Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 13
Ray Transit Time (Contd.)

Hello student, for this in this course which is Physics of Linear and Non-Linear Optical Waveguide. So, today we have lecture number 13and in this lecture we will continue with the concept of Transit Time.

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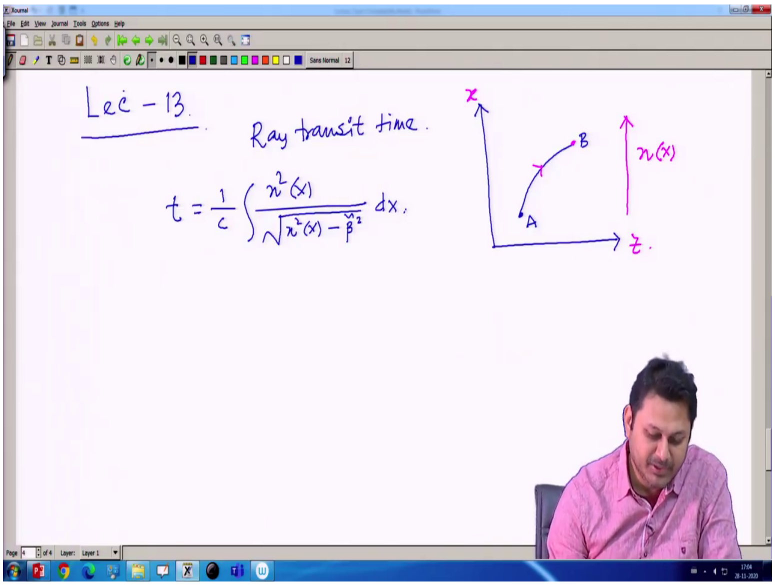

Today, we will have lecture number 13. So, in the last class so, we calculated the expression of ray transit time. So, the concept was something like this if I launch a ray it goes from some point A to B with the condition that I have a varying refractive index here.

In this direction, this is my x axis this is my z axis. And, the question is how to calculate the ray time the ray should take from going A to B. And, the expression that we derived was something like t is equal to 1 divided by c integration of n square function of x, divided by n square x minus beta tilde square and then dx. So, if I know the variation of n x explicitly, then we can calculate the value of t.

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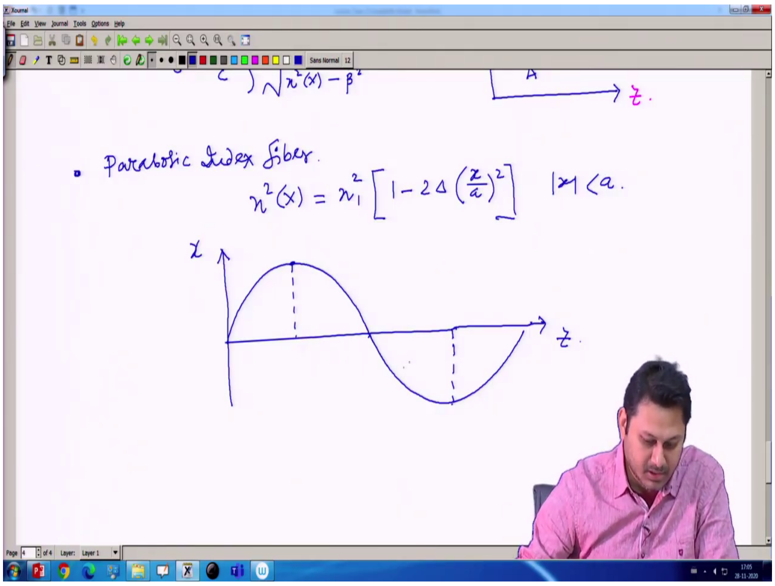

Now, let us try to understand what happened for graded index qualitatively what happened for graded index fibre. So, say parabolic index for parabolic index fibre, we had the refractive index profile something like this. And, then we find that the ray is following a sinusoidal wave this is a 1 half and another half in this direction.

Now, you can see that the ray is moving along this part. And, this is the point, where the amplitude become maximum. And, if you calculate this point by the symmetry of the problem, one can find what is the; what is the full period, what is the time taken the ray to complete the full period.

