Physics of Linear and Non-Linear Optical Waveguides
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Module - 02
Basic Fiber Optics
Lecture - 11
Ray Equation (Contd.)

Hello student to the course of Physics of Linear and Non-Linear Optical Waveguides. So, today we have lecture 11. And, today we will going to continue the concept of Ray Equation and try to find out the ray path for a graded index profile.

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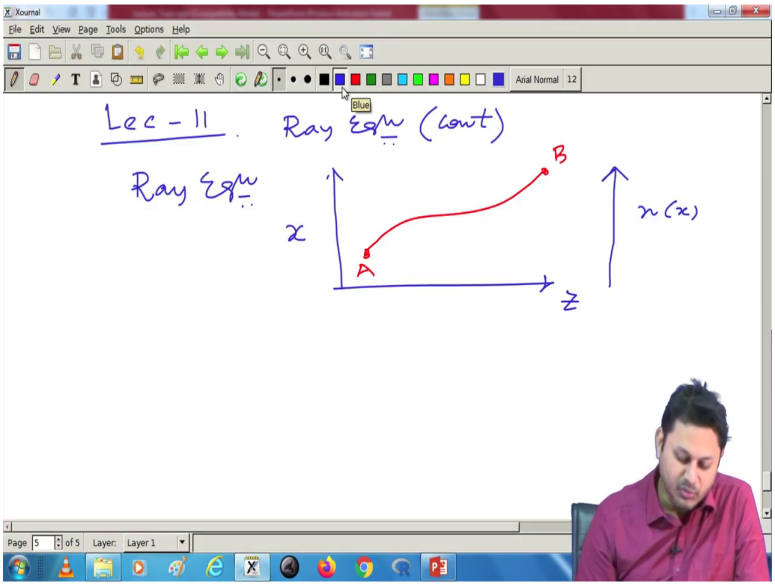

Lecture number 11 so, ray equation is continued, it is a continuation of the previous concept ray equation. Well, what was the ray equation let me write with once again. If, I have z and x a, 2 dimensional refractive index variation like this, if the profile is given. Then based on the profile a ray will follow certain path going to some point A to B. And, I can have an differential equation, which tells us how the ray will follow the path?

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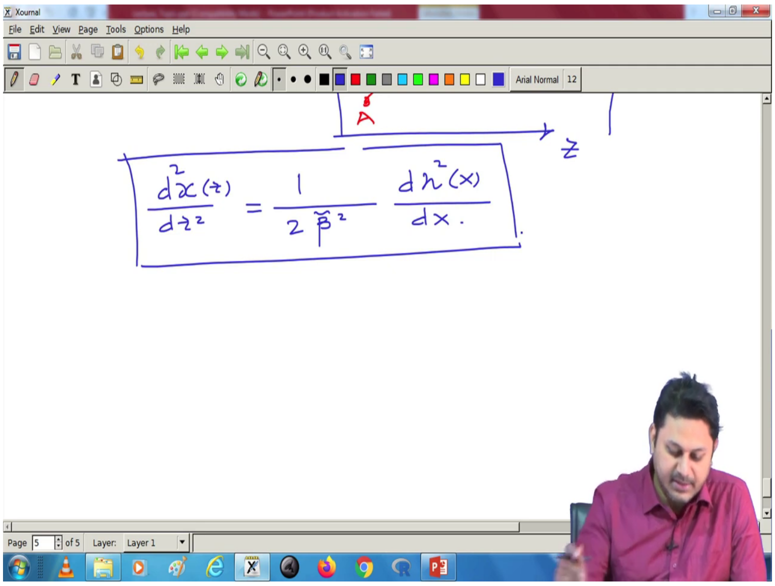

And, this differential equation was something like d 2 x d z square is a function of z is equal to 1 divided by 2 of beta tilde square. This quantity beta tilde is a ray path constant, we know that and this is a constant quantity; equal to d n square X d X. So, that was our ray equation.

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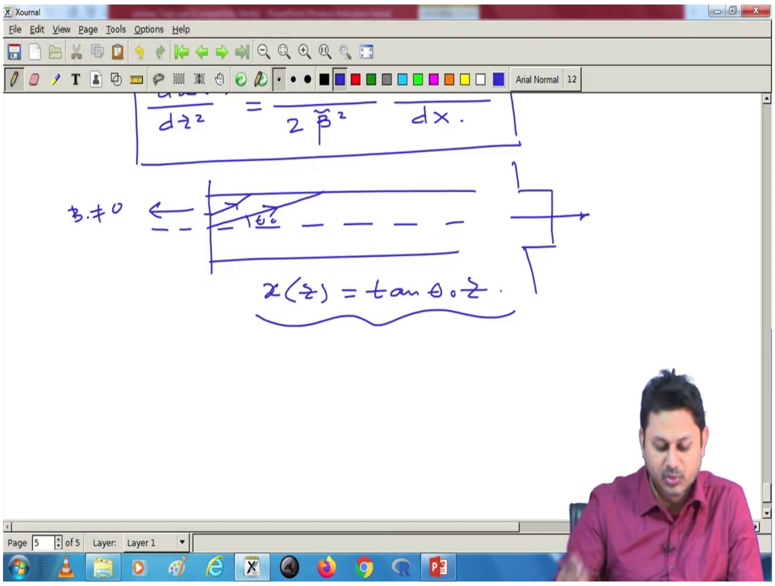

And, we will going to use this equation to find, now, a different previous in the previous class we find out the condition for the path for graded step index profile. So, that was the step index profile. And, we find from this equation, that the rays are moving in a straight line, like this. And, the expression was x function of z is simply, if this initial angle is given theta 0, then tan theta 0 multiplied by z a simple expression like this.

