Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 14
Material Dispersion

Hello student. To the course of Physics of Linear and Non-Linear Optical Waveguides. Today, we have lecture number 14. And, in this lecture we are going to start the concept basic concept of Material Dispersion, which is very important in wave guide theory.

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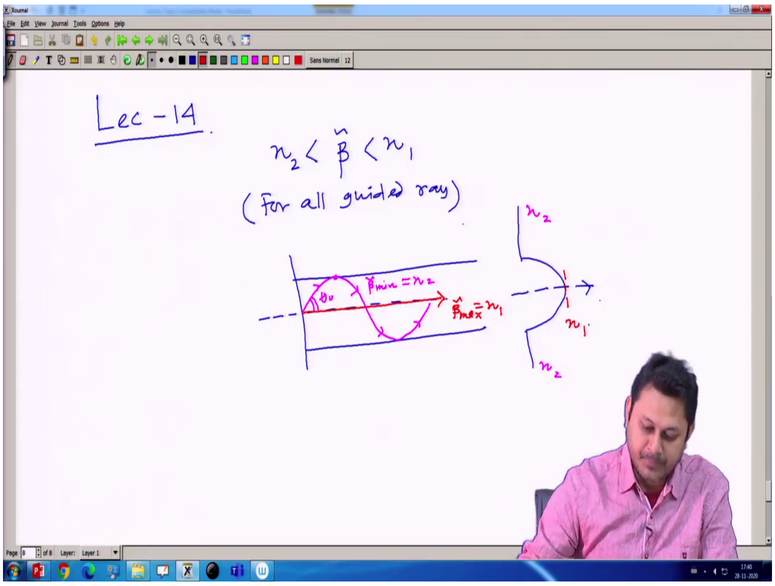

So, today we have lecture number 14. So, let us recap what we have done, that beta in the last class. That the beta tilde has some restriction, that it should be less than n 1 and greater than n

2 for all guided rays. For all guided rays inside a wave guide, these are the restriction of the beta. So, this is the fibre structure and this is the parabolic index profile. And, we find that the ray that is passing along the axis have the value of beta tilde maximum, which is equal to n 1.

And, the ray that is moving like this, touching this point, the transit point, having an angle theta 0 for that beta tilde is minimum and that value is equal to n 2. Where n 2 is this 1, n 2 is this 1, and n 1 is this peak value.

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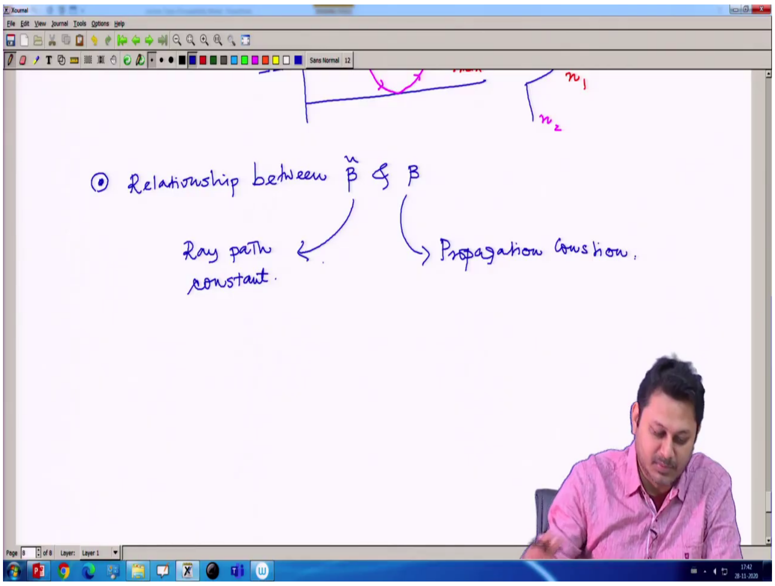

So, today we will going to understand another thing before going to the material dispersion part. That the relationship between relationship between beta tilde and beta. What is the relationship between beta tilde and beta? This is by the way is ray path constant and this is propagation constant. Where, beta tilde do not have any kind of dimension, the propagation constant beta has dimension 1 by length, that you should always remember.

