Physics of Linear and Non-Linear Optical Waveguides

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Module – 01
Basic Optics
Lecture – 01
Wave Equation, Maxwell's Equation, Plane Wave

Welcome student to the first lecture of the topic Physics of Non- Linear and Non-Linear Optical Waveguides. Today, we will going to have our lecture 1. In lecture 1, we will going to cover three important topic. One is Wave Equation, then Maxwell's Equation and the concept of Plane Wave. However, these topics are not very new, but still I feel before going to our main course which is physics of linear and non-linear optical waveguides.

It is useful to brush up all the basic concept that we know regarding the wave propagation or wave motion. So, electromagnetic wave motion is mainly governed by Maxwell's equation. So, let us start with that Maxwell's equation and then gradually we build up the topic one by one.

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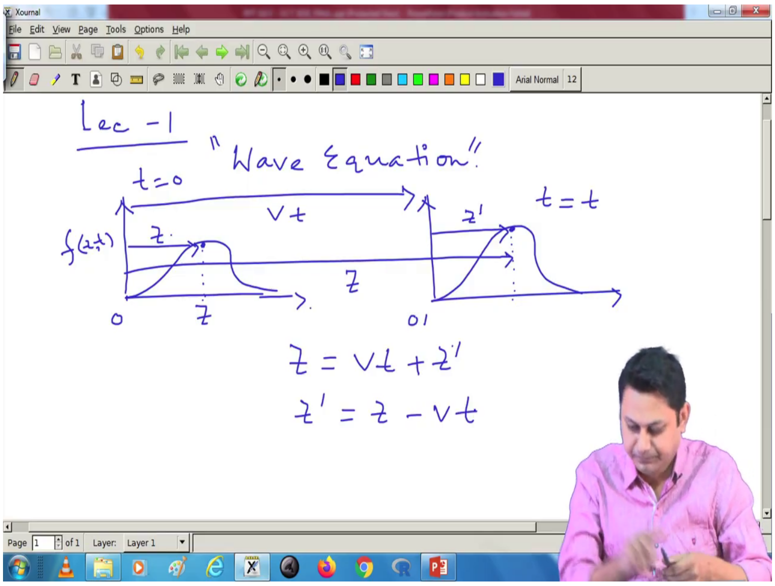

So, the first thing today we have lecture 1. So, first topic that we will going to learn today is "wave equation". Well in the wave equation let us have a coordinate frame and I have an arbitrary wave like this. This is my f z t along y axis and along z axis I have z along this axis I have z, this is say at o frame.

I have another frame say, o prime and now wave is moving. So, this wave whatever the wave we have here in frame o, which is at rest. Now, this wave is moving at certain velocity. And, this o prime frame is attached with that wave, which is moving.

So, if I now try to find out the coordinates of some particular point over the wave for example, this one. So, this distance from here to here is my z, in this frame the same point what we have is z prime. And, now it is moving with a velocity v so, at some time t, so, it suppose this is at t equal to 0. So, these things at t equal to some point t. This from here to here this distance should be v t, where v is the velocity of the wave as well as the frame, because the frame is moving at same velocity that of the wave.

Now, if I try to understand what is the coordinate of this point? In this frame as I mentioned in a fixed frame this coordinate is defined as z. So, this is my this distance is eventually z. So, now if I try to understand these three coordinates, whatever the coordinates we have z z prime and then another coordinate t is associated with that.

So, with simple expression I can write z is equal to v t plus z prime. That is the coordinate z that is associated with another coordinate z prime which is moving with the velocity V. Now, if I want to write z prime in terms of z and t it should be z prime is equal to z minus v t.

