Physics of Linear and Non-Linear Optical Waveguides

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Module - 02
Basic Fiber Optics
Lecture - 12
Ray Transit Time

Hello student, for the course of Physics of Linear and Non-Linear Optical Waveguides. So, today, we will going to discuss something called Ray Transit Time.

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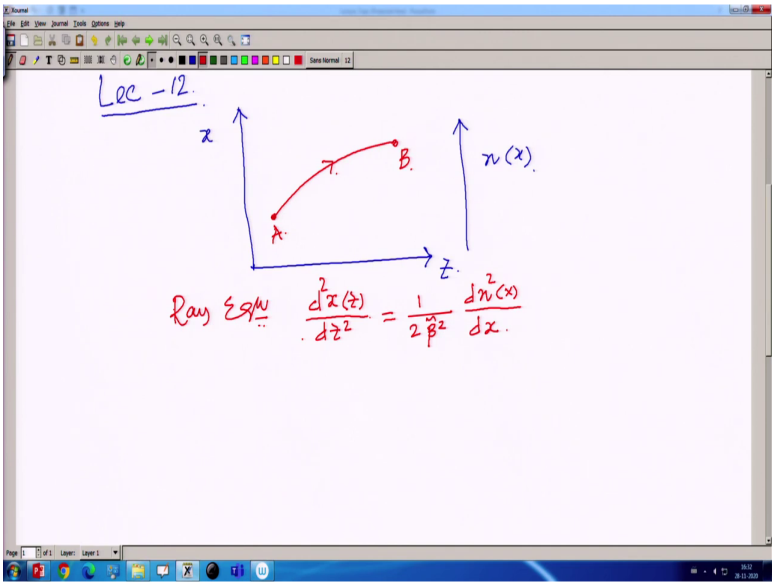

Then, gradually, we will understand what is the meaning of ray trajectory etcetera. So, if we have this is x, this is z and along this direction, if we have a refractive index variation. Then, what happened if I launch a ray? It should follow certain path. It goes to point A to point B following certain path and one can calculate this path equation and if we calculate this path equation or ray equation, it should be like d 2 x which is a function of z dz square is equal to 1 divided by 2 beta tilde square d of n square which is a function of x and dx. That was our ray equation.

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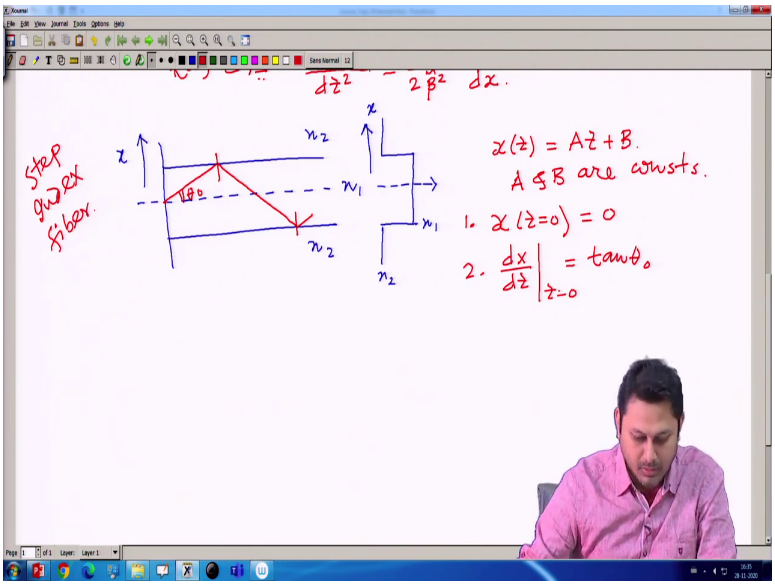

Now, if I have a fiber structure like this, refractive index n 1, n 2, n 2 and it is a step index kind of fiber. This is n 1, this is n 2, this is along x. Now, for this structure, if I launch a light, it should follow a straight-line path. If this is my launching angle theta 0, we calculate using the ray equation, we calculate the path follow a general equation should be AZ plus B; where, A and B are constants, where A and B are constants and can be evaluated using the boundary condition.

What was the boundary condition? The boundary condition was x at z equal to 0 is equal to 0, that was the first boundary condition. And second boundary condition was d of X dz at z equal to 0 is equal to tan of theta 0. That means, it depends on the initial angle theta 0; the value of A depends on the initial angle theta 0. Another case, so this is for simply step index fiber. What happened for a graded index fiber?