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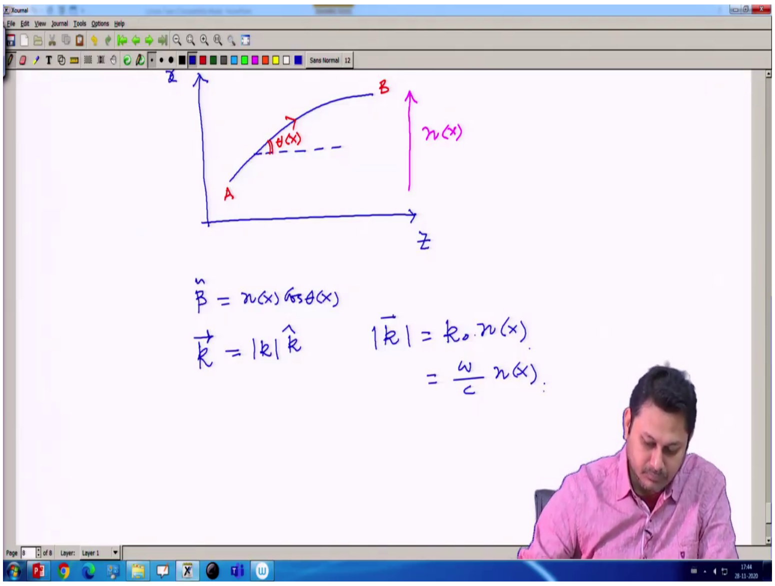

Well, let me again draw this two dimensional plane, where the ray is passing like this. So, when the ray is passing like this. In this path from point some point A to B every point this theta is changing. So, at some point say it is theta x and as usual along this direction we have the refractive index change of refractive index in x. So, beta tilde is n x cos of theta x true always.

Now, what is k the propagation constant? k is in general is a vector quantity, it is mod of k and k unit vector is the direction along which the ray is propagating the k is direction of that part, that direction. And, mod of k is k 0 multiplied by the refractive index, because refractive index is changing every time. So, n should be a function of x. So, that is the so, this value is omega divided by c n x. What is k z? So, the z component of so; that means, if the ray is passing like, if the ray is passing in this path.

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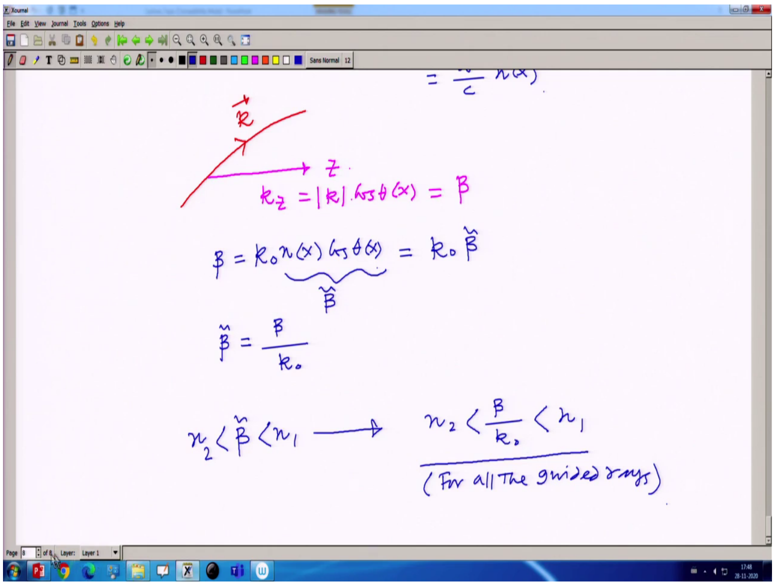

So, this is the k vector we have. If I want to find out the component of the k vector along this direction this is along z. So, this is k z component. And, what is k z component? It is nothing, but mod of k, then cos of theta function of x, which is by the way my beta.

So, beta is a propagation constant along the direction of z and k is the propagation constant, so, beta is the specific. So, it is the z component of the propagation constant. So, beta here so, propagation constant is a there is a component of the propagation constant, the component along, the propagation direction z so, I.

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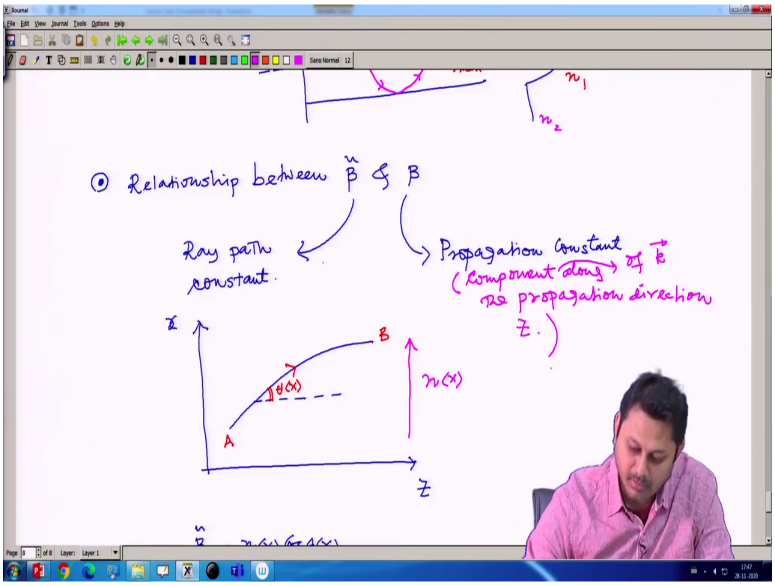