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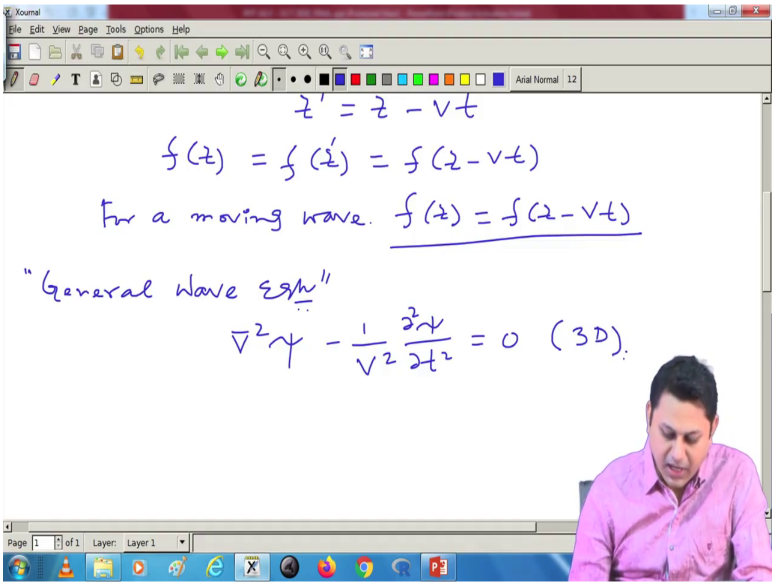

So, I have my function f z here, which is equivalent to f z prime and that thing is equivalent to f of z minus v t. I can write this entire thing in this way. So, for a moving wave when the wave is moving for a moving wave I have a relationship like, f z in one dimension, like f z minus v t. That is a relation I just find from whatever the simple geometry I have. So, let me try to understand once again. We have a fixed frame which is o and the wave is initially t equal to 0 the wave is at this fixed frame o.

Now, wave is moving with respect to time it is moving to some other place, in one dimension along z direction suppose. I have another frame which is also moving with the same velocity that of the wave. And, this velocity o prime, in this frame o prime the wave is not moving it is stand still.

So, now what I am doing I try to find out the relationship between the coordinates in prime frame and non prime frame. In prime frame the space coordinate can be related to the non prime frame z, and t and that is the relation we have z prime is equal to z minus v t.

But, the function is same whatever the value of the wave we have here the same value, we are having here. So, these functions are same. So, for a moving wave I have a relationship with the coordinates and this relationship can be represented in this form. So, that is the first thing, we have in our hand. Next thing, we need to write the general wave equation, so, "general wave equation".

So, if I write the general wave equation it should be something like this. This is the form and this form in 3 D. I have a more simpler form in 2 D as well and in 1 D as well, if I write in 1 D it should be something like this.

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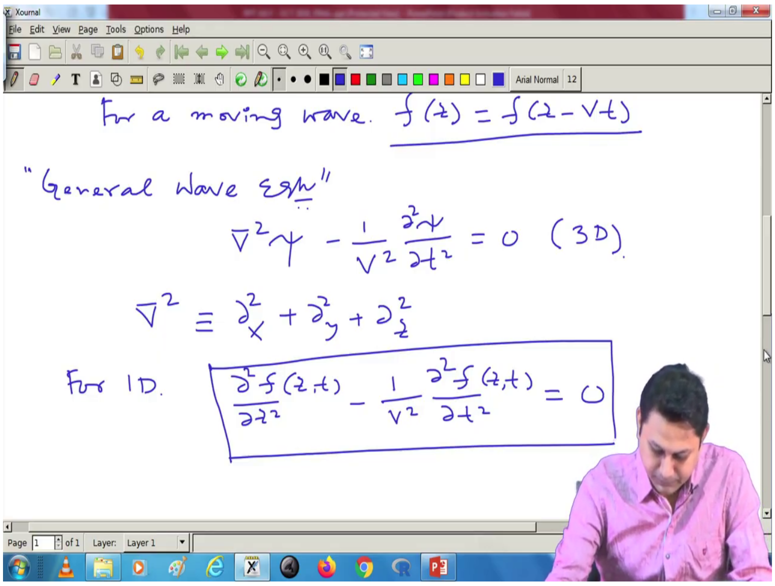

So, this is a Laplacian operator grad square operator. We know I can expand this Laplacian operator like del x square plus del y square plus this is a partial derivative with respect to x, y and z. That is basically this operator in Cartesian coordinate system.

So, in one dimension if I just want to find out what is the form of this wave equation in just one dimension to make life simple. I can have a equation so, for 1 D, I should have an equation like, d 2 f function of z t divided by d z z square minus of 1 by v square d 2 f this is again function of z, t divided by z t square equal to 0, this is for one dimension. So, let us. So, for one dimension I have this equation. So, now, let us concentrate on this equation and try to find out that what kind of solution we can have for that ok.