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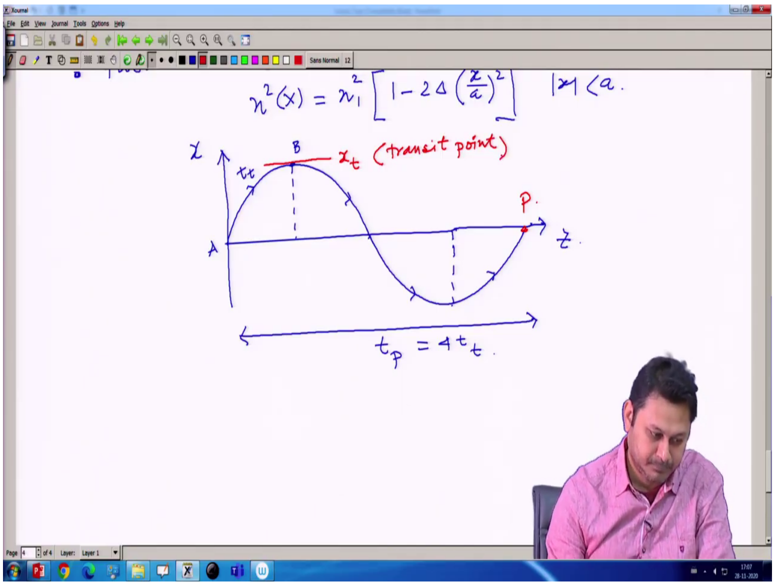

If, the full period is say t p, then it is nothing, but 4 t t. What is t t? t t is if A and B is there, so this is the time the ray should take to reach the point B. B is the point where from where the ray is turning back, initially it is going fine. But, at point B this is the point from where it is now turning back. So, that is why I just write this time as t t.

So, it will fourth time it will happen. So, if I calculate this value then if I multiply it with 4 then whatever the value I will going to get is the time that the ray should take to start from A to some points P. It will take some what is the amount of time the ray should take to complete this complete period can be calculated, if we able to calculate this. So, this is very very special point. So, this point I say called this point is called x t, because this is specific point. And, this point is let me call the transit point.

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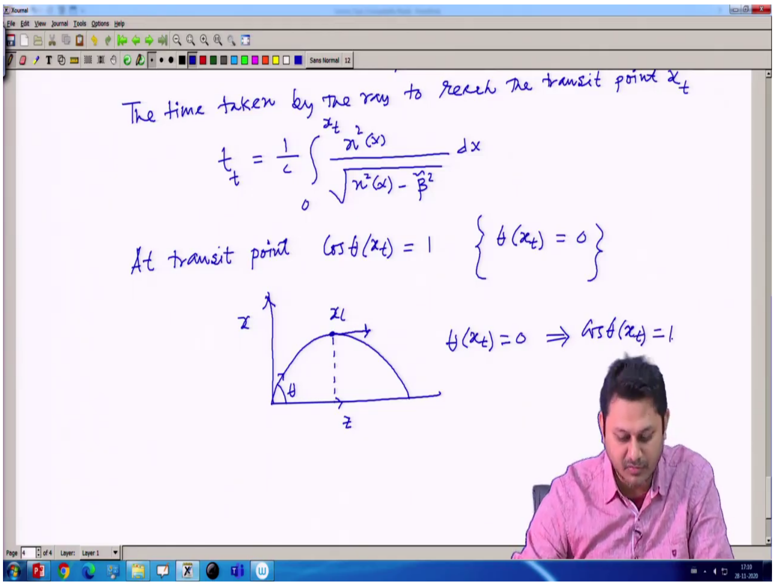

So, the time taken by the ray to reach the transit point x t is t t equal to 1 divided by C, integration of 0 to the point, which is x t and then n square x d x. Now, few things we need to note, that at transit point, the value of cos theta at transit point is equal to 1, why? Because theta at transit point is equal to 0.

So, this is the structure we had and exactly at this point what happened? This is my theta and theta is making an angle this is a ray theta is making an angle with respect to the z axis, this is my x.

So, what happened here? At x t the ray direction of the ray and the direction of the z axis will be in the same direction. So, I have theta at x t is equal to 0, which gives me cos theta x t is equal to 1 that is an additional information we should know for this transit point whatever the transit point we define.