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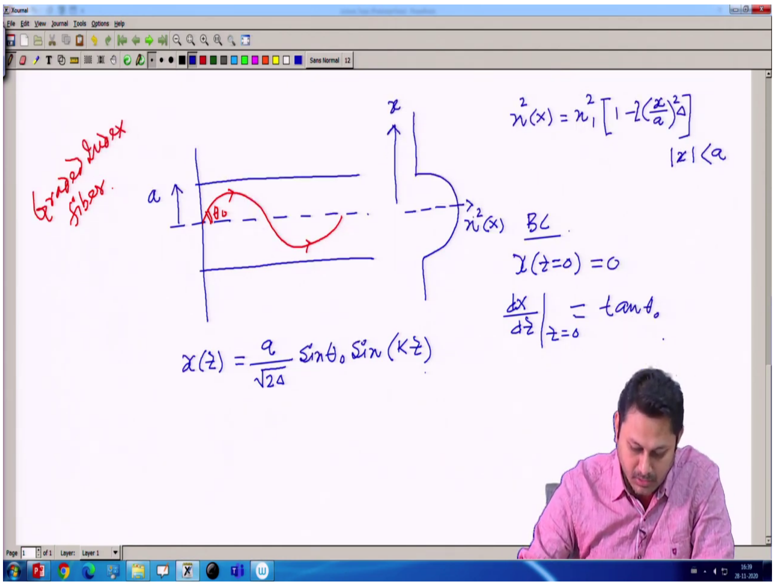

For a graded index fiber, we have a geometry like this. Here, the refractive index profile is parabolic. And we have a form of a refractive index that is n square X was n 1 square 1 minus x by a whole square and then, there was a term 2 delta. So, it was actually 2 of this into delta. This is for when x is less than a, and this from here to here, this is a. So, this is my n square X, the variation and along this direction suppose we have x. In this particular fiber, where we have a refractive index having this particular form which you called the parabolic index.

Then, we find the ray that is passing is following a sinusoidal path. If this is my initial angle theta 0, then whatever the equation we had is having a sinusoidal form, if that that was having a sinusoidal form. So, if I write let me write it here that x of z was a divided by after putting the boundary condition, sin theta 0 and then, sin of K of Z. Mind it, boundary condition was same that x at z equal to 0 is 0 and d X d Z at z equal to 0 was dX dZ at Z equal to 0 was tan theta 0 depending on the initial angle, launching angle this theta 0.

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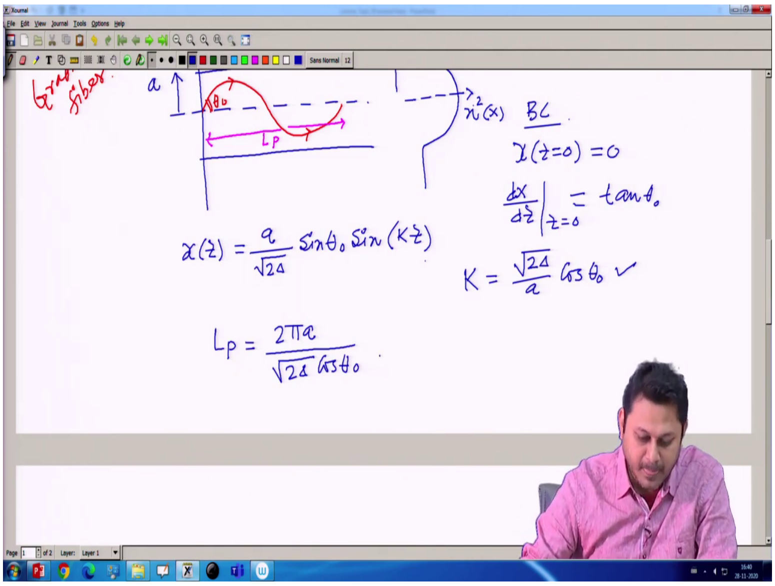

And the value of the K, that we calculated last time was root over of 2 delta divided by a and cos theta 0 that was the value of K depending on the initial angle theta 0, depending on the initial angle theta 0. Now, also we calculated the period; once we know what is K, then we also calculated the period and L p was the period, where L p is this amount. From this, this is my L p. So, L p was 2 pi a divided by root over of 2 delta cos of theta 0. So, L p now again depending on the theta 0 and that means, if I so my amplitude.