So, now I just put this value so, my beta which is a z component of the propagation vector k is 2. So, here I should write the component. So, I should write clearly component of propagation vector. So, component so, here I should write it component of what so, component of k along the propagation direction z.

So, beta is k 0 so, let me erase this part. So, my beta is how much? k 0 n x cos theta x, that is all. Now, this n x cos theta x is my beta tilde. So, it is eventually k 0 multiplied by beta tilde. So, beta tilde is nothing but beta divided by k 0. So, the restriction we had last time is beta tilde for guided mode, beta tilde is less than n 1 greater than n 2 can be rewritten in terms of beta as a propagation constant like, n 2 is less than beta divided by k 0 is greater than n 1.

So, this is a very important restriction of all the guided for; all the guided for all the guided rays. This important restriction, that we will going to explore more in our future classes ok.

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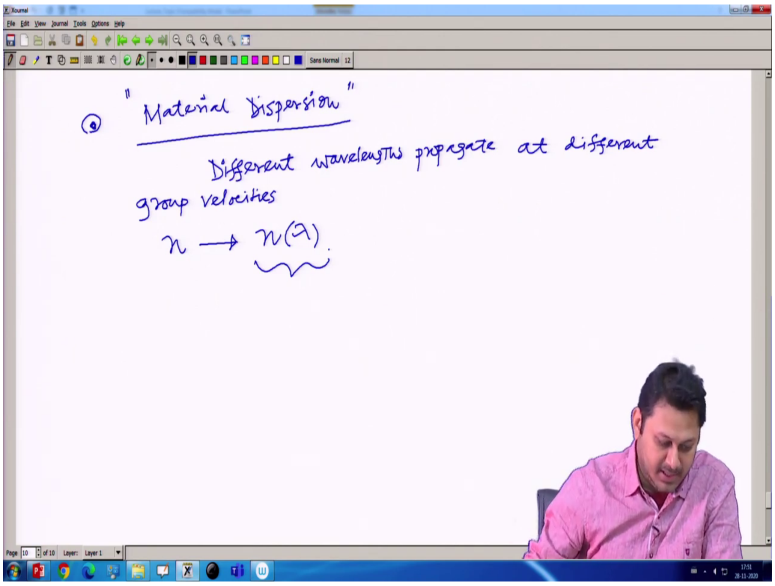

After, that we will directly jump to one of the major property of waveguide, which is called the material dispersion, material dispersion. So, quickly we brush up the material concept of material dispersion. So, different wavelength is nothing but the different wavelength propagates at different group velocities. So, different wavelength propagate at different so, that happens because n is a function of lambda. So, that is the main issue here. So, the refractive index n is a function of lambda.

So, that is the reason due to which the wave having lambda 1 and the wave is having lambda 2, has 2 different group velocities. So, that we are going to explore in a more detailed manner.

But, before that we need to find out how n is a function of lambda? What is the functional form?

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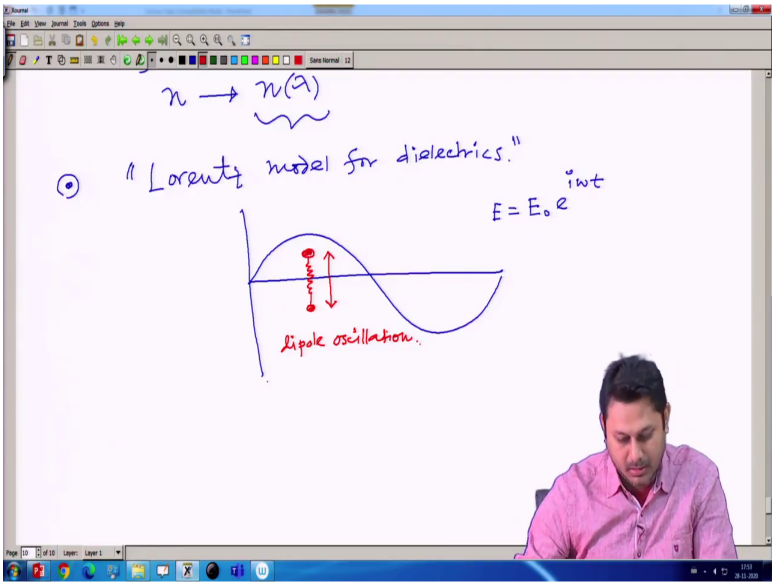