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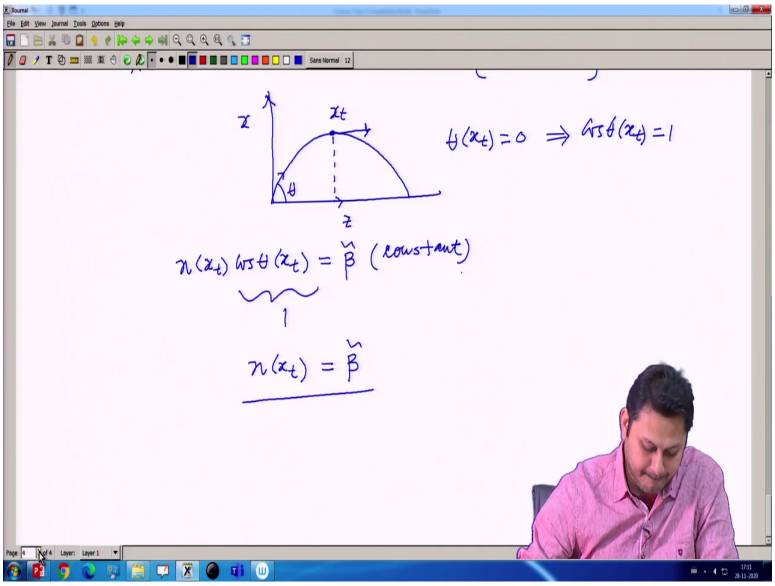

Now, n at x t cos theta at x t, it should be this quantity is always constant. So, this constant we all know it is beta tilde the ray path constant. So, this equation should valid. So, this is constant.

Now, this quantity is 1. So, at transit point we have an expression which is this is a special expression we have or there is a special relation we have with the refractive index to the ray path constant. That a transit point the refractive index at the transit point is equal to the value of beta tilde well.

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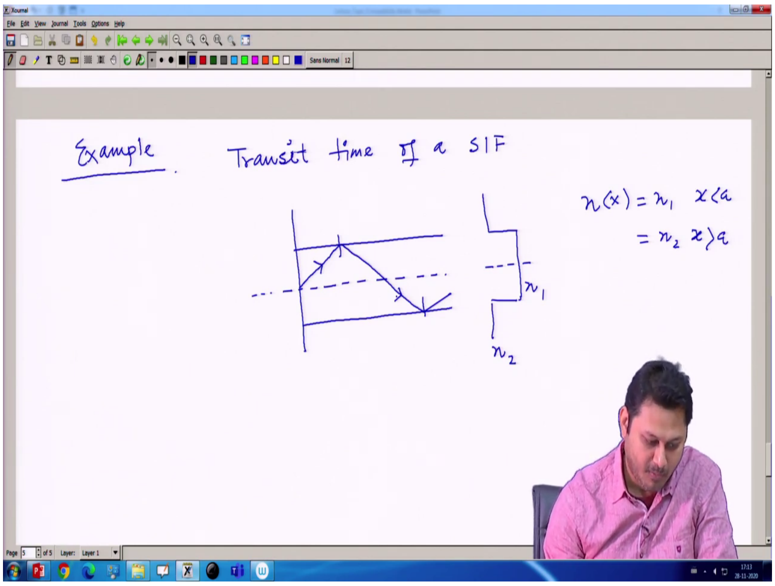

Now, after that we will take the example, that how to calculate. A very very simple example and it looks very trivial, example that transit time of a step index fibre having step index profile. So, let me again draw these things, this is the cross sectional view of an optical fibre having step index profile I am drawing the profile here. So, this is my n 1 and this is my n 2 where n x is equal to n 1, when x is less than A equal to n 2 when x is greater than A. So, these are constant.

Since, this is these value of n 1 and n 2 are constant, my calculation will be much simpler and let me calculate that. So, what happen if I launch a light here? It should hit here, and then have a total internal reflection and hit here and so on. Because for step index fibre the ray will follow a straight line path. So, it is just following a straight line path.

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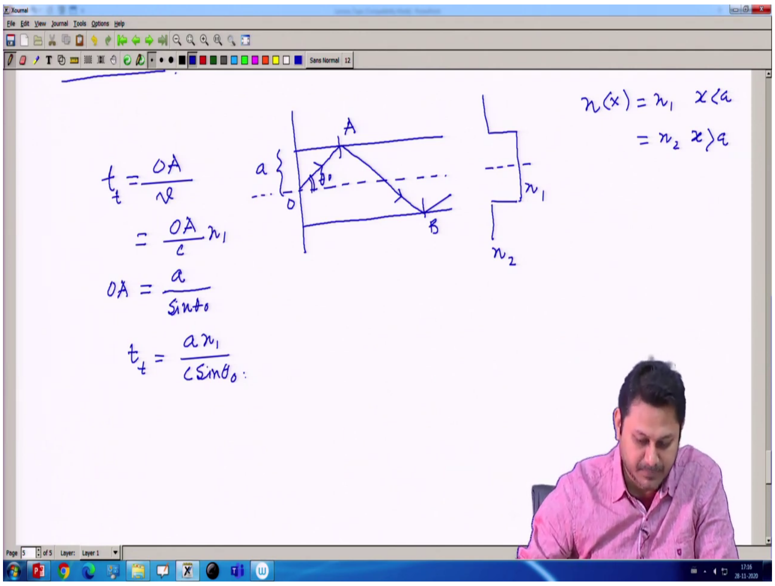

Now, I are going to calculate my transit time; that means, if this is O, this is A, so suppose this is B. So, what is the time the ray should take going from O to A? And, very very simple calculation is already there. So, we will check that, if I if this is n. So, the time is precisely O A divided by, we do not need to use at all the expression of the transit time the integral, expression of the transit time.