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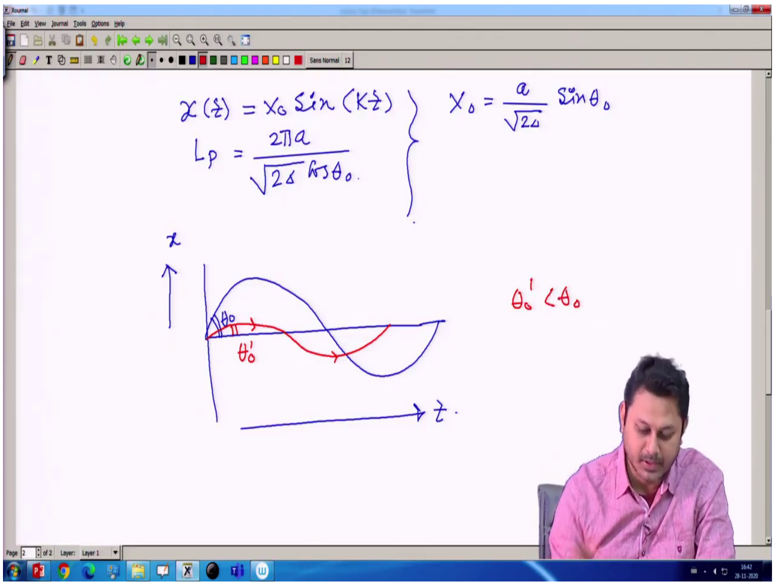

So, let me write once again x, which is a function of z was X 0 sin of K Z and my L p was 2 pi a divided by root over of 2 delta cos of theta 0. From this information, we can and X 0, by the way X 0 was the amplitude and amplitude was a divided by root of 2 delta and then, sin of theta 0. So, if I launch a light, if I launch a light along this direction, we have x and this is my Z direction. So, if I launch a light with larger theta 0, my period is also be larger because if theta 0 is large, cos theta 0 is small; cos theta 0 is small, that means, L p will be large.

So, this is for certain theta 0. Now, if I launch a smaller theta 0, then I should have a smaller period because if theta 0 is smaller, then cos theta 0 again will be higher and I should have a lower L p. So, in first case whatever the L p I calculate, in second case the L p will be different because of the launching angle. Now, this launching angle is suppose this launching angle is theta 0 prime; when theta 0 prime is less than theta 0. Now, if we also find that if somehow this launching angle is small.

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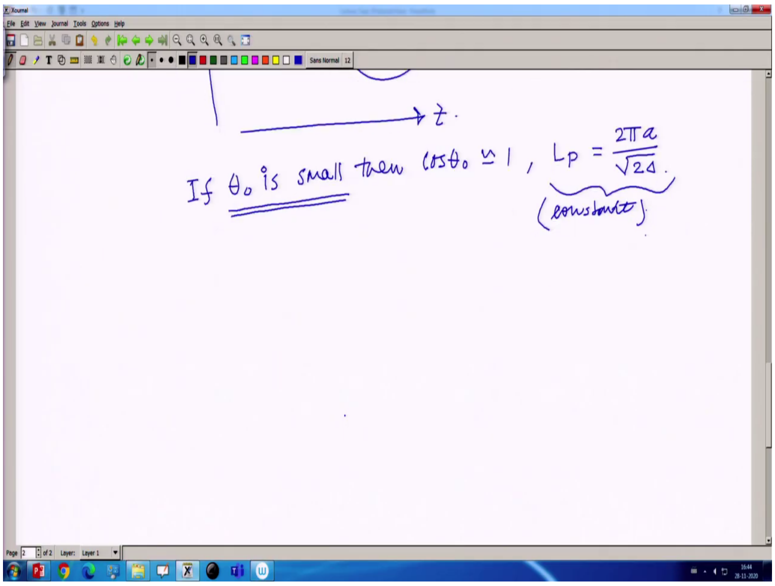

If theta 0 is small, then we can, we can say cos theta 0 is nearly equal to 1 and for that, I have L p is equal to 2 pi a whole divided by root over of 2 delta and that is independent of that is almost that is constant and independent of if any theta 0. So, we can have all the, all the ray is in the same period, but with the condition that this has to be theta has to be small; this is the condition, theta is small. This is a specific condition under which we have this, ok.