If I launch by the way if I launch the light in other points suppose here, then I need to add that as well with the boundary condition. Suppose I launch a light here. So, it can also follow a straight line path, but I need to add something it is like the B is not equal to 0, in this case. In this case the constant B is not equal to 0 as simple as that. You just need to put the boundary condition and find it out.

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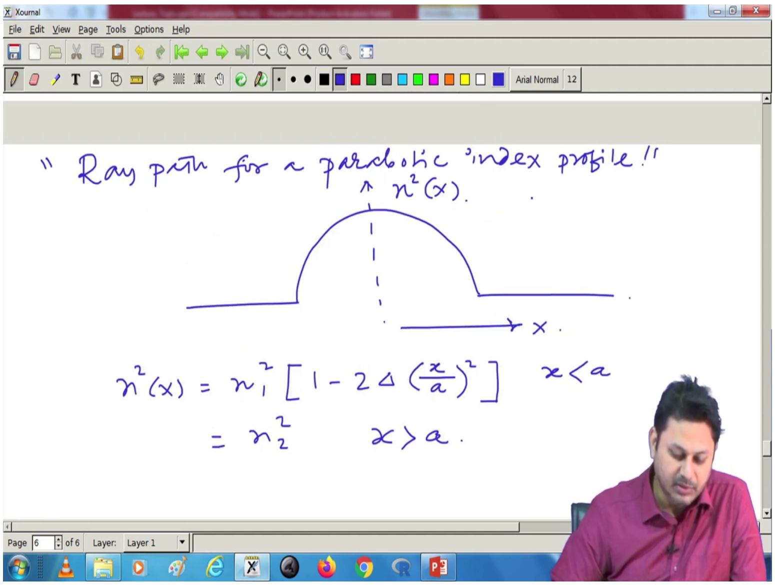

However, today we will going to find the ray path for a parabolic index profile. This parabolic index profile is not new I already introduced this in one of the classes, previous classes and the profile looks like this. Along this direction it is suppose n square x this is the direction of X. And, mathematically it was n square X is equal to n 1 square 1 minus 2 of delta x divided by a whole square for this region.

And, equal to n 2 square for this region. That was the mathematical definition of the refractive index profile for this parabolic structure, parabolic refractive index profile. Now, I know what is my n square for this profile? And, I need to find out how the ray path will look like, ray equation is my in is in my hand.

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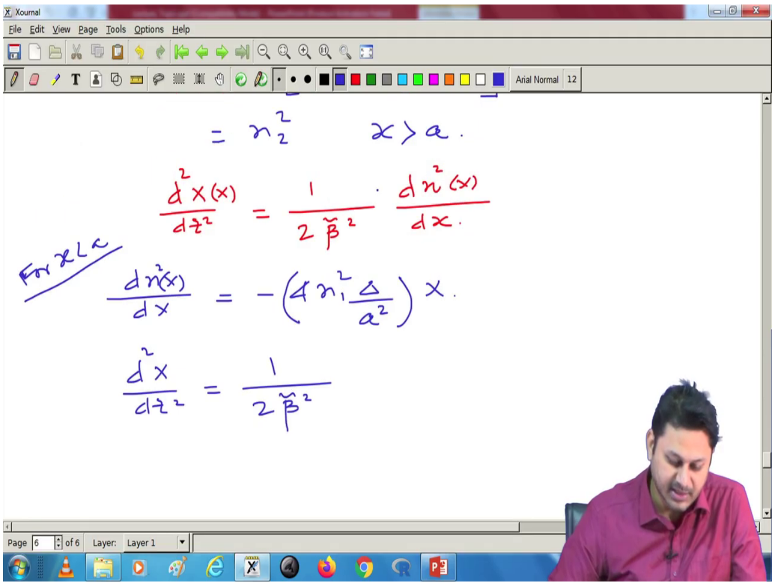

So, let me write down my ray equation once again. So, my ray equation make it red my ray equation is d 2 X, d z square is a function of x is equal to 1 divided by 2 of beta tilde square, d of n square X, then d x, which is this one. And, now n square is given as a function of x. In the step index profile it was constant, but here you can see it is not constant anymore it is a function of x, x is sitting here.