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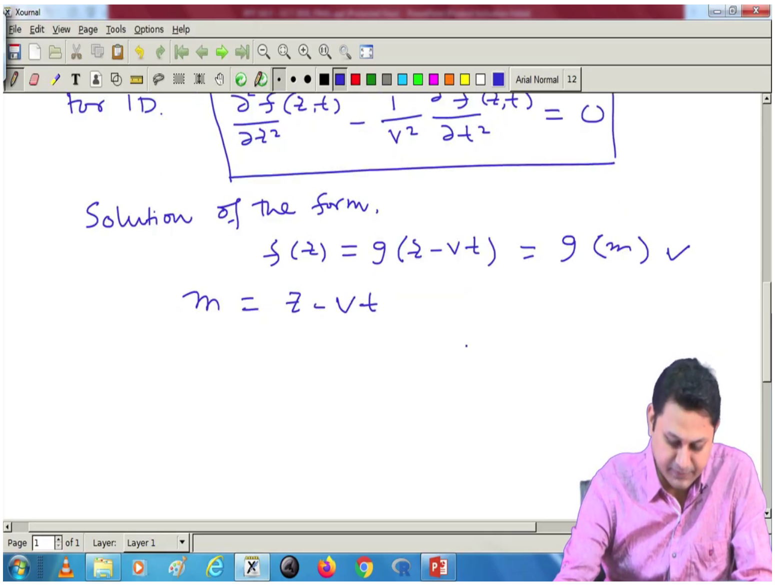

So, let us consider the solution of the form f of z is equal to g some g function z minus v t. So, if you look very carefully to this solution you find that I am using the same concept that I just developed. That if the wave is moving in certain reference frame, then the coordinate can be represented in this compact form the space coordinate can be represented in this compact form.

So, I am going to use this particular form as a solution and check that whether really it is a solution or not. If, it is a solution, then we can confirm our self that ok, whatever the equation I have is really give us the solution, that is a moving wave. So, I write it as another variable m. So, my m new variable is z minus v t. So, this is my new variable. Now, I am going to use this expression this relation whatever the relation to find out the solution. So, I demand that this is my solution.

So, what I will do I will put this thing into the equation and try to find out really it is a solution or not. So, in the equation if I go back to the equation my first term is the second order derivative of f with respect to z, second order derivative with respect to z.

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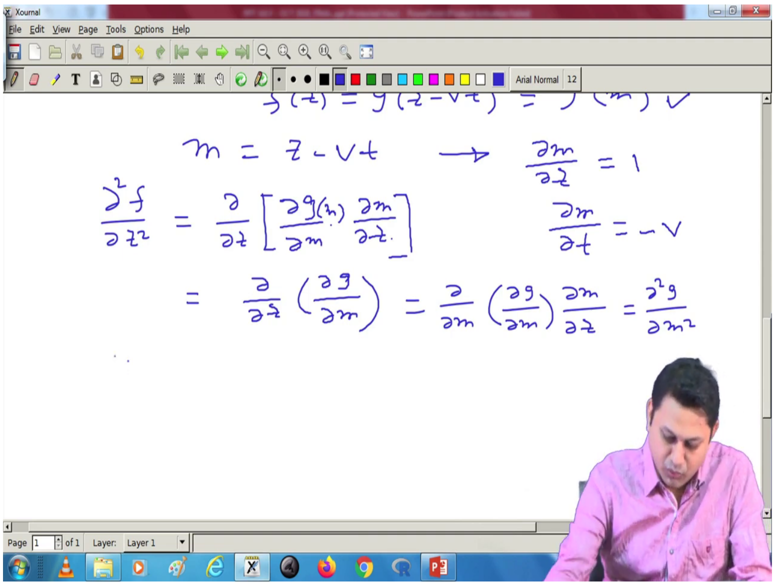

So, let me start with this term del 2 f del z square, which is del 2 del z square and if I put the solution there. So, let me do that one by one, then it will be easier. So, then what I do I put the solution. So, it should be del g del m and because g is a function of m. So, I am using the chain rule and del m del z. So, I write this del 2 f del z square and I then I put the solution g m, after putting the solution g m because g is a function of m. So, I need to derive in this form, using the chain rule.