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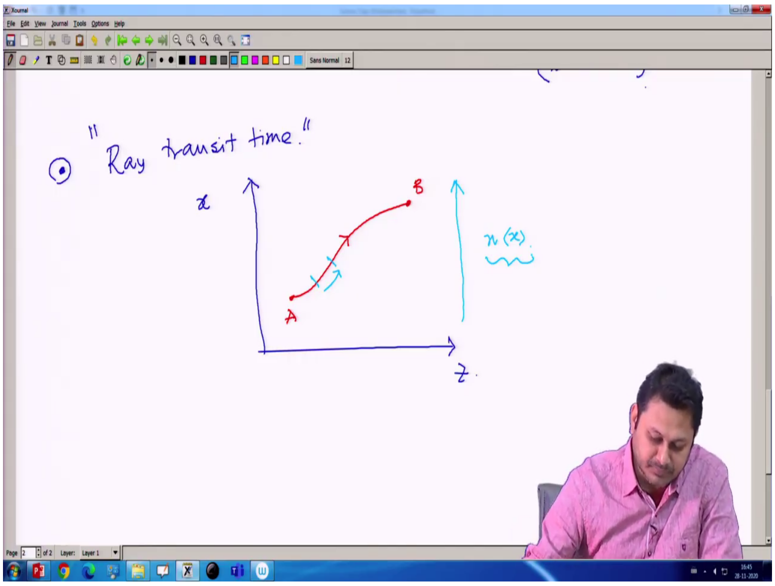

So, now today, we will extend our calculation to find something and that is called the ray transit time; ray transit time. What is ray transit time in a? So, let us try to understand this first. Again, we have a two-dimensional coordinate system through which array is passing, following certain path, going to point A to B under the condition that I have a refractive index variation along this direction ok, which is a function of x.

So, the. So, if the refractive index is vary, constantly varies, then what happened that at small distance the time taken will be calculated. Having with the can be calculated using the value of the value of n x because this value is constantly changing. So, delta t will be calculated using the small distance divided by the velocity.

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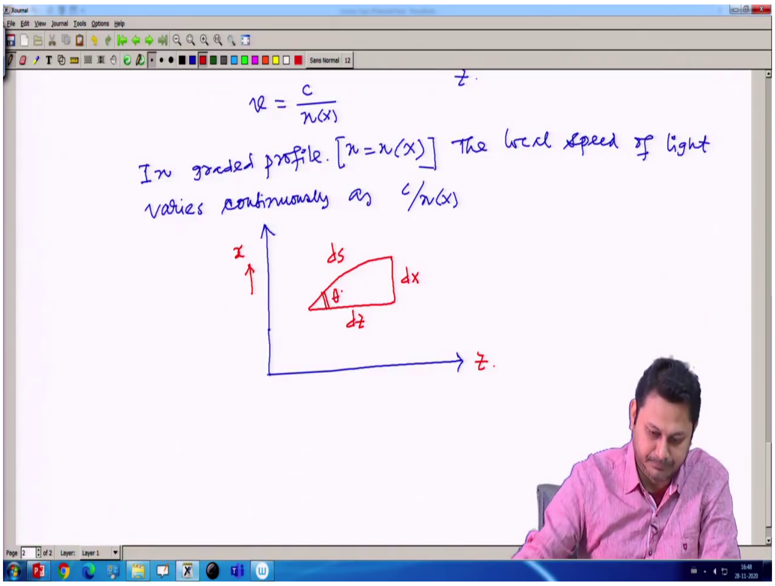

And now, this velocity whatever the velocity, it is we know. It is c divided by n. In this case, it should be n x. So, this velocity will changes it will not will be it will not going to be same in all the paths. It will change. In some point, it will be higher; in some points, it is lower depending on the value of the n x. Now, I can calculate that and in order before calculating, so let me write it. So, in graded profile, that means, when n is a function of x in two-dimension.

In graded profile, the local speed of light varies continuously as c divided by n x because at every point, the local speed of the light at every x point will vary because n x is continuously changing. In order to address this problem, let me again draw this. This is ds, this is dz, it is dx because along this direction I have x and this is z, this angle is theta.

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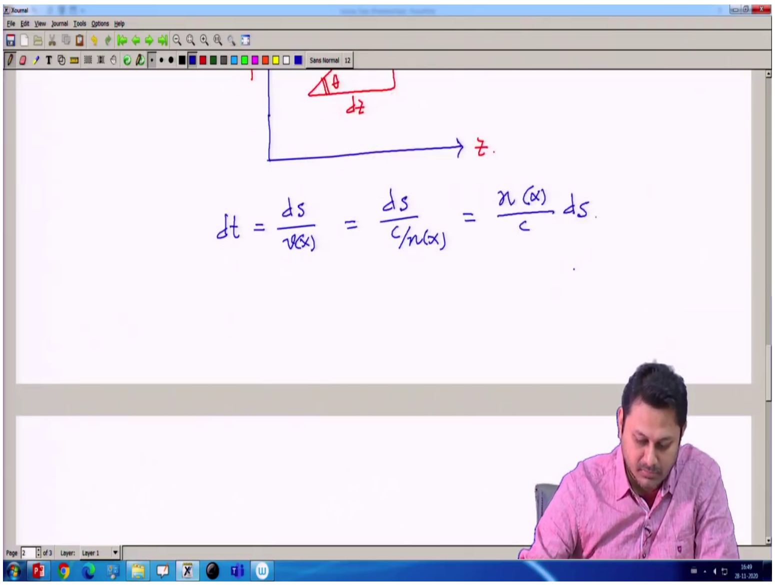