And, in order to do that we going to use are going to refresh the "Lorentz model for dielectrics". So, Lorentz model for the dielectrics we try to understand, this is not a very new thing, but still I feel that, it will be useful for you to understand the concept of dispersion. In a dielectric if I launch an electric varying electric field like this, there is the varying electric field, electric field, I can write as E 0 E to the power of i omega t.

So, some frequency omega the electric field is launched having some frequency omega. So, what happened the dielectrics this dipole will going to oscillate, under the varying oscillating external electric field. So, there should be a dipole oscillation. Now, we know what is the polarization? Because, the refractive index is the consequence of the polarization of the system, especially for dielectric system.

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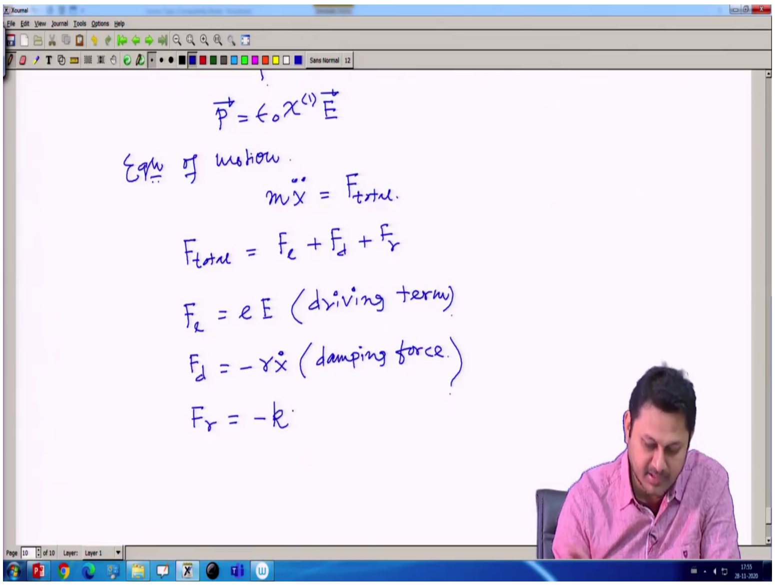

So, polarization P in vectorial form if I write it is this quantity. If I consider only the linear part of that linear polarization, then this is the expression. Obviously for non-linear cases there will be higher order term.

So, we are not going to the non-linear domain right now. For linear case P is simply epsilon 0 chi 1 E. Now, the equation of motion, if I want to find out what is the equation of motion of this system? So, the equation of motion is m double dot is equal to the total force.

Now, what is the total force? So, the total force experienced by this tiny dipole is like that, F total is a combination of the 3 term F e, F d and F r. What is F e? F e is the driving term due to the external electric field. What is F d? F d is minus of gamma x r this is velocity dependent term and this is basically the damping term so, damping force actually. So, the F e is a driving force or driving term, F d is a damping force and F r, which is minus of because we are considering that is this spring mass like a spring mass system dipole is oscillating, it is k x is a restoring force.

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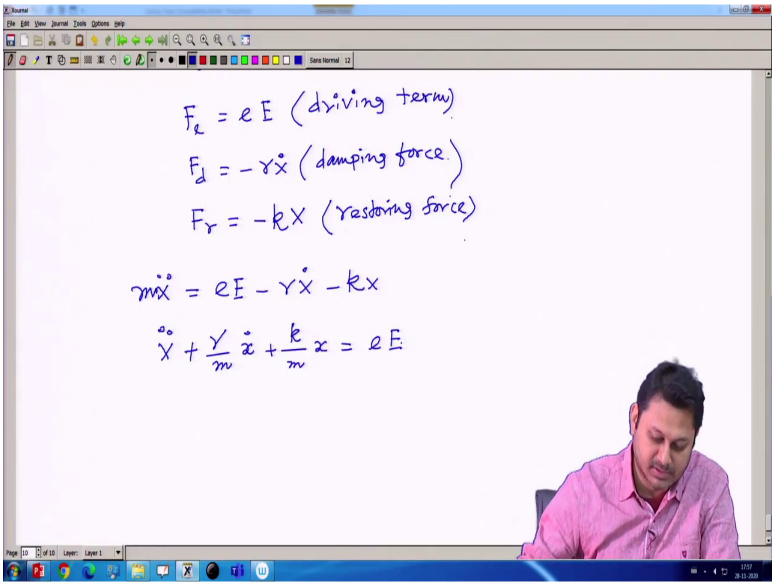

This all these things are well known so, I do not need to explain much, I just need to extract the expression of the refractive index out of this model. And, then it will be readily evident that how the refractive index is a function of lambda? That is all that is the goal of this treatment. So, I just put the values like this and then x double dot is equal to ok. So, let me write few step, then I will going to calculate that.

