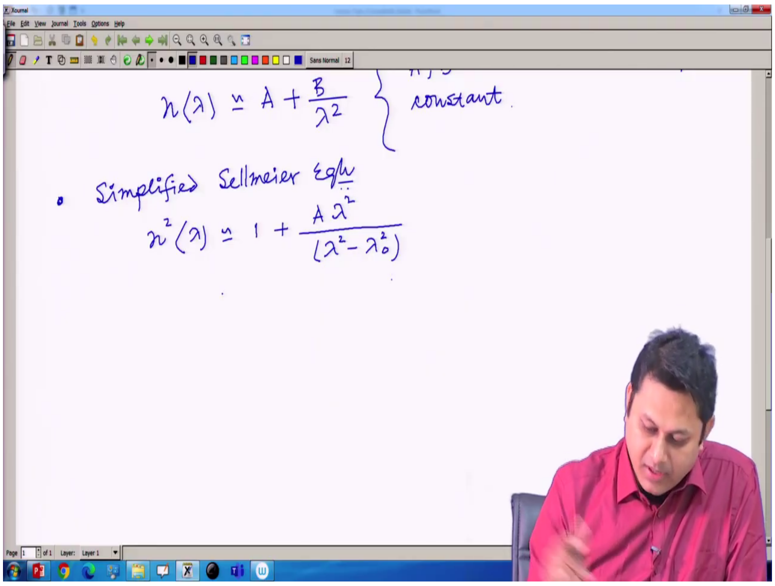

So, we can simplify the Sellmeier equation as well. So, simplified Sellmeier equation is something like this, taking only the first term there is a summation sign. So, I am taking the first term in the summation, and I can write the expression in this way. So, this is the simplified Sellmeier equation.

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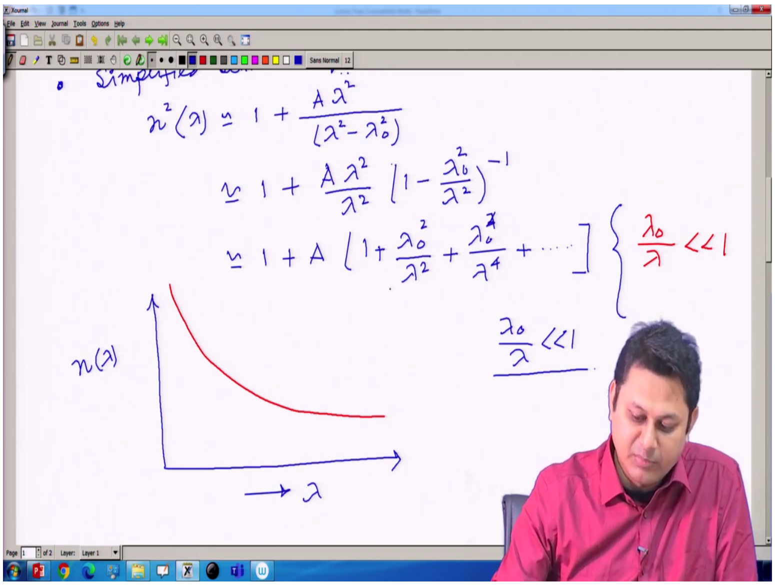

And, now I can write this expression as 1 plus A lambda square divided by lambda square, 1 minus lambda 0 square divided by lambda square minus 1. I take the lambda square common out of that and then I put it as minus 1.

So, this equation is simply 1 minus lambda 0 square, it should be plus if I expand this to the power 4 divided by 4 and so on. With the condition this is one can write this condition, that I can do that when lambda 0 divided by lambda is very very less than 1. That means, my working wavelength is far away from the resonance wavelength. In such condition I can write in this way.

Now, if I want to plot the refractive index as a function of lambda in this regime when lambda 0 divided by lambda is very, very less than 1 so; that means, far away from any resonance frequency or any resonance wavelength. So, in this direction I will going to plot n as a function of lambda, and this is lambda as usual. Then, the curve will be something like this. This is the curve that with respect to lambda it will gradually decrease, generally that is the, that is the form.