Rather we just use this is the path and this is the velocity, that the ray will have to travel from the point A to B. And, since n 1 is constant this velocity is not a function of x anymore and that makes my life much simpler. And, I simply have O A, divided by C multiplied by n 1.

And, if this angle is known suppose this is theta 0. And, this is my a I can calculate that so, this is my t t because I am calculating eventually the transit time at point, where the ray is bending to this point.

So, O A, I can write as because this is theta this is a. So, O A is simply a divided by sin theta 0. Because, sin theta 0 is a divided by O A so, O is A divided by sin theta 0. So, now, I am done.

So, my t t will be simply a n 1 divided by c of sin theta 0. Using without using any kind of expression, I can readily calculate, because in this case n 1 is constant. But, it is not that easy when we have n as a function of given function of x for example, A graded index or parabolic index fibre, it is not that easy to calculate, but anyway.

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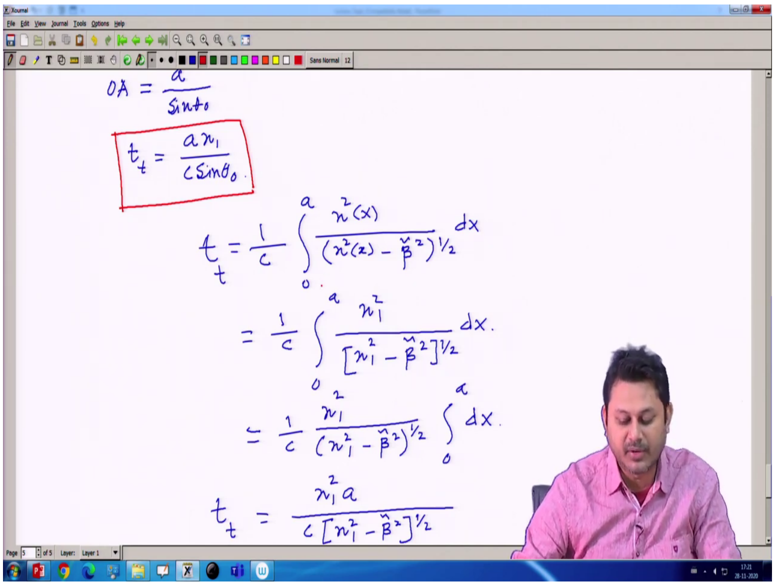

Now, we are going to test that whether my expression will give the same result or not. My expression suggest, the general expression suggest, it is this n square x minus beta tilde square whole to the power half dx. That is the value of my and now I need to go from transit, when I calculate the transit time I to go from 0 to a. Because, this is the transit point this a point is eventually my transit point.

So, I can calculate 1 divided by c, integration 0 to a, this is my n 1 because, here it is constant and this is n 1 square minus beta tilde square whole to the power half and d x. You can see this entire term is constant n 1 square divided by n 1 square minus beta tilde square, this is not a function of x at all. If, this is not a function of x then I can put it out. So, it should be 1 by c n square divided by n 1 square minus tilde whole to the power half this I put outside. And, then the integration is simply integration 0 to a dx.

And, this quantity is a, so, eventually I have n 1 square a divided by c, multiplied by n 1 square, minus beta square, whole to the power half. Well, I am already there now I need to use few things, because this result is not matching with whatever we have my t t is here is a n divided by c sin theta 0. So, I need to introduce this theta 0 in the expression to find out, so, that we can do.