Now, dt will be ds divided by v of X. d t is the time that should take the ray going from this point to this point. In order to travel the ds path, what is the time taken by the ray is d t here. Now, this quantity is d s divided by c n x or is simply n of x divided by c ds. So, I need to evaluate this right hand side, in order to find what is the value of t, the entire time.

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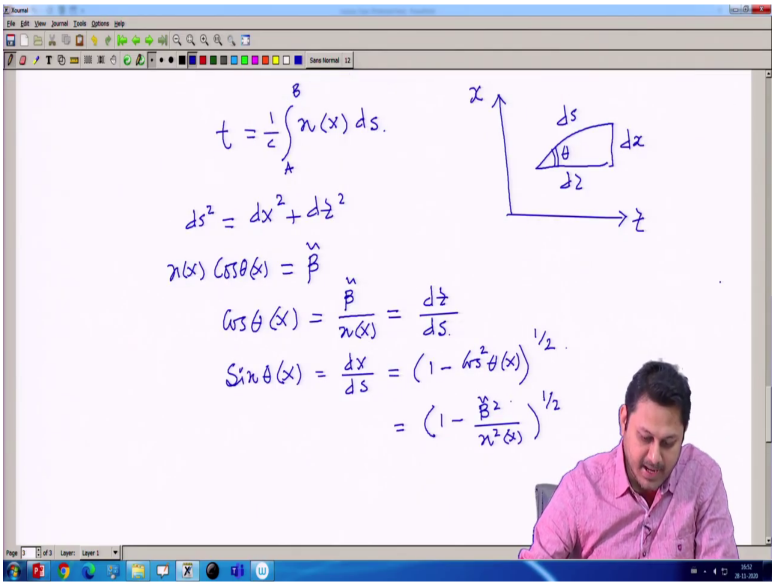

Well, now my t, as I mentioned should be simply integration of 1 by c because c is a constant should be come outside n X d s, that integration from one point to another point say point A to point B, I need to calculate. This is this should be the recipe to calculate the time. Now, d s square I can modify slightly. This expression is equal to dX square plus dz square because this is the structure. So, let me draw this side so that this is dz, this is dx, ds, this angle is theta.

So, mind it, we already have n X cos theta X is equal to beta tilde that is my ray path constant that we always have. This quantity is always true and we always have this. So, I can write cos theta X as beta tilde divided by n of X and from this geometry, I can write cos theta as dz divided by d s. Using that, I can also write sin theta X which is according to the geometry dX divided by s and that quantity is 1 minus cos square theta which is a function of X whole to the power of half and in terms of this ray path constant, this quantity is 1 minus beta square divided by this.

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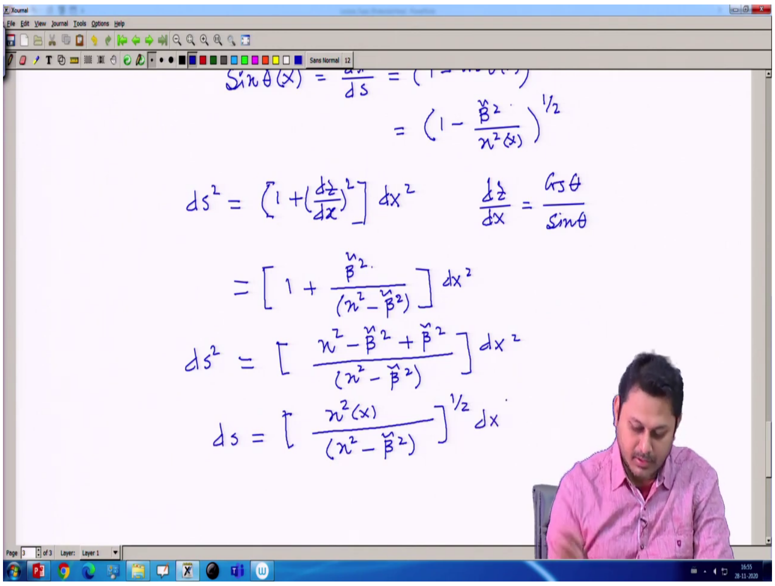

Now, already I have ds, so d s square I can write it at this ds square is 1 plus dz dx whole square of this. Let us put a third bracket; third bracket dX square. Now, this quantity d z, d z d X is nothing but cos theta divided by sin theta because tan theta is dx by dz; tan theta is dx by dz. So, dz by dz, dz by d X will be 1 by tan theta.