So, x double dot plus gamma divided by m x dot plus k divided by m x is equal to the driving term e E. Mind it k here is a spring constant not the propagation constant, this is different things. So maybe the notation looks same, but the k is the spring constant usual, coming usually from the Hooke's law.

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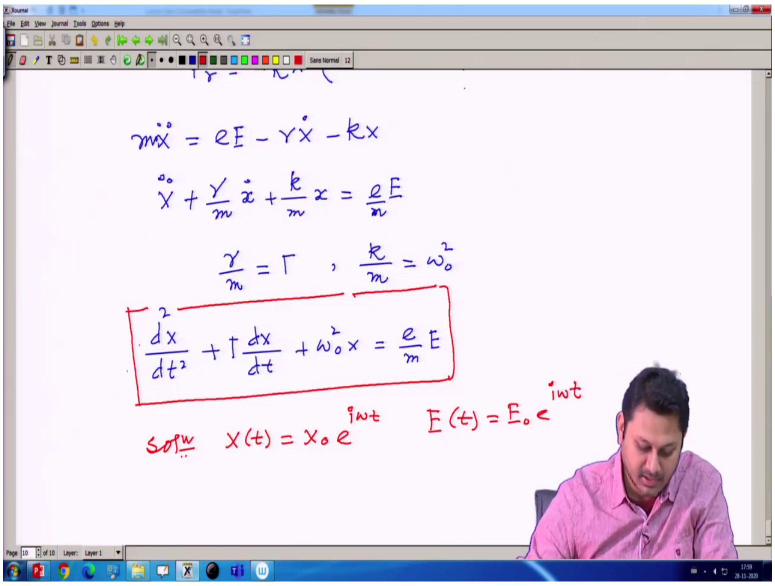

Now, I defined a gamma divided by m is a damping term as big gamma and k divided by m with our usual omega square, which is a resonance frequency of the system, that we know. That in a spring mass system, the resonance frequency is nothing but root over of k m that I am just using this that one. So, my differential equation becomes simply d 2 x d t square plus gamma d x d t plus omega 0 square x is e ok, when I divide it should be e divided by m e m E.

So, this is the differential equation now I need to solve. So, this differential equation is nothing but the force damped oscillation expression a very well known x differential equation nothing special here. And, we know how to solve this. So, the solution I want in this form. So, solution I should have in this particular form, e to the power I of omega t. Because, the electric field against the electric field, which is e external electric field the x will also follow the same vibration frequency. And, I am expecting the solution in this form.

Once, I have this solution and this assume that the solution in this form, I can readily calculate my x 0 just putting this solution here in this equation, putting this solution in this equation. I can readily find out my x 0 as e divided by m E as usual divided by now, I put this value here.

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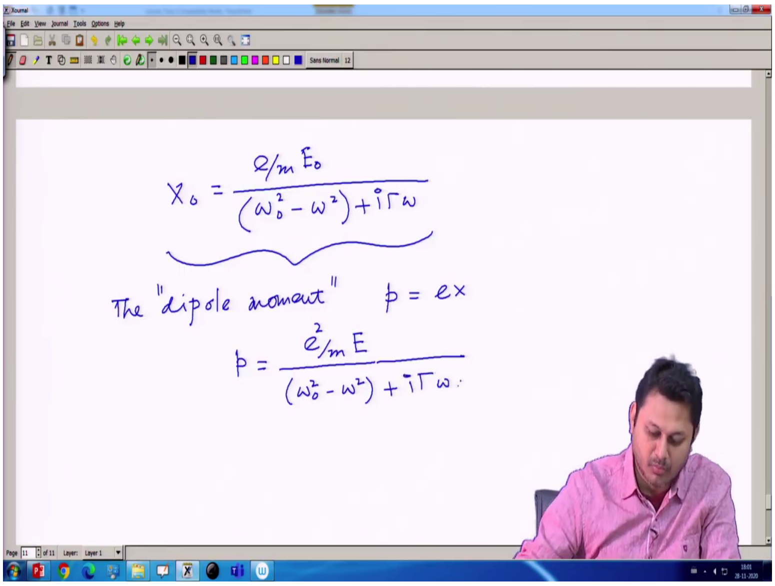

So, double derivative when I make a double derivative there should be a minus of omega square. So, an omega square is already here. So, it should be simply omega square minus, omega square plus i gamma omega and it should be E 0, because I am calculating x. So, from this expression, I can calculate the "dipole moment". The dipole moment is simply p the dipole moment is simply charge multiplied by the distance x.