So, it is a function of x. So, first I need to calculate this. So, d of n square X, d X if I make a derivative of in for x less than a in this region, if I make a derivative. Then, this is minus n square is here. So, this derivation the derivative becomes simply minus of 4 n 1 square, because this 2 and then due to this derivative I have 1 2. So, this is 4 negative sign is there n square is here, delta is sitting here a square is also constant.

So, it should be simply delta divided by a square and then this quantity multiplied by X. So, this is a function of x so, this x will be there. So, I can put it here in the ray equation. So, the ray equation becomes d 2 X d z square will be simply 1 divided by 2 of beta tilde square and this quantity, d 2 d n square d x I just replace this here whatever the value I got.

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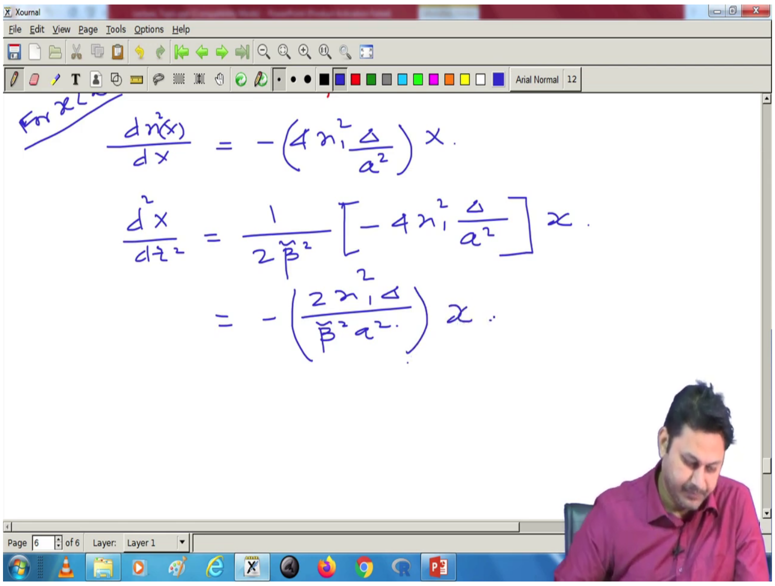

So, it should be minus of 4 n 1 square delta a square and then x, this portion is constant. So, whatever so, let me simplify this is minus of 2 of n 1 square delta divided by beta tilde square a square this entire term is a constant term multiplied by x.

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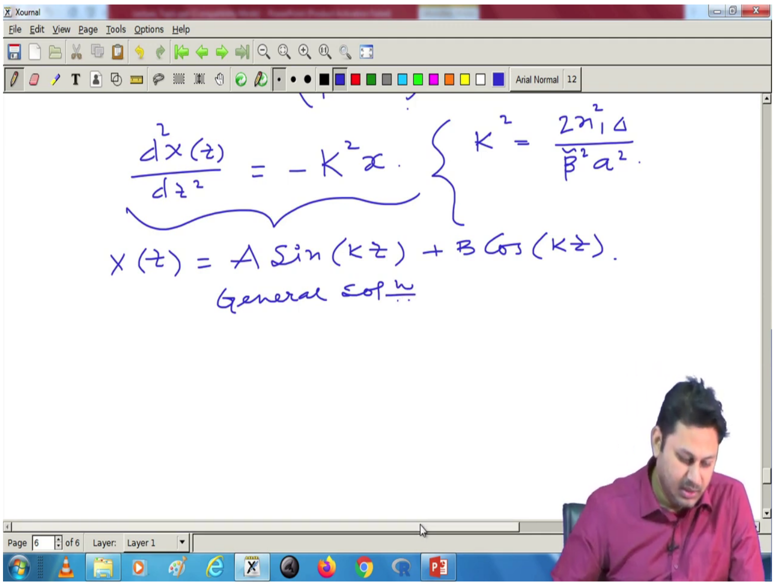

So, I can simplify this differential equation and this differential equation becomes d 2 X d z square is equal to minus of say K square, which is a constant x, where my K square is simply 2 n 1 square delta divided by beta square a square, this is my K square. So, I have a well known differential equation, this is this differential equation is well known.

And, we know that for this differential equation one can expect a sinusoidal solution. So, I can readily write down the solution here one can readily find the solution here and it looks like this. A constant A sin of K Z plus B sorry, B it should be, it should be cos of K Z. So, this is a general solution, this is a general solution. So, I have a general solution of this differential equation. Now, the next part is to put the boundary condition to evaluate the value A and B that is all.