Now, what is this del m del z? So, I have a relationship so, I can find it out set that del m del z if I do I will find this is 1 from here. Also del m del t is minus of v that is another piece of information I have from this relation whatever the relation I have with this. So, this thing I can write at del del z and this is 1.

So, simply del g del m. In the similar way the first derivative I can resolve like, del del m, then del g, del m, and then del m del z. This quantity again is 1 del m del z. So, I can write it is del 2 g del m square. So, this quantity del 2 f del z square become del 2 g del m square. Now, the next thing I need to do is to find out, what is the derivative of the function with respect to t?

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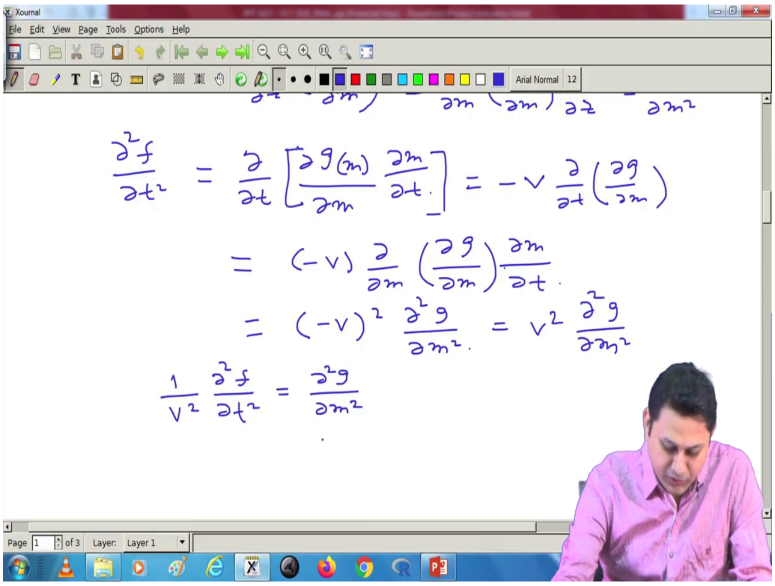

So, I have del 2 f del t square in the right another I mean as another derivative in the wave equation. So, I can exactly the similar way, I can resolve it like del del t and then g is a function of m. So, I put the solution g here. So, I need to derive with respect to m and here I need to put like this term del m del t.

Now, this quantity I already derived here. If you look carefully del m delta t is equal to minus v I already derived. So, this thing I can use and this value is minus v and I have del del t of del g, del m. Now, I use the second derivative as well.

So, I have 1 minus v then again del del t, I can replace as del del m using the chain rule and I have like, del m del t del m del t again minus v so, another minus v should be here. So, I have minus v square of that and then del 2 of g del m square. So, this thing is nothing, but v square multiplied by del 2 of g del m square. So, I can write this as 1 by v square del 2 of f del t square is equal to del 2 of g del m square.

So, previously I find del 2 g del m square if you remember previously I find del 2 g m square here, as del 2 f del z square this one. So; that means, this quantity is equivalent to del 2 f del z square, which is my wave equation.

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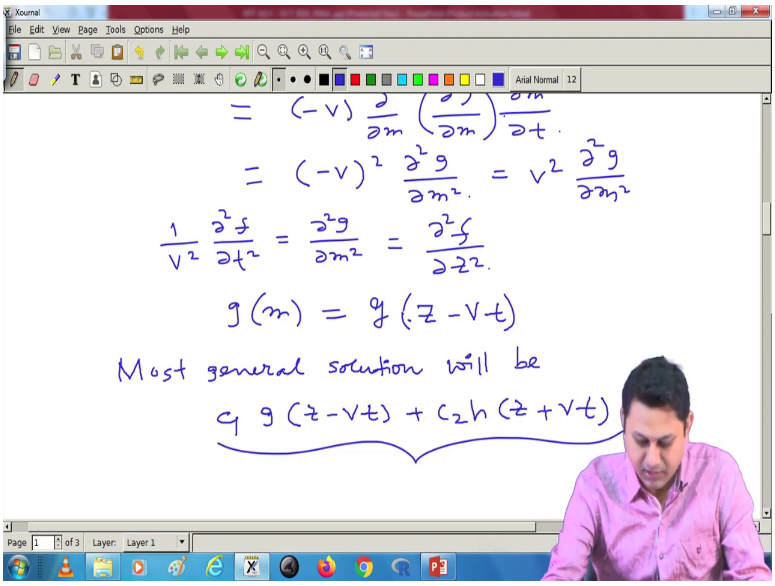