So, here it should be p equal to e square divided by m, then e whole divided by that quantity omega square minus, omega square omega 0 square minus, omega square I gamma omega.

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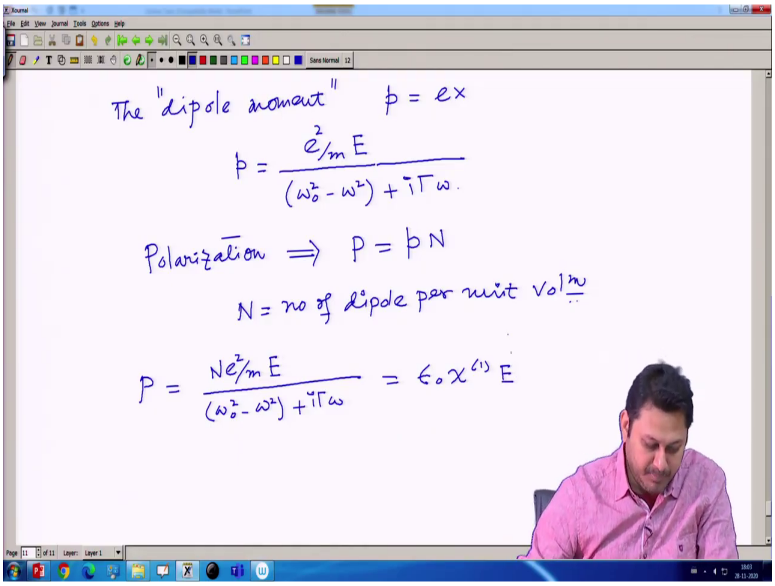

Polarization, if I want to calculate, because I started with the expression of the polarization. Because, the refractive index the information is of refractive index is hidden here. So, I need to extract that this portion, I need to extract that. So, the polarization is P, which I can write dipole moment. And, n is a number of dipole per unit volume. So, dipole moment per unit volume is my polarization. So, n here is the number of dipole per unit volume.

So, I can write my P in terms of small p n and my small p already I calculate. So, it is the expression is simply N e square divided by m, then E divided by omega 0 square minus omega square plus i gamma omega, which is basically epsilon 0 chi 1 E.

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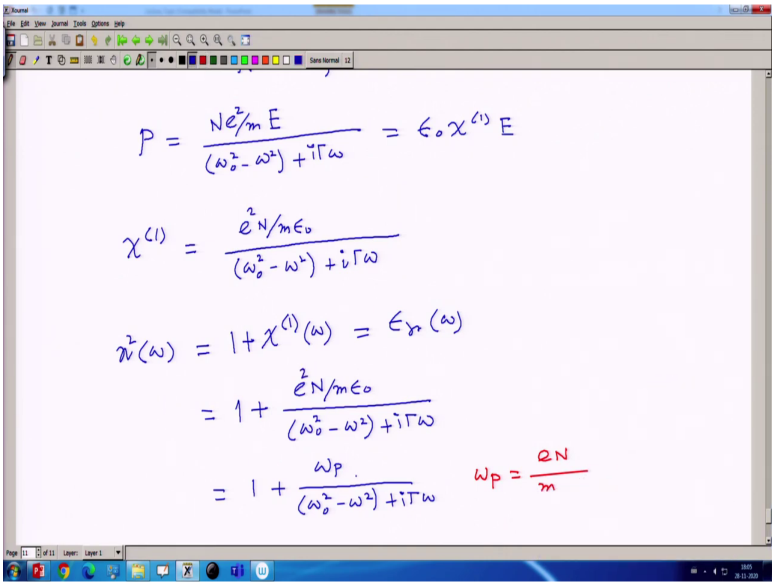

So, now, comparing this to expression I can extract the value of chi 1. My chi 1 susceptibility first order susceptibility is e square N divided by m epsilon 0. I just divide this epsilon 0 and then compare that, whole divided by whatever the term I already have omega naught square minus omega square plus i gamma omega. Now, my n square which readily become function of omega, because it is related to chi as 1 plus chi i. Chi i I can find this is a function of omega.