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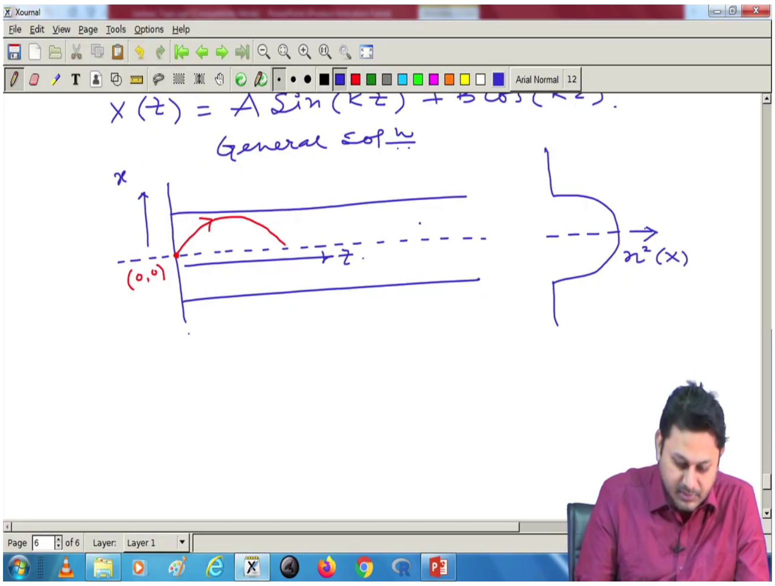

Again, I will going to draw the structure. And, here the profile, this is a parabolic profile. So, this is n square as a function of X and along this direction I have my x, and along this direction I have Z. Since, it is a sinusoidal kind of wave, I can launch the wave exactly at this point and if I launch here it should follow a path like this. Now, note it here I launch at this stuff at 0, 0 point. So, this point is 0, 0 at origin.

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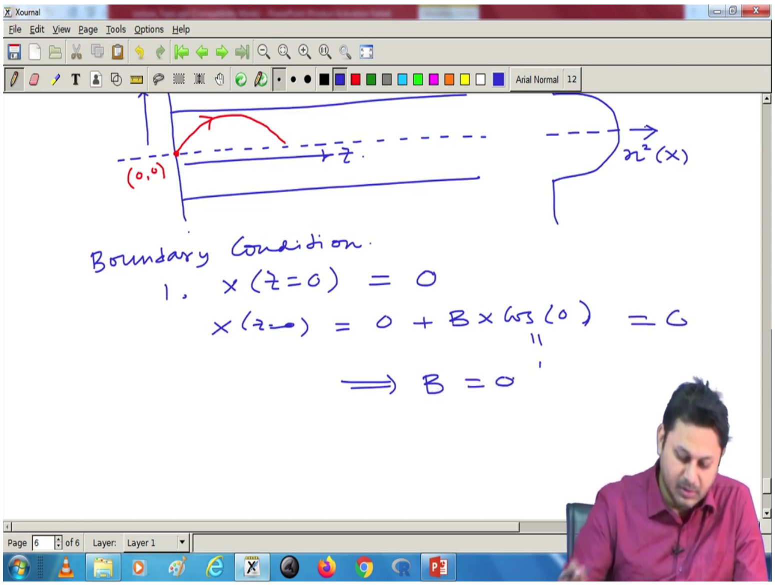

So, one boundary condition so, boundary condition 1 is X at Z equal to 0 is equal to 0. So, here I if I put that boundary condition I can have X Z equal to 0 is 0 plus B into cos of 0, I should not put degree it is simply 0, which is one, this is one. So, I can have readily B, so, this quantity is 0. So, B is equal to 0.

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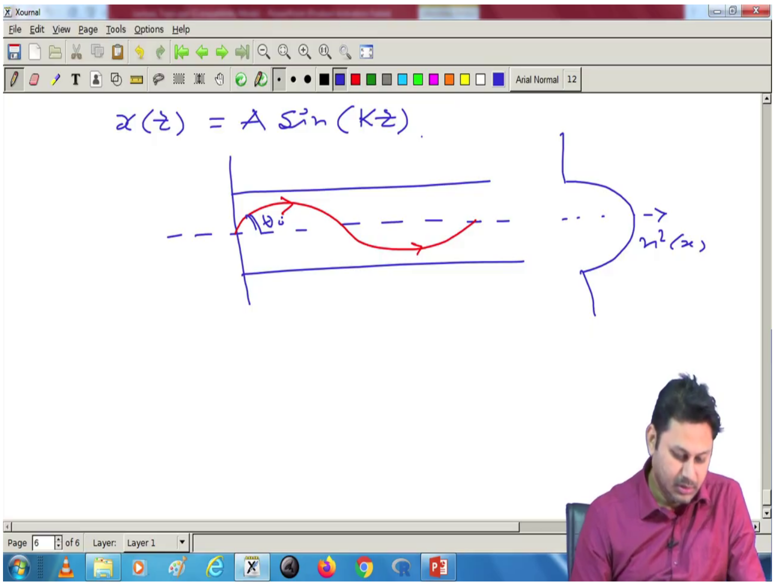

So, I simply have x is some constant A sin of K Z. So, it is simply a sinusoidal wave, if I launch, if I launch this wave exactly at this point for a parabolic structure, then it should follow a sinusoidal path. A very interesting thing that I find, for step index fiber it was a straight line, but in parabolic index profile, this is a parabolic index profile. So, the path it will that will going to follow is sinusoidal in nature.