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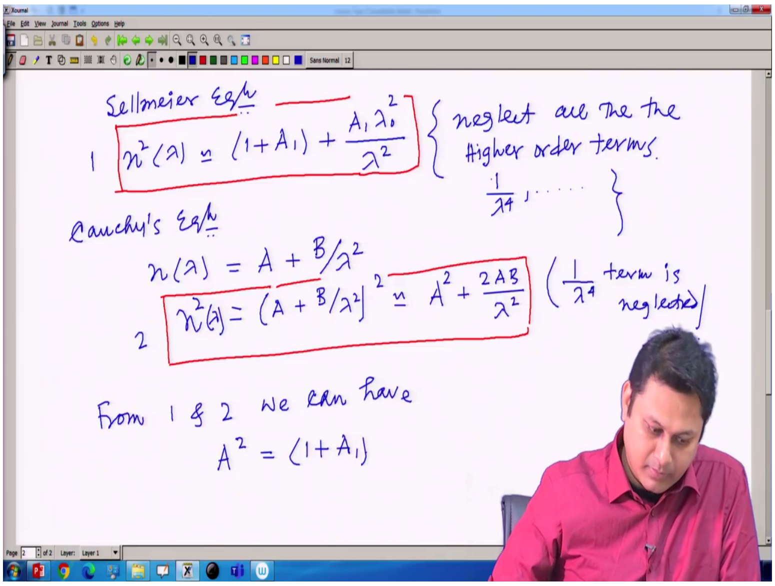

Now, from Sellmeier equation I can write n square lambda is nearly equal to 1 plus A 1 plus A 1 lambda 0 square divided by lambda square, taking only this lambda square term. There are many other term like lambda to the power 4 and all these things, and you can see that this simplified expression of Sellmeier equation under the condition that lambda 0 divided by lambda very, very less than 1, can merge to the Cauchy's equation. So, it just try to correlate that the Sellmeier equation and the Cauchy's equation together.

So, now here we neglect all the higher order terms like, 1 divided by lambda, lambda to the power 4 etcetera. Now, from Cauchy's equation I have n as a function of lambda as A plus B by lambda square. Also I am not going to take any higher order term only just take the first term lambda square, and then if I want to find out what is n lambda n square, then it should be A plus B lambda square whole square. Which can be approximated as A square plus 2 A B divided by lambda square. Again, I neglect 1 by lambda to the power 4 term is neglected.

Now, if I compare these 2 equations side by side, whatever the expression I have here this is the expression I have exploiting the Sellmeier relation. Another equation I have this one exploit exploiting the Cauchy's relation and essentially these two things are same. So, if I compare then I can have say this is 1 and 2. So, from 1 and 2 we can have A square is equal to 1 plus A 1.

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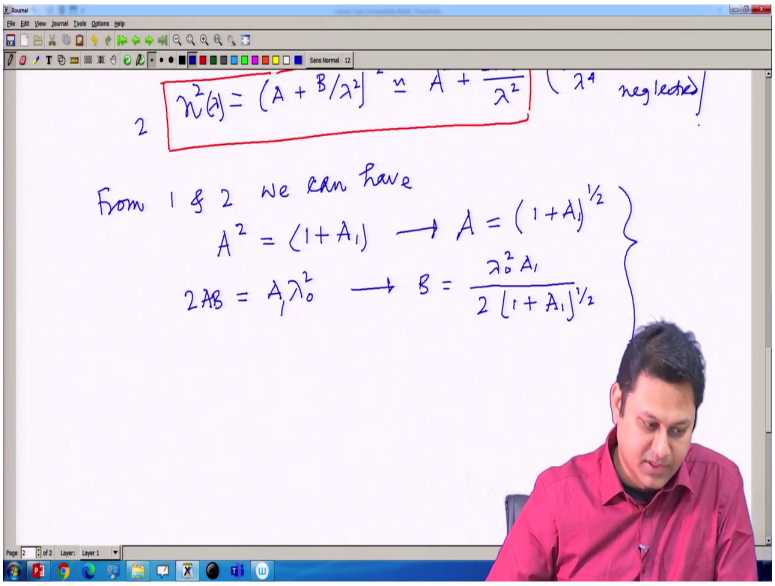

Which gives me A as 1 plus A 1 whole to the power half and, 2 AB is equal to A lambda 0 square, because the coefficient of 1 by lambda square is A 1 lambda 0 square. So, it should be

A 1 lambda 0 this is lambda 0 square. So, from that I can have the value of B as lambda 0 square A 1 divided by 2 of A and A already I figure out so, this one.