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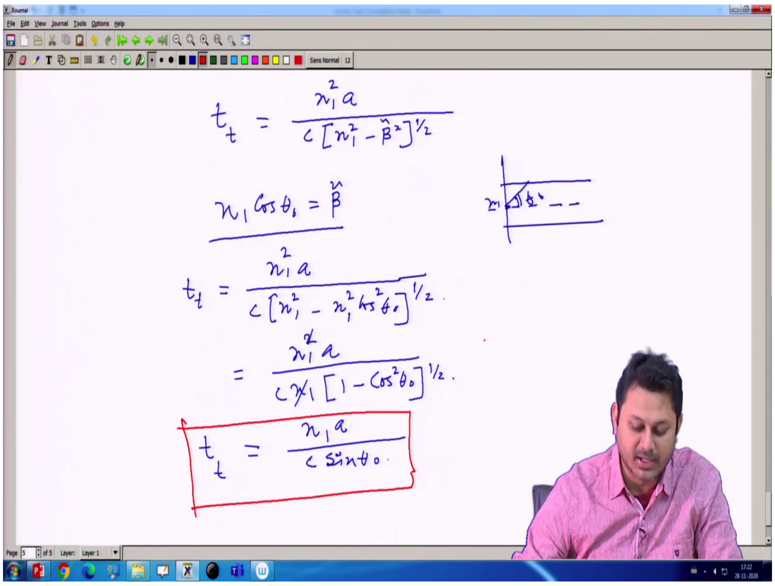

So, n 1 now I can write it n 1 of cos theta 0 is my beta tilde this is always true. And, in this fibre from here I am launching the light, and this angle is theta 0, so here the value is n 1. So, at this point n 1 and cos theta 0 should be equal to beta tilde.

And, now I need to put this value there. So, my t t is eventually n 1 square a divided by c n 1 square minus beta tilde square I can put at n 1 square cos square theta 0 whole to the power half. I can have n 1 square a divided by c n 1 1 minus cos s square theta 0 whole to the power half.

1 n 1 it will cancel out so, eventually I have n 1 a divided by c sin theta 0 which is my original result, which we already derived without using any kind of integral form. So, from this simple expression, I find that whatever the value I calculate using is straightforward concept that the ray is moving from O to A point, what is the time it should take.

I know the refractive index n 1 which is a constant, which is not varying as a function of x. And, I can derive a value, like a n 1 divided by c sin theta 0, when the sin theta 0 is the initial angle.

I can have the same expression using our general integral form I can have the same expression, but the important thing that you should know that, when we have a parabolic index profile, then this integration will not be that easy. For step index profile this integration looks very easy, because n is not a function of x. So, we can take this outside and then this integration become very very simple here integration 0 to a d x.

But, it will not be true for graded index or any kind of given profile and then you need to calculate the integration. So, I will not going to do that. So, I will give you the result may be in the next day. So, you should try it out by yourself, that whether it is really coming to whatever the result given to you or not ok. So, now, after that we should move on to another very important conclusion and these things I should write here.

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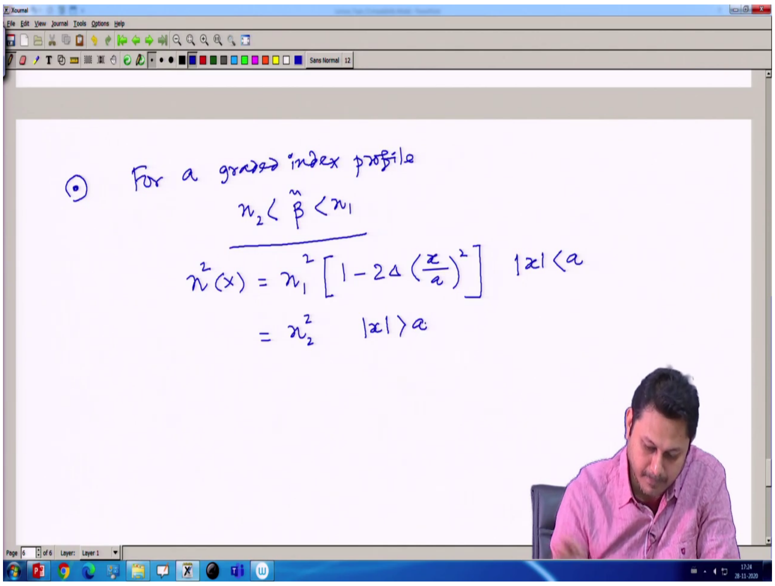

For a graded index profile we can always have this. So, the be the ray the ray path constant beta tilde should be less than n 1 and greater than n 2, for a graded index profile or in index profile. So, this is the restriction, I am putting some kind of restriction over beta. And, we will go we are going to check that what happened, if we have a refractive index profile with a parabolic form.