So, it should be simply like this, for graded index. Now, again another boundary condition so, this is the amplitude A is amplitude of this sinusoidal wave. So, I can also find out this amplitude A in terms of another boundary condition. So, in the exactly in the previous way, if this angle is theta 0.

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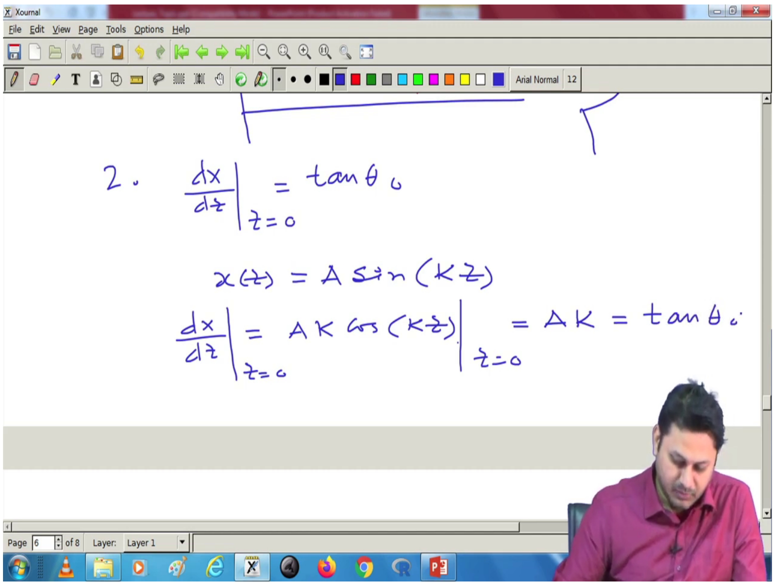

Then, I have second boundary condition that, d of X d Z at Z equal to 0 point; that means, at here is simply tan theta 0. This quantity if I calculate from whatever the expression I have. So, now, my x, now my x Z is A of sin of K Z. So, if I make a derivative with respect to if I make a derivative with respect to Z it should be like A of K of cos of K Z. And, I want to evaluate these things at Z equal to 0, at Z equal to 0, this value is simply AK and that thing at Z equal to 0 according to my boundary condition is tan theta 0.

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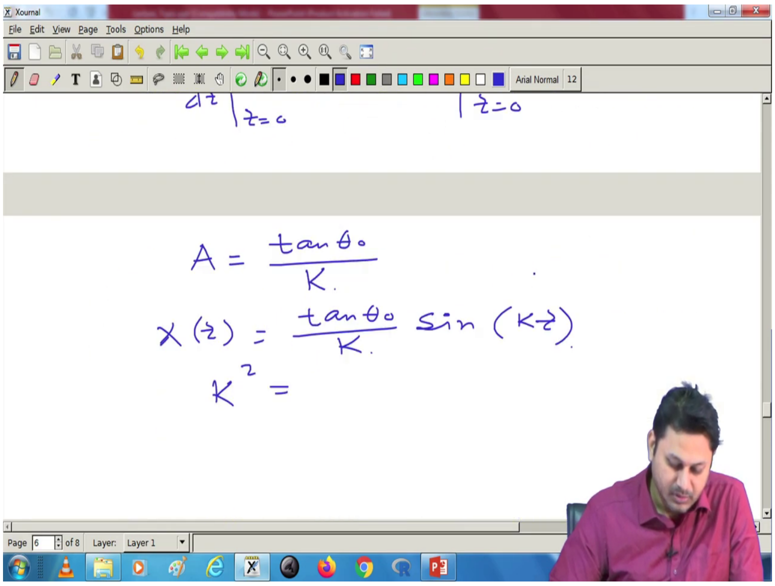

So, I can figure out what is my A with this boundary. So, my A will be simply tan of theta 0 divided by K. So, in totality my path will be something like tan theta 0 divided by K sin of K of Z. So, I can have a set of sin sinusoidal wave by the way K value I should also write it. So, K square is I what I calculated here. So, let me go back to the value it is 2 n 1 square delta divided by beta tilde square A square here.

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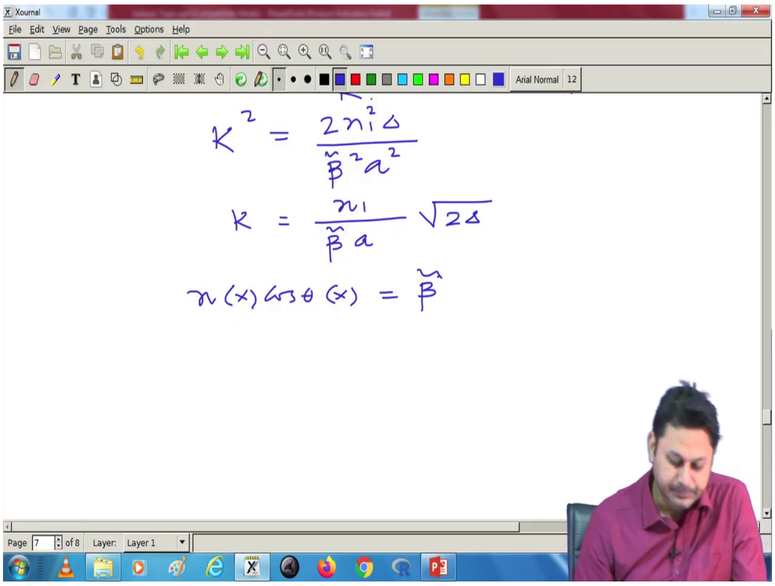