So, if I know the value of lambda 0 and A 1, then I can correlate this lambda 0 and A 1 with the Cauchy's constant A and B. So, this relationship between the Cauchy's constant and the constant that one can have from Sellmeier equation.

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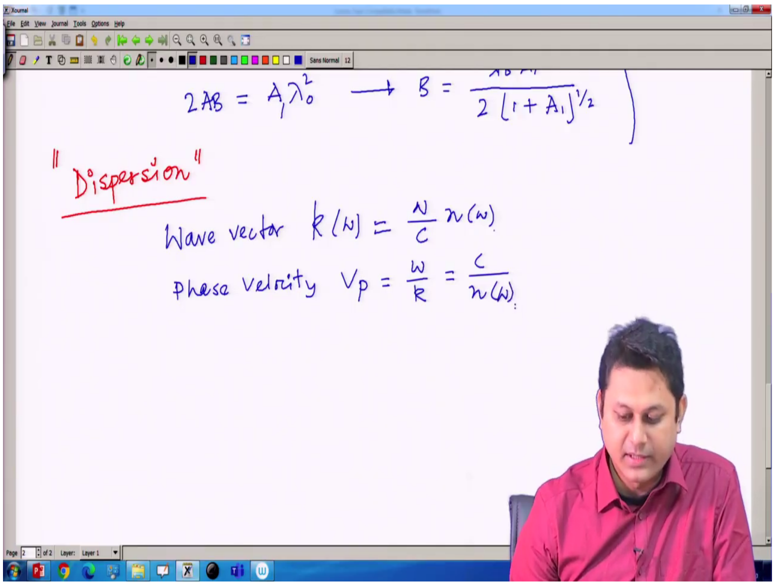

Well, after that the next thing that we will do to find out the dispersion, once more, in terms of group velocity phase velocity etcetera, how one can define dispersion that we going to learn. So, first the wave vector. The wave vector k is essentially a function of frequency and that is the main reason to have the dispersion. So, k should be written in this way k is omega divided by C multiplied by n omega, where n omega is a refractive index.

Now, what is phase velocity? This is V p is equal to omega by k and omega by k is nothing, but C divided by n omega, that is the expression of the phase velocity, that we know this is nothing new.

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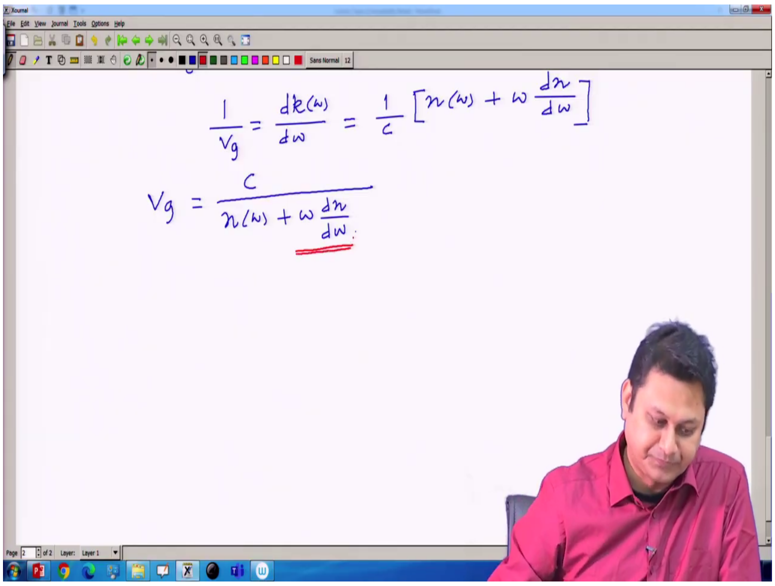

Now, what is group velocity? This is V g equal to d omega d k, that is by definition this is our group velocity. So, 1 by V g inverse of group velocity is d k which is a function of omega and d omega. It is convenient to write 1 by group velocity, because we know the expression of k explicit expression of k omega, k omega I already defined here wave vector it is omega divided by C then n omega.

So, this quantity is 1 divided by C n of omega plus omega d n d omega. Just make a derivative with respect to omega of whatever the k I have and I am going to find, what is my V g? So, V g is essentially, C divided by n function of omega plus omega d n d omega.