So, let us write it in this way. So, this is a fundamental restriction that a ray constant should follow, should follow for a graded index profile. So; that means, beta tilde is related to the guided wave for all the guided waves, the value of beta tilde which is the ray path constant will be in between the n 1 and n 2, where n 1 and n 2, the boundary refractive; in refractive index of the core and cladding. This is a peak refractive index of the core and cladding. So, let me write it in this way less than a and this is equal to n 2 square 1 x is greater than a.

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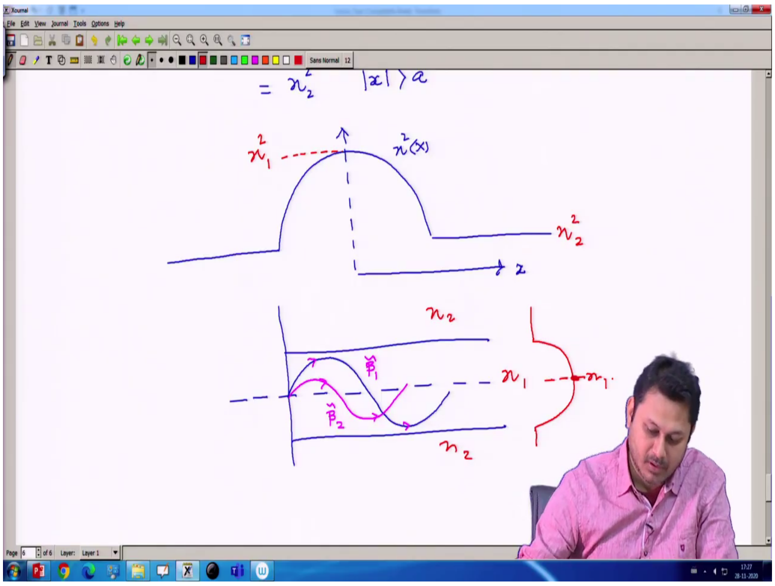

So, if I draw that sorry this is not x axis this is my refractive index as a function of x I am plotting the refractive index. This is x and this peak value is n 1 and this value is n 2.

Since, I am plotting the square of that so, let us put in this way this is the square. So, the peak values and 1 square and whatever the beta propagation. So, for this case I have the ray path that we already calculated that for this kind of profile we have a sinusoidal kind of ray path. So, this is the path 1 path, this might be another path. So, for all the paths all the paths I have beta tilde constant.

So, here I have say 1 constant beta tilde 1, here I have another constant beta tilde 2, because for different paths I have different beta tilde, but that tilde beta tilde should be constant throughout the path. And, here we try to show that, that this beta tilde, the restriction of the beta tilde is such that, it should be less than n 1 and greater than n 2, where n 1 and n 2 is this

1. So, n 1 is the refractive index of the core and n 2 is the refractive index of the cladding. Refractive index of the core part refractive index of the this is n 1 ok. So, quickly try to find out.

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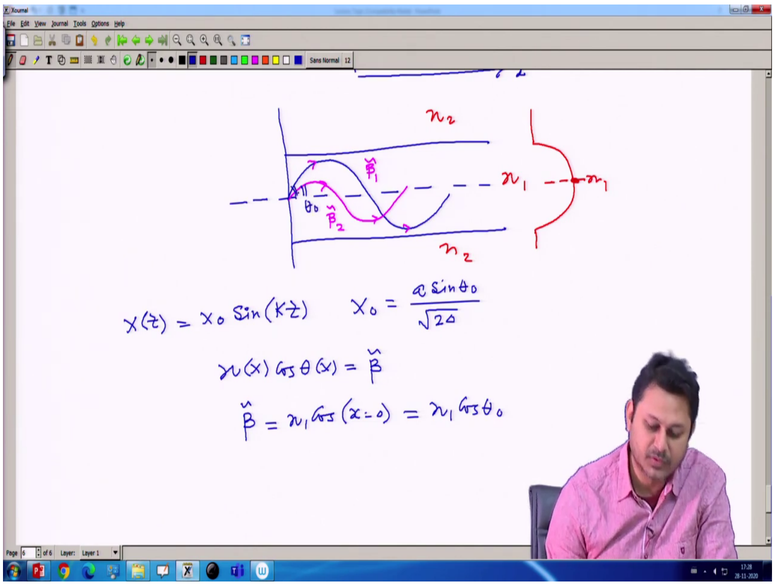

So, we already have my x function of z for this case as x 0 sin of k z, where x 0 is a sin theta 0, divided by root over of 2 delta. And, also we have a restriction that n function of x cos theta function of x is equal to beta tilde.