So, in the wave equation, so, using g I find that this is basically the solution of the wave equation. And, what is the form of the g? The form of the g is g m, if I expand, if I write the full form of m, so, g equal to a sorry. So, I just need to write the value of m here. So, it is like z minus v t. So, we find that this specific form become a solution of equation, which is wave equation and this is indeed forming a wave, it is a propagating wave.

So, if I want to find out the most general solution, then the most general solution can be represented as a linear combination of these things. Because in the when the v when the wave is propagating in this direction, I say this is v. If the wave is moving in the opposite direction it should be minus v.

So, if I start with that with opposite the wave is moving in the opposite direction still I can have a solution. So, that is why the most general solution will be the most general solution will be something like c 1 g of z minus v t plus c 2 say h of z plus v t a new function.

So, this is the form general form of a solution of a wave equation and we know beforehand. Because, just after this thing we will directly jump to the Maxwell's equation and try to derive the wave equation from the 4 Maxwell's equation.

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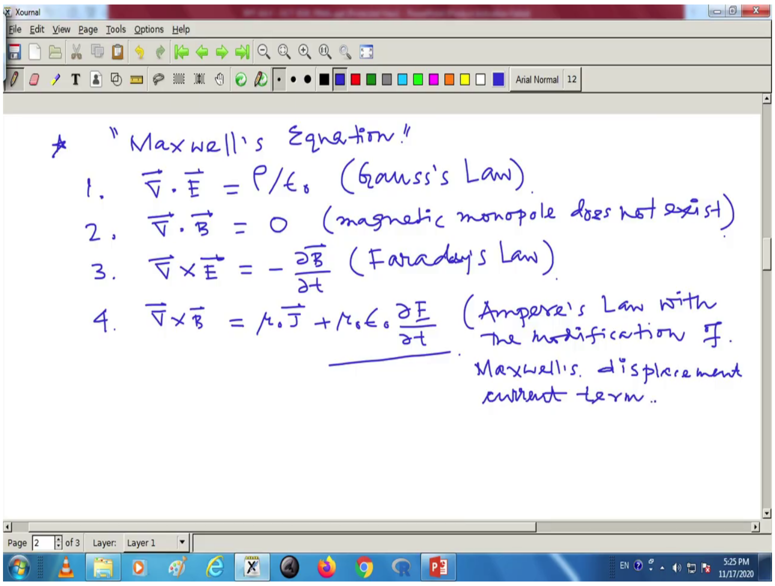

So, let us try to do that now. So, directly let us directly jump to Maxwell's equation. So, the next thing is Maxwell's equation. So, let me quickly write the 4 Maxwell's equation that we know, 1 grad dot E is equal to rho by epsilon 0, that is my first Maxwell's equation which is nothing, but the Gauss's law. 2nd, Maxwell's equation is divergence of B is equal to 0. And, this basically tells us that magnetic monopole does not exist, there is no magnetic monopole.

Third equation is curl of E is equal to minus of del v del t, which is our Faraday's law. And, finally, we have curl cross B is equal to mu 0 J plus mu 0 epsilon 0 del E del t. So, this is Ampere's law with Maxwell's modification. This is the modifications of the Maxwell the next term mu 0 epsilon 0 that we know this is the modification of the Maxwell's. So, this is the modified Ampere's law. Ampere's law with the modification of Maxwell's current displacement current term.

This last term you know that is coming due to the displacement concept of the displacement current. So, Maxwell's this is Maxwell's modification of Maxwell's displacement current term ok.