This next part is interesting. In the phase velocity we have C divided by n omega that is true, but here for the group velocity calculation I have C divided by n omega, but additional one term is there, which is the variation, that gives me the variation of n with respect to omega. That is important here this term. Now, if I want to find out what is the expression V g in terms of lambda?

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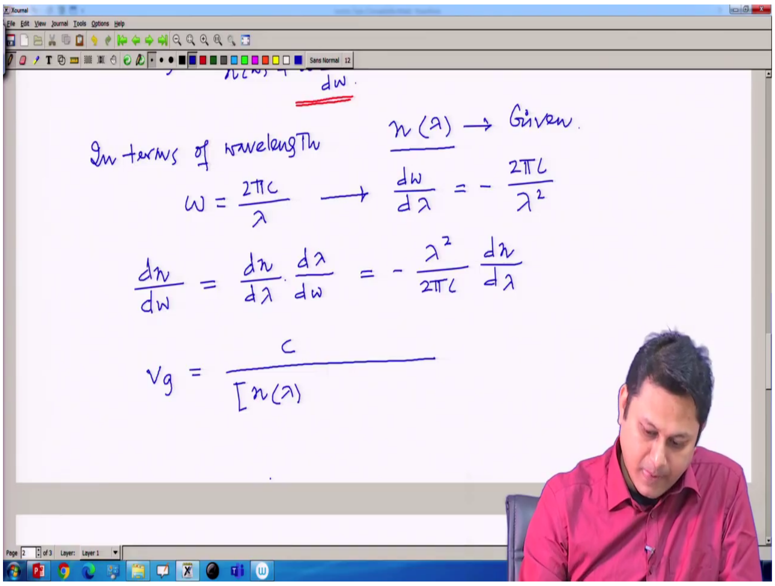

Then, so, in terms of wavelength how one can write that, because normally why I am doing that. Because n normally given as a function of wavelength n as a function of lambda is normally given, using Cauchy's law or Sellmeier equation n as a function of lambda is known. So, it is expected that V g should be represented in terms of omega as well. How the group velocity we will going to change with respect to lambda is meaningful, because n as a function of lambda is given to us.

So, in order to use, in order to find the value of V g in terms of lambda, we need to use the relation of omega and lambda. Omega is 2 pi C divided by lambda, which directly give me the relation d omega, d lambda as minus of 2 pi C divided by lambda square just make a derivative with respect to lambda.

Now, d n d omega I need to replace it as d n d lambda. And, using the chain rule I can write this. So, d n d lambda d omega is simply minus of lambda square divided by 2 pi C into d n d lambda. And, essentially V g becomes C divided by in the denominator I have n omega so, I replace as n lambda, then I have plus omega here.

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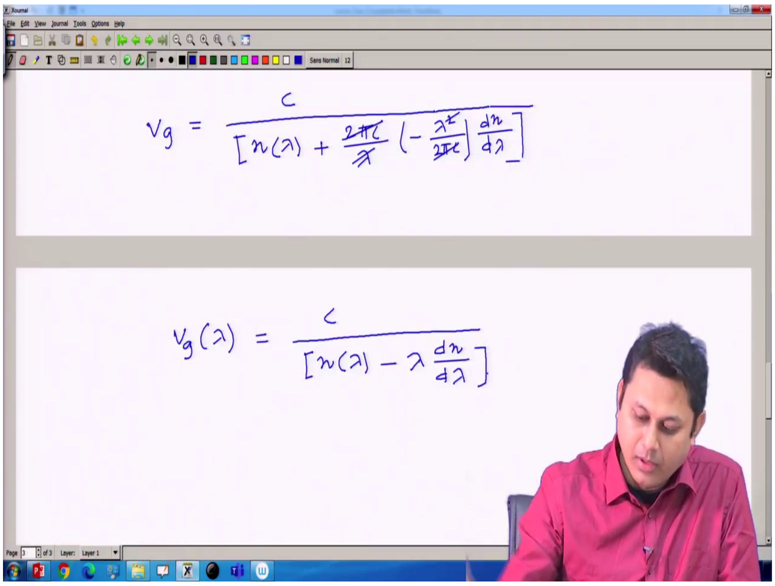

So, this omega I can replace as 2 pi C divided by lambda there should be a plus sign. And, now, I replace d n d omega as this value. So, it should be minus of lambda square divided by 2 pi C and d n d lambda.