So, through chi 1 I can have the value of n square, which becomes a function of omega. So, I can again this quantity is nothing but the relative susceptibility, relative permittivity. So, now, I have 1 plus e N divided by m epsilon, whole divided by omega 0 minus, omega square plus, i gamma omega. This quantity normally written in a more compact form so, let me write it compact form in terms of plasma frequency. So, this is like this is a constant ok. I think, I am missing 1 e square here.

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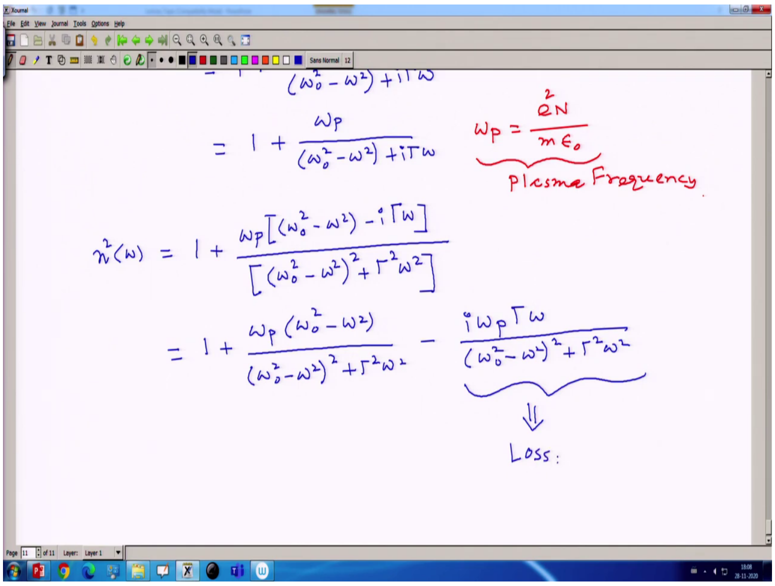

So, this is e square and then m epsilon 0. This is a plasma frequency term. Well, N of omega, I can now find one interesting thing, that n square omega is complex. And, if I just remove try to write it as standard form a plus i v x plus i y form or a plus i b from. So, here I think, I should have a negative sign. And, the denominator I have multiply the numerator and denominator by omega 0 square minus omega square minus i gamma omega.

Then, this will have term like 1 plus omega p, omega 0 square, minus omega square, this is the real part I am extracting out. And, the denominator it should be simply omega 0 square, here I should have a square minus omega square, whole square, then plus of gamma square, omega square. And, then minus of i omega p gamma of omega whole divided by omega 0 minus omega square whole square plus gamma square omega square. So, this is a part containing the complex term.

And, this term is basically due to gives us a loss. The refractive index, if I have a complex term it gives us a loss. So, the refractive index contribution is coming through the first term, that is interesting and that I need to just mention. So, now, this gamma is loss. If I neglect the loss and I can neglect that when the frequency is far away from the resonance frequency of the systems.

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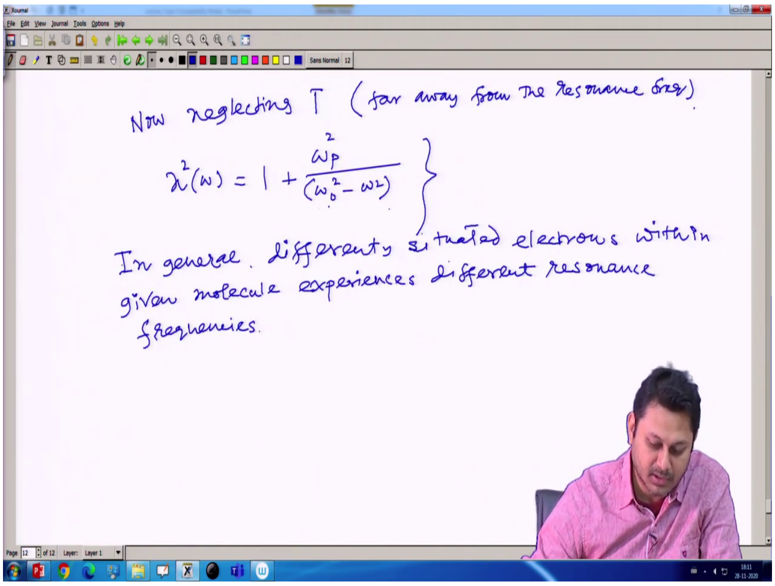

So, in that case now neglecting sorry gamma, so, which is far away from the resonance frequency then, I can write n square omega as 1 plus omega p square divided by omega 0 square minus omega square ok. So, this is roughly the expression and from here we can readily see that how the refractive index is a function of omega? But, that is not the end.