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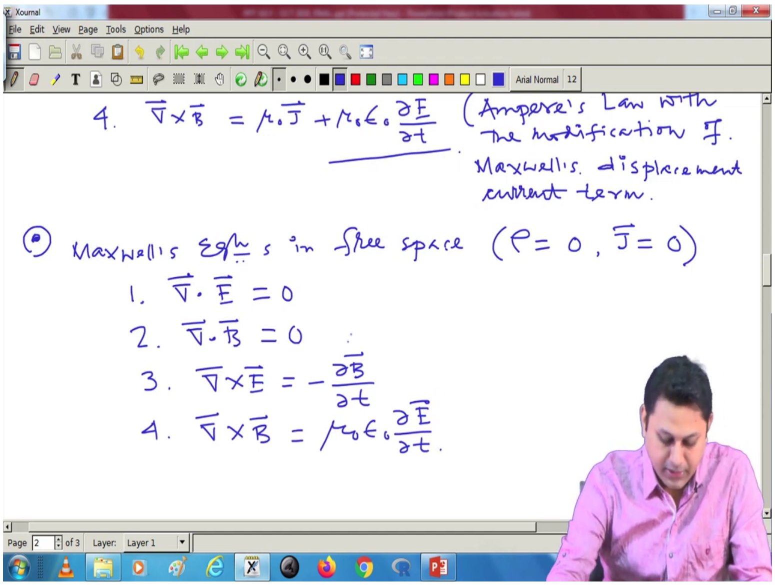

So, we know all these 4 equation. The next thing that we do is to find out the same equation in free space or vacuum. So, let me write Maxwell's equations in free space, Maxwell's equation in free space. In Maxwell's equation in free space I should write as dot E equal to 0, because in the free space what is the special thing? I have rho equal to 0 and the current density the current density and the charge density both are 0; that means, all the source terms are not there.

Then, the next term grad dot B is equal to 0 and then the next term curl cross E is equal to minus of del B, del t. And, finally, curl cross B is equal to mu 0 epsilon 0 del E, del t. Well, now after having this 4 equation, I can play with that and try to find out the Maxwell's wave equation which is easy.

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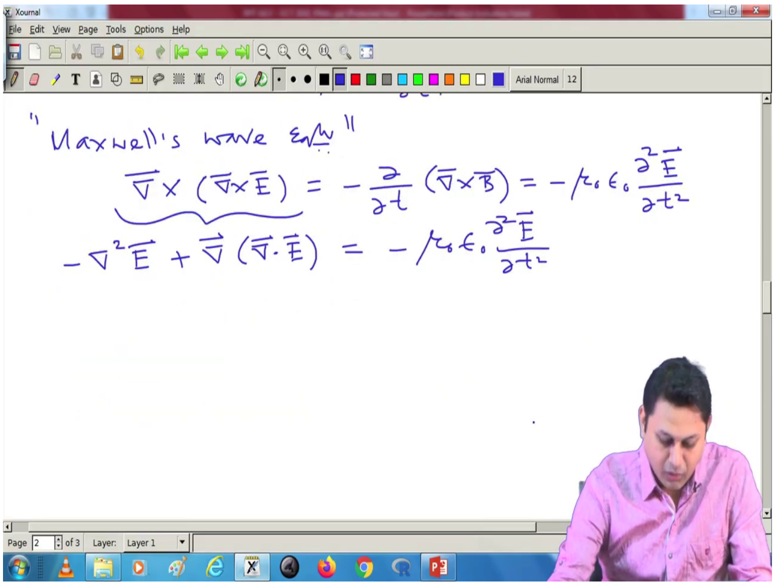

Maxwell's wave equation. What is the wave Maxwell's wave equation? I will, so, I will take this equation 3, whatever the equation I have in 3. So, Faraday's law and I will make a curl over that, curl of curl of E is equal to the right hand side also I like to make a curl.

So, if I make a curl, it will be something like curl cross B which is again curl cross B using the equation 4, I can write it is minus of mu 0, epsilon 0, del 2 E vector, del t square. Left hand side I know curl of curl of E, the vector identity. And, the vector identity is minus of grad square E plus grad dot E, gradient of divergence of E, that is equal to minus of mu 0, epsilon 0, del 2 E del t square.