So, eventually this 2 pi C 2 pi C is we will going to cancel out and lambda and lambda square one term will cancel out. So, finally, I have V g which is a function of lambda now, that for different lambda I have different group velocity. It should be C divided by n as a function of lambda minus lambda d n d lambda this is one form.

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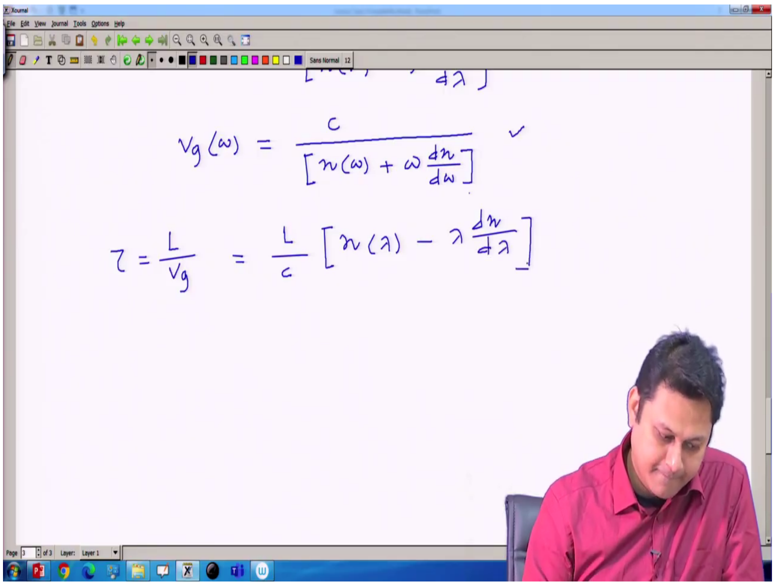

And, if I want to write this in the frequency, in terms of frequency it is V g C, which is the velocity of light divided by n of omega plus omega, d n d omega. So, these two form I can have if n is a give with the function n the refractive index is given as a function of lambda or is given as a function of omega, based on that I can use the V g in this way well.

After having the V g in order to calculate dispersion we need to find out what is the time lag actually, that we calculated in earlier classes. So, here also t tau which is a time taken by a wave, the time taken by a wave to move a distance L can be calculated as simply L divided by V g, V g is a group velocity. Now, it should be simply L divided by C multiplied by n of lambda minus lambda d n d lambda ok. So,far it is fine.

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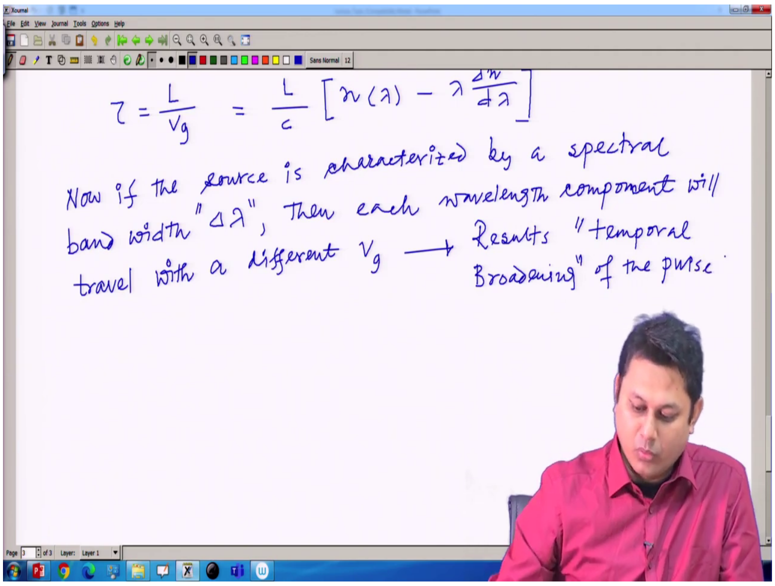

Now, if the source is characterized by a spectral bandwidth say delta lambda, then each wavelength component rather will travel with a different V g, different group velocity. This results something a temporal broadening of the pulse.