So, beta tilde is equal to n 1 cos of x equal to 0, which is n 1 cos theta 0. So, this value is true for cos theta 0 at; that means, the initial point at also. So, this is my theta 0 initial so, this is also some theta 0 prime like that.

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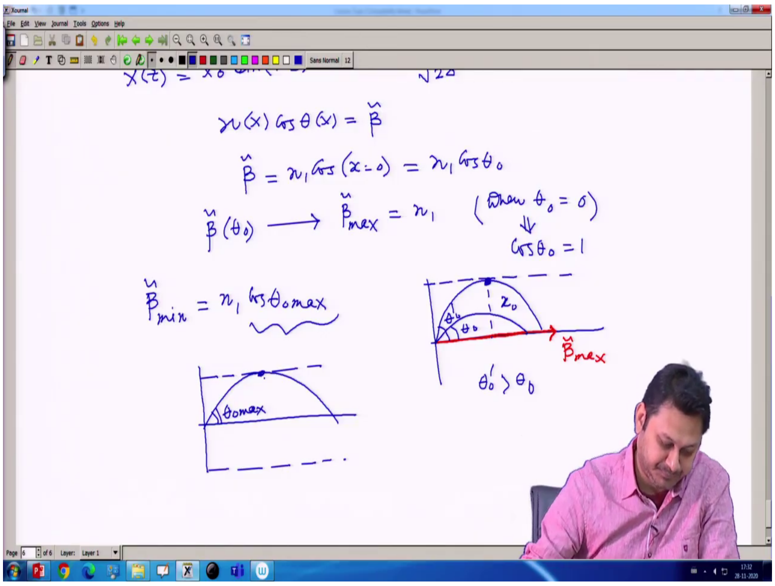

So, from this expression, I can we can find that depending on the so, beta tilde is basically a function of theta 0. And, from here, I can readily find that beta tilde maximum value from this expression is nothing, but n 1.

Because, it will be maximum when theta 0 is equal to 0, because in that case the cos theta will have the maximum value and maximum value of the cos theta will be 1. So, cos theta becomes for this cos theta will be 1 and I have max equal to d.

Now, what is beta minimum? So, beta tilde minimum, it should be n 1 cos theta 0, I should put a suffix here that beta 0, theta 0 max. Why it is that? Because if you look carefully. So, this is the ray for which the so, the ray for which I have maximum beta is this one, which is directly going over this axis, on the axis.

So, the ray passing along the axis is having the value of beta tilde max. So, gradually if I launch with an angle, then what happened every time, suppose I am launching this one, I am launching this one. So, what happened, when I launch with a different angle this angle will going to increase and if this angle increase, then the cos theta value is also decrease.

Because, if theta increase then I can have a maximum cos theta value for which this these things is minimum. And, then I can have this quantity. So, if I look carefully this is the boundary I have. So, theta minimum will the point the value say theta 0 prime is my theta minimum here this is the value, where theta maximum here.

So, the point where it reaches to this point. So, if I say this is x 0 from here to here. So, when it reaches to x 0, then that that value is basically my theta maximum. So, let me draw it once again. So, I have this profile this is the bound boundary of that and I launch a ray and it ray reaches here. And, this angle it should be theta 0 max for which this ray all the theta 0 less than that will never reach this point.

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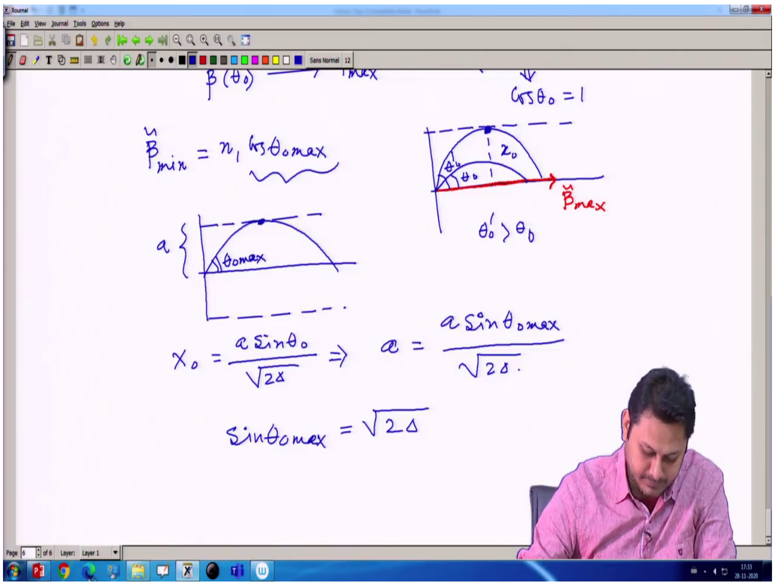

So, this is my a. So, I have my x 0 is equal to a sin theta 0 divided by root over of 2 delta. Now, I know that this value from here when x 0 is my a, the right hand side is a sin theta 0 max divided by root of 2 delta. So, I can find from this expression I can find what is my theta 0 sin theta 0 max. So, sin theta 0 max is eventually root over of 2 delta.