So, it is 2 n 1 square delta divided by beta tilde square a square. So, my K is simply n 1 divided by beta tilde a, then root over of 2 of delta, this is my K. And, beta tilde by n 1 can also be written in terms of theta 0, because I know that n of X cos of theta X is beta tilde that we know.

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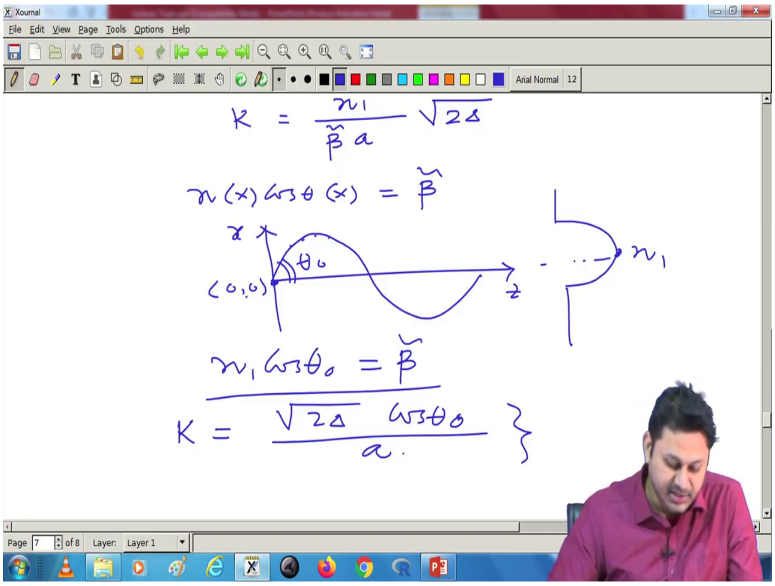

When, we launch the light, if I go back to the picture once again the ray will follow this kind of path. So, here this initial angle is theta 0. So, and this value, the peak value, if I draw the parabolic profile so, at this peak value it was n 1. So, n 1 cos of theta 0 is equal to beta tilde, because beta tilde will be same for all the points, it should also have the same value at this point, which is the origin 0 0, this is x and this is z. So, this condition is also valid at x equal to z equal to 0, or x equal to 0 point.

So, my K, I can write my K as n 1 divided by beta tilde I just replace to the function cos theta. So, it should be, it should be root over of 2 delta, then cos theta 0, cos theta 0 divided by a, that is my K in terms of theta 0. Because, my goal here is to find everything in terms of theta 0, because theta 0 is a initial angle that is given to me.

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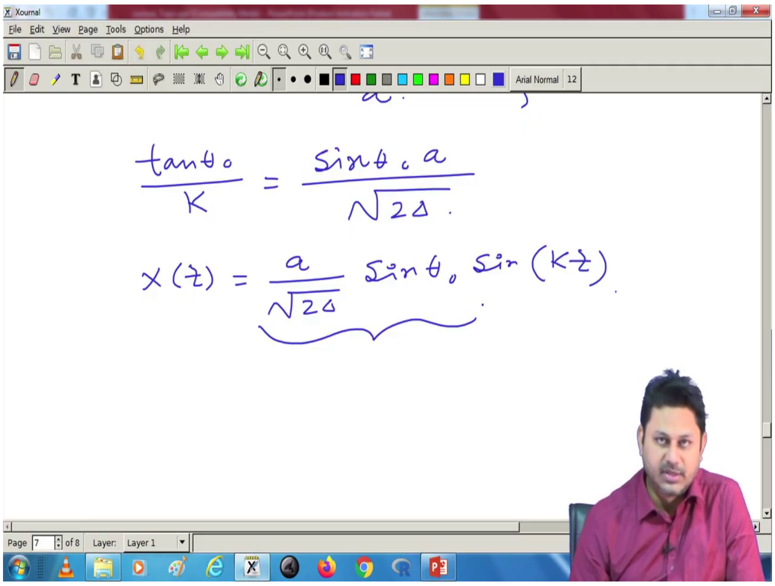

So, tan theta 0 divided by K, that term we had earlier tan theta 0 divided by K, which is A. So, that thing will be simply when I put the value of K 1 cos theta will cancel out. So, it should be sin theta 0 multiplied by a divided by root over of 2 delta.

So, my final expression for the ray, X Z is equal to a divided by root over of 2 delta, sin of theta 0 sin of K Z. So, it should be a sinusoidal wave; it should be a sinusoidal wave, but at the same point it should have an amplitude here and this amplitude will depend on the initial value of theta 0. If, I change my theta 0, I have different paths.