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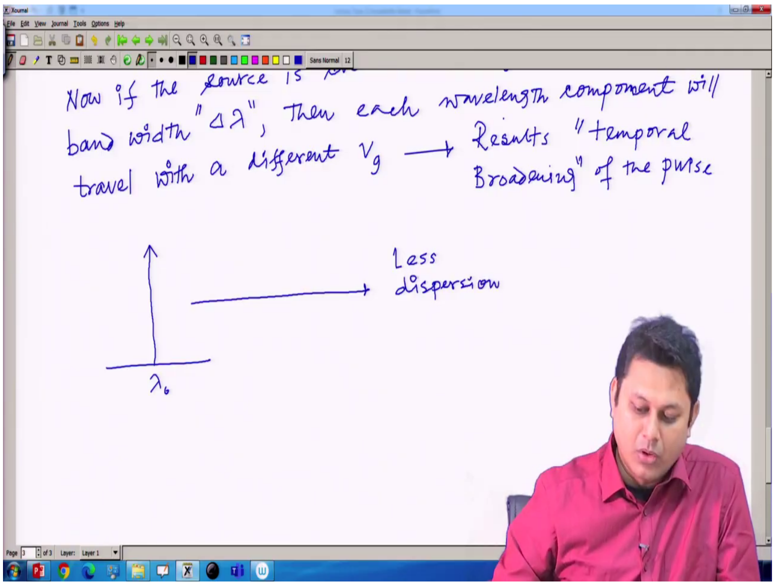

So, what is the meaning of that? So, suppose I have a source very soft source. So, normally this is the way to write a soft source. So, this is a very specific lambda say lambda 0. And, when it is passing through then since there is no frequency component extra frequency component is a very, very soft source lambda 0.

So, the time taken by this so, d n d lambda there is no such d n d lambda, because only 1 lambda are there. So, I should not have any kind of dispersion for that, because this is a single wavelength so, less dispersion.

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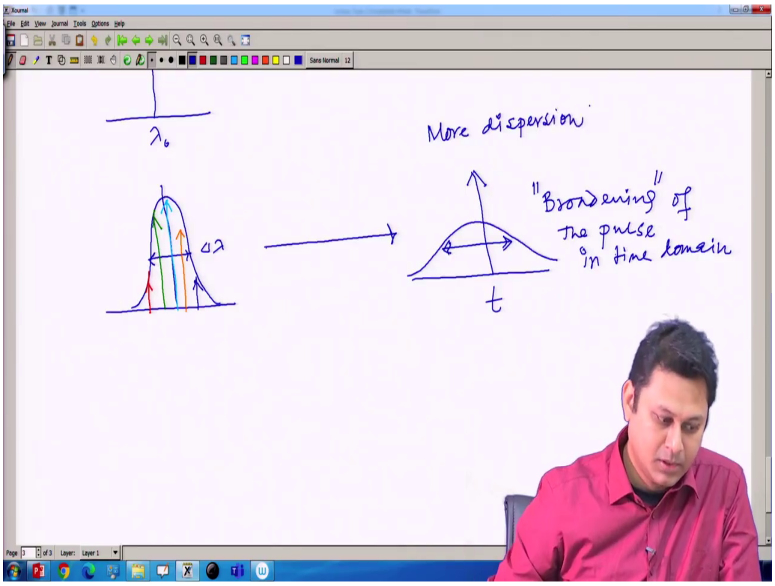

Now, instead of having 1 lambda if I have a range of lambda; a range of lambda means if I have a width bandwidth say delta lambda 0. That means, not only 1 lambda, different lambdas are present here. Say 1 lambda is here, another lambda is here, another lambda is here. So, there are there is a spectrum with different colors like that.

So, when I have a spectrum and when it is propagating. So, what happened that in temporal domain there is a broadening. So, I should have a broadening in time domain, this is broadening. So, there should be temporal broadening of the pulse.

Because, different component now we will going to travel at different group velocity and when it is travelling with a different group velocity. So, there will be a stretching of the pulse depending on the value of the dispersion, coefficient, and we will going to have a broadening. So, that is. So, we here we have dispersion, more dispersion.