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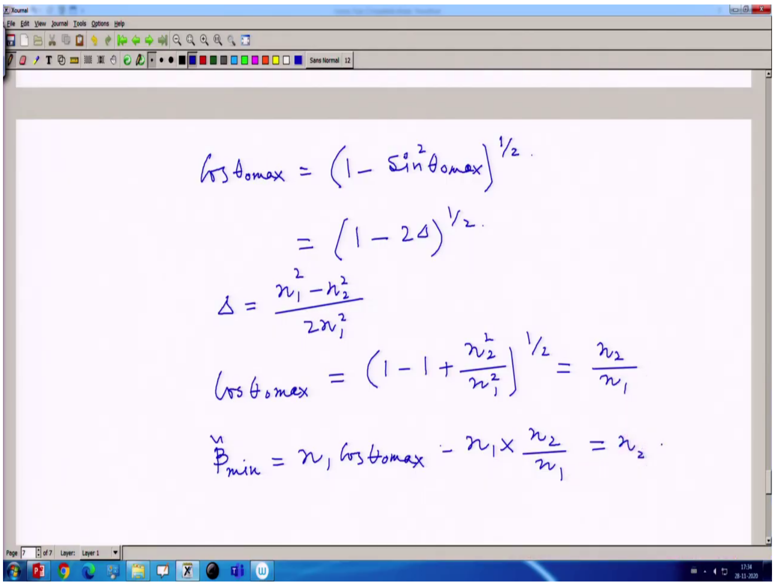

Once, we have sin theta 0 max I can readily calculate cos theta 0 max, which is my goal here, which is 1 minus sin square theta 0 max whole to the power half. Which is 1 minus 2 delta whole to the power half and delta, we know it is n 1 square minus n 2 square divided by 2 n 1 square.

So, from here I can have, if I put this value here, then it should be cos theta 0 max should be 1 minus 1 plus into square divided by n 1 square, whole to the power half which is equal to n 2 divided by n 1. So, my beta max beta min, which is n 1 cos theta 0 max is equal to n 1 multiplied by n 2 divided by n 1, which is equal to n 2.

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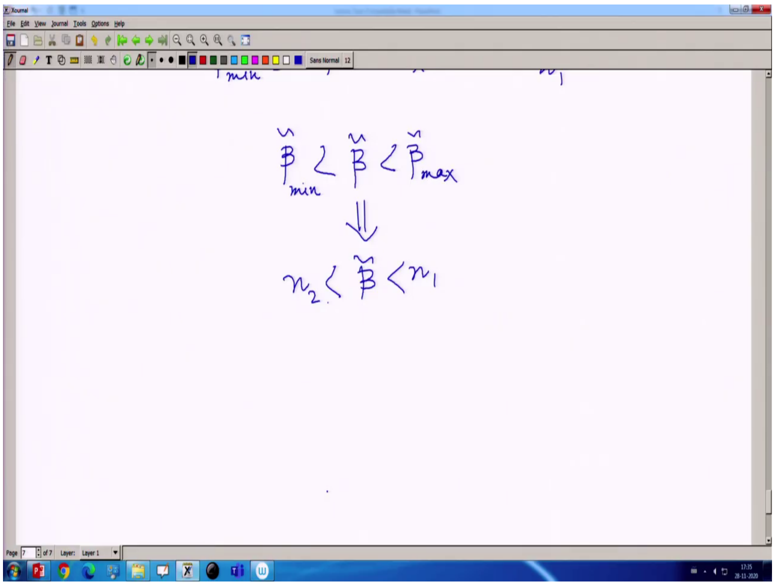

Hence, I have a restriction over beta, so I find the beta max. So, beta tilde should be in between beta max and beta min. So, this basically tells me that, my beta tilde should be less than equal to this. So, with this note I like to conclude today, because we do not have much time.

So, in the next class we will try to find out the relationship between the beta tilde which is the ray path constant with the beta, which have propagation constant, what is the relationship between that. And, also start the concept of material dispersion with that note let me conclude here.

Thank you for your attention.