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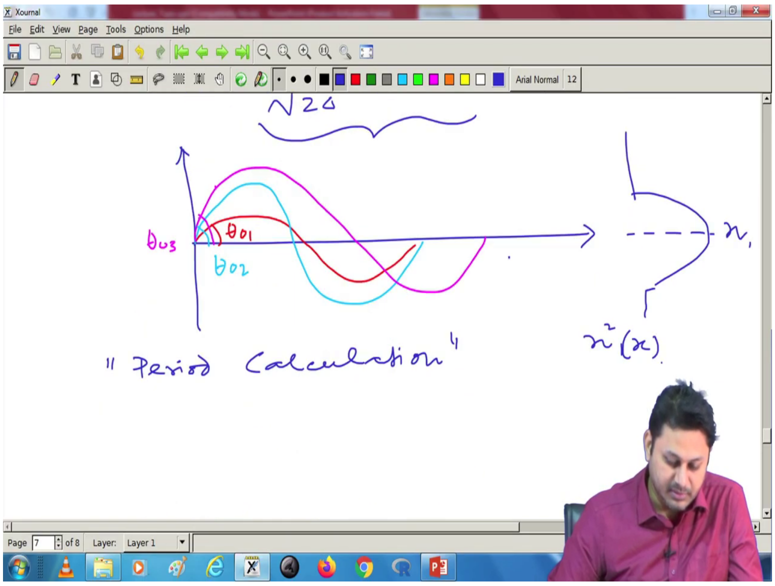

So, let me draw finally, how the path will look like for a parabolic? So, I need to draw this drawing is slightly I need to make it better. So, this is the profile I have and here at this point I have n 1. So, this profile is this is a n 1 square x profile, parabolic profile and the path the ray will follow is this. This is one path that I follow, another path let me draw it another line, and another path it can follow something like this.

Mind it this depending on the value of this angles for example, here this angle is say theta 0 1, this angle is initial angle is theta 0 2, for another ray this angle, this initial angle for this ray is theta 0 3 and so on. The amplitude will also change because in the amplitude I have sin theta 0. So, if theta 0 1 is less than theta 2 0, then there will be this ray will this amplitude is small compared to that one.

And, also in the inside the period, I have the value, because K is cos theta 0, so that should also play a role in the period, so, that we need to find out. So, let us try to find out the period, because I draw that in just to show how this thing will look like, but at the same point we have to be careful about the period.

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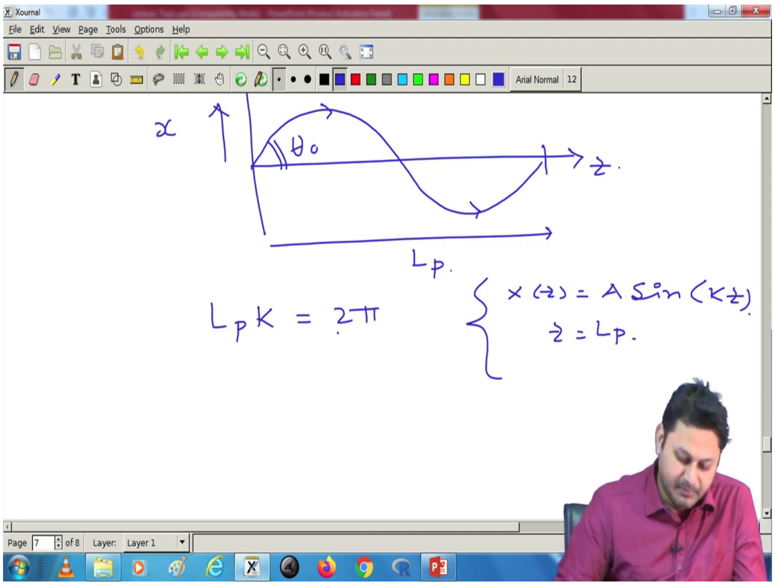

So, "period calculation", the period of the sinusoidal path I need to calculate what is what should be the period? Then, I can rectify this. So, this figure I think there is I mean when I draw a larger amplitude, then theta 0 is high. When the theta 0 is high, then in the period if I look carefully, in the period K is sitting here.

So, in the period what happened? The K value, which is associated with this cos theta, this cos theta will decrease, because theta if theta increase the cos theta will decrease. If, this is decreased, then I can have, I can have the period a larger period, so that we will going to find out. So, let us do that quickly. So, this is the path and for this path I have this theta 0 here.

This is x and this is z and the period is from this point to this point, this is the period actually. And, I write this length as L P, because L P is the period of this sinusoidal ray path. So, simply L P multiplied by K should be 2 pi because my path equation it is sin K Z. So, my path equation let me write it once again, what was my path equation? So, X of Z is simply A A I only evaluated, it is already there I write it in this form this one.