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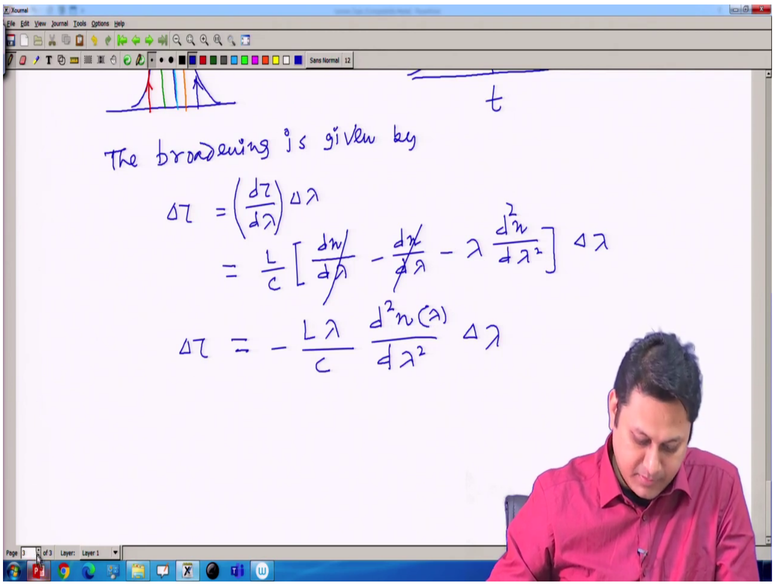

Now, quickly we calculate the dispersion coefficient. So, how the broadening is given? The broadening is given by delta tau, because delta tau is a measurement of the broadening that, how these things we will going to broaden. And, one can write this delta tau as d d t d lambda delta lambda. So, delta lambda is a width and because of that one can have different time and this is basically the measure of the temporal broadening.

So, if I know what is tau, because I calculated that, it is L divided by V g and expression is L divided by C multiplied by n lambda minus lambda d and d lambda. So, I want to make a derivative of this quantity first.

So, if I do I simply have L by C there is a inverse. So, d n d lambda minus d n d lambda minus lambda the second term d 2 n d lambda square multiplied by delta lambda this term, this term we will going to cancel out. So, eventually I have minus of L lambda divided by C d 2 n, which is a function of lambda, d lambda square multiplied by delta lambda. This is my delta tau the measurement of the measurement of the temporal broadening.

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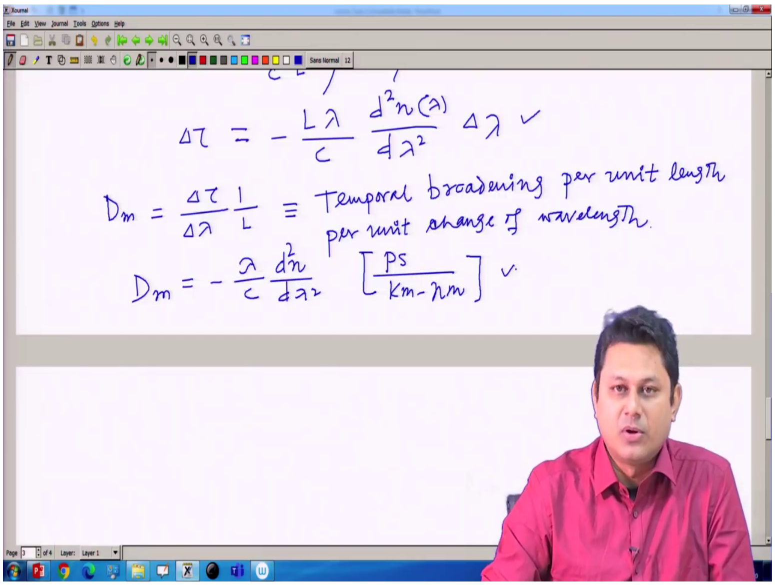

Well, I can define something like del m, which is delta tau divided by delta lambda 1 by L. Which is temporal broadening this quantity is nothing, but temporal broadening per unit, change per unit length per unit change of wavelength.

So, by definition this is the change of this is the temporal broadening happening per unit length 1 by L and per unit change of wavelength delta tau. So, this quantity we call the dispersion coefficient. Delta m is eventually minus of lambda divided by C from this expression I can write, d 2 n d lambda square. And, in unit it should be picosecond per kilometer nanometer. This is a very, very important parameter which we call the dispersion parameter.

So, today we do not have much time to discuss more about this. So, in the next class we will going to discuss more about the dispersion coefficient and how one can calculate that and another kind of dispersion coefficient also be defined. So, with this note I like to conclude this class.

Thank you for your attention.