So, simply this is the quantity and that we already evaluated. So, this because sin theta and cos theta already we evaluated. If I put this, it should be 1 plus beta tilde square divided by n square minus beta tilde square and then, it should be dX square. Just replace this. So, sin theta is n square minus beta tilde square, which I put it here divided by n square and this is n square and cos square theta is beta square divided by n square. So, this n square theta, n square x and in this n square x will cancel out. So, eventually, I have this quantity.

This is again n square minus beta tilde square plus beta tilde square divided by n square minus beta tilde square d of X square. This is my d of s square. So, finally, I have d s is equal to n square function of X divided by n square minus beta tilde square whole to the power half dX. So, in this calculation, my goal is fine. Mind it, in this calculation, my goal is to find out d s in terms of n in terms of function of x. So, that I can I can execute the integration.

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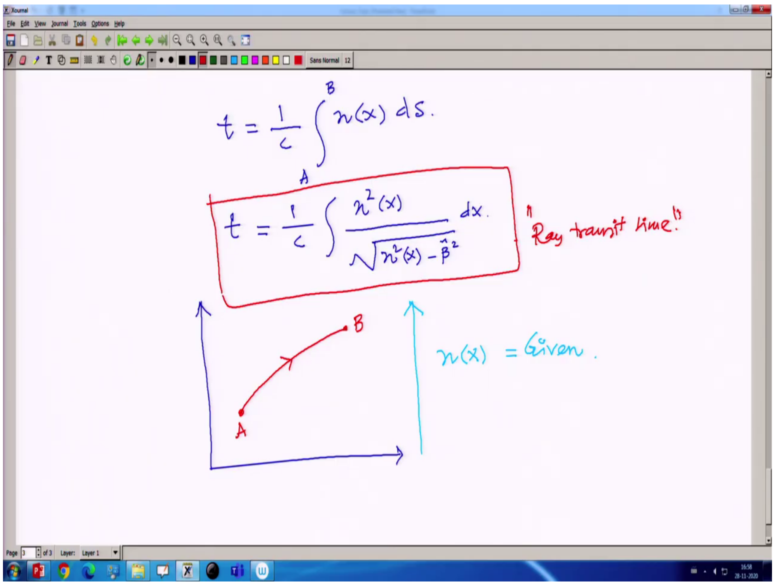

So, finally, if I put this d s because my t was 1 divided by c integration of n of x d s that that was my t from some point A to B. So, now, I know what is my ds. So, I will put this and finally, I have n square x; one n will come from d s and another n is already sitting. So, that side should be n square divided by root over of n square x minus beta square and dX.

So, now, from this expression, if a variation of n is given, then you can calculate the time taken by the ray going from one point to other another point. So, suppose I have a variation of refractive index; variation of refractive index n X and it is given, it is given; the explicit form of n x is given. The next thing one can ask that can we calculate the time taken the ray to go from the point A to B, the ray will going from the point A to B and what should be the time?

And the answer is this. Yes, definitely we can find out what is the time, the ray should take from going from one point A to B; only thing that we need to know to execute this integration as n as a function of x in explicit manner. If we know that, then we just need to integrate it. Mind it, beta square is a constant.

So, I need to integrate over x for the point A to B, final initial point and final point and then, eventually, I have the value of t in our hand which is called the transit ray transit time. So, this is basically the expression of ray transit time. So, so in the next class, we will start from this point and try to find out the ray transit point in certain cases. We start with a simple cases, but in principle you can calculate also from complicated cases; only thing is that you need to integrate it properly, the integration should be doable.

Then, you can find out what is the value of the transit time t. It is useful to understand few things that how much the ray will going to take going from one point to another point in a wave guide and if you know the what is the refractive index profile, then you can calculate by using this integration whatever is done to them. So, with this I like to can con conclude today's class. So, we will meet in the next class.

Thank you for your attention.