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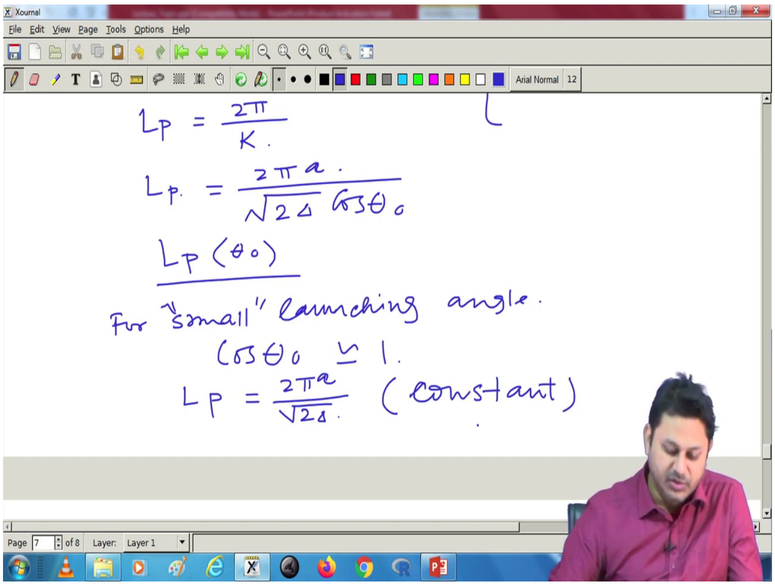

So, period will be at Z equal to L P I have the period. So, my period L P will be simply 2 pi divided by K. Now, K value I already calculated here. The K is this one root over of 2 delta cos theta 0 divided by a. So, I will put this value here. So, 2 pi it should be 2 pi a divided by root over of 2 delta cos theta 0.

Now, you can see the L P is a function of theta 0. So, L P is eventually a function of theta 0 that is the important thing. So, if I launch a light; if I launch a light with different theta 0, then L P, the period will also going to change. If, theta 0 is small, then I have a larger period if theta 0 is large, we have a smaller period and so on.

So, for launching for small launching angle; however, for small launching angle this cos theta 0 will be nearly equal to 1. Then, my L P will be 2 pi a divided by root over of 2 delta. Now, you can see when the launching angle is small so, that the cos theta 0 is of the order of 1, then this L P value become a constant quantity. It will not going to depend anymore with the launching condition.

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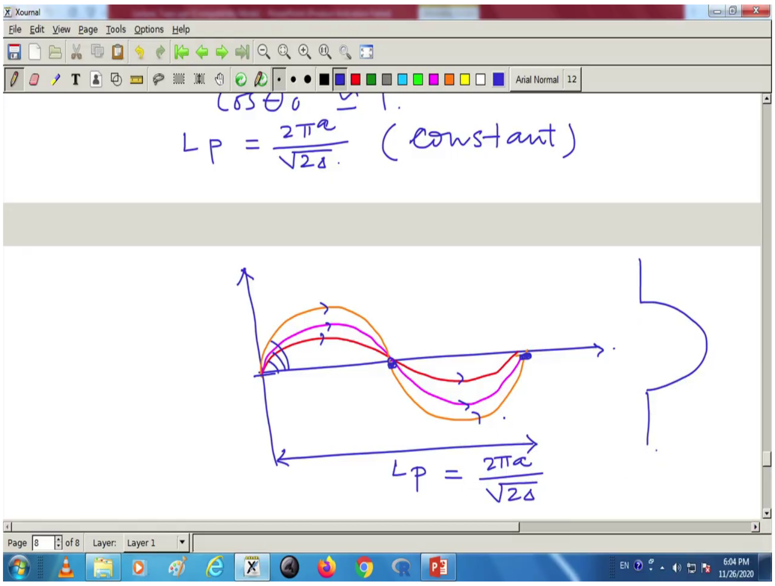

So, under such condition what happened, all the rays will follow the same period. So, for small launching angle so, I can write it, I can draw this in this way. This is one ray for nearly equal launching angle, I have another ray like this, for another launching angle I can have another red line with the same L P. So, if this is small I mean very; however, in according to my drawing it not it is not looking as a it does not look very small.

But, if it is small then what we find an extra information that L P for all the rays is constant. And, whose value depend on only the value of the fiber parameter, which is the delta and a. It does not depend on whatever the angle, what angle I am in which angle I am launching. In all cases it will meet here and meet here.

So, when the rays are meeting at same point; that means, the dispersion value is less. So, that is an added advantage, that we can reduce the dispersion by making a refractive index profile like a parabolic profile. So, that if I launch a small angle, then these rays are having a almost similar L P.

So, with this node I will like to conclude today's class. So, thank you for your attention. In the next class, we will learn more about these ray paths, whatever the ray paths we have that, what should be the time it should take to reach certain points and when the refractive index is constantly varying. Then, how it reaches to a certain point what should be the expression etcetera, we will going to calculate in the next class with this note.

Thank you very much for your attention and see you in the next class.