In general what happened, differently situated electrons within a given molecule, experiences different resonance, resonance frequencies. Because, all the in this calculation, we consider the resonance frequency for all the electrons is same, which is not true.

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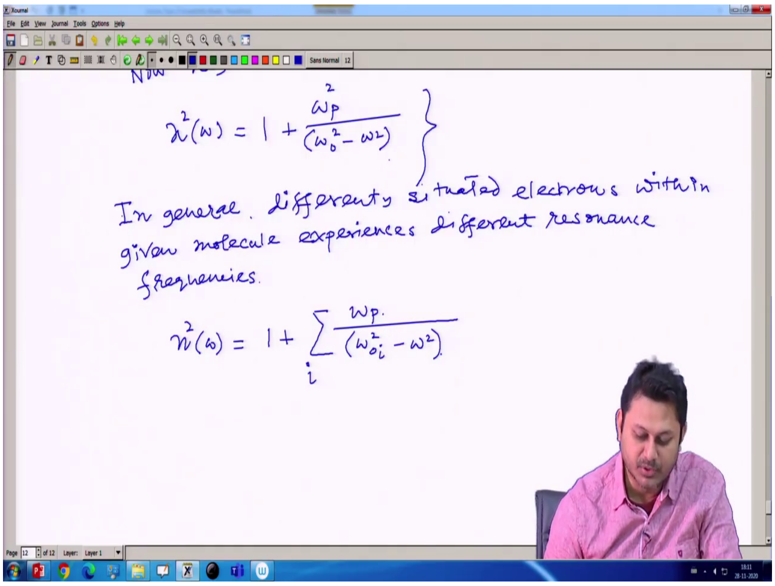

So, I need to consider that as well. So, n omega should be written in this particular form. For every the resonance frequency is now different for different differently situated electrons. So, I need to take account that as well by putting the summation sign. So, for different I have different resonance frequency.

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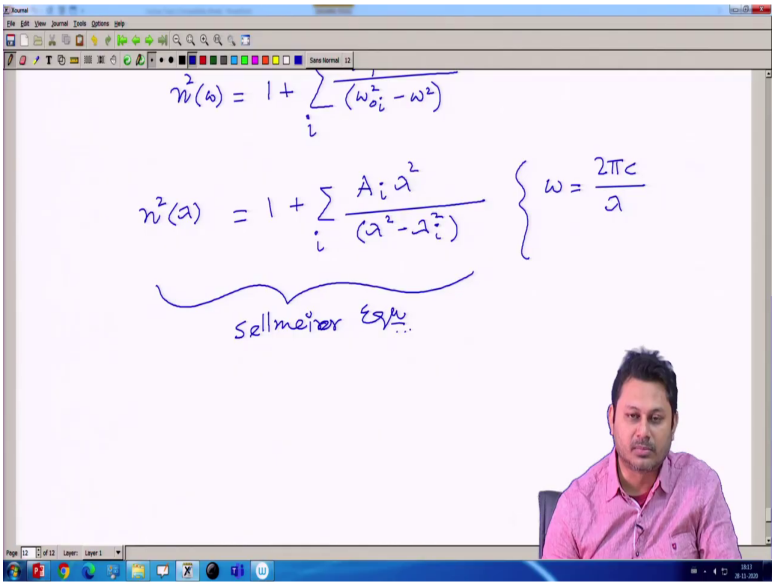

And, then I can have an expression, if I now write this expression in terms of lambda, I have something like this. Some constant say, A i lambda square divided by lambda square minus lambda i square using, the expression of the relationship between omega and lambda. This equation is basically called the Sellmeir equation. So, Sellmeir equation is the equation through which you can understand how the refractive index will going to vary with respect to lambda.

So, with this note I like to conclude, because today we do not have much time. In the next class, we will go on with this concept. So, now, from today's class we understand that a refractive index is a function of lambda. And, that is why when we launch a light with different frequency component, it will pass through the system with the different velocity group velocity, because n is changing as a function of lambda. And, that is why there is a time lag between these two component frequency component of the wave and eventually we have the dispersion.

So, we will discuss these things and also like to learn how because of the dispersion there is a broadening of a optical pulse. So, let us do in the next class all this calculation.

So, thank you for your attention and see you in the next class.