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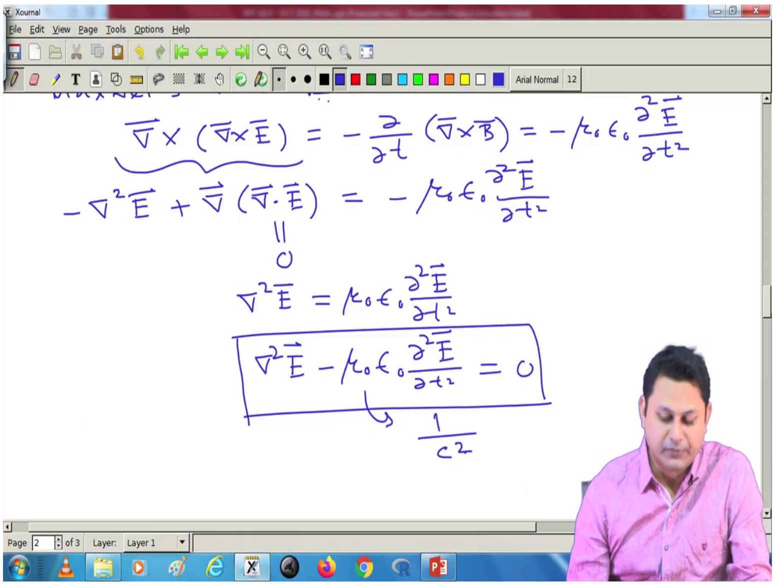

So, now, we know that this term is 0, because grad dot E is equal to 0. So, eventually whatever the equation I have is grad square E is equal to mu 0, epsilon 0, E del t square. Or grad square E minus mu 0 epsilon 0 del 2 E del t square is equal to 0. So, this equation if we look carefully is nothing, but a wave equation.

And, that is the greatest achievement we have due to the modification that Maxwell make in the third, fourth equation Ampere's law. And, we have readily we have the equation, which is a wave equation and if you calculate this value, it will be very close to this. So, the velocity term 1 by c square.

So, as soon as Maxwell got this equation and he calculated this value mu 0 epsilon 0. And, find it very close to 1 by c square you readily understand that the light is nothing, but electromagnetic wave.

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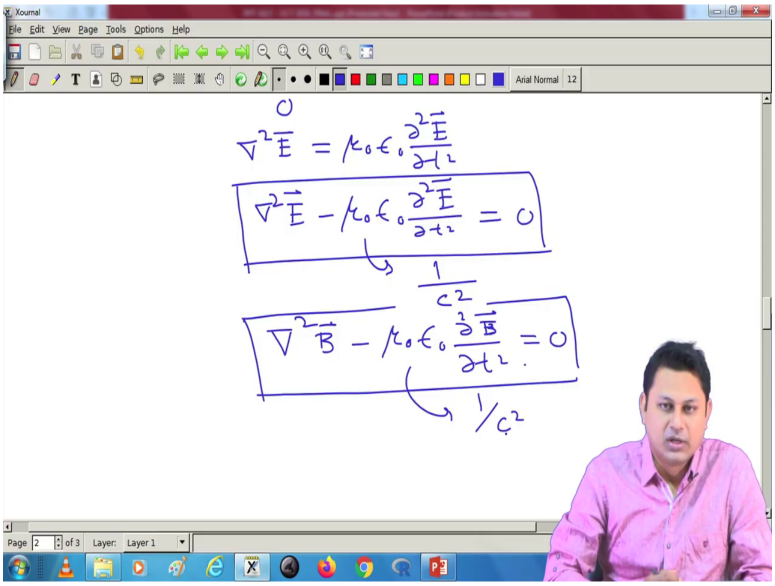

In the similar way, if you calculate the B you will find that this equation for B is coming like this. So; that means, electric field E and magnetic field B both are following the wave equation in a similar fashion. And, in both cases the velocity term 1 by c square is coming in this way. So, that means, electric and magnetic field both are propagating with the same velocity and this velocity in the vacuum is c the velocity of light. With this note today I like to conclude.

So, today we learn the concept of wave equation once more and then derive the Maxwell's from the Maxwell's equation we derive the wave equation for electric field and magnetic field. In the next class we will try to find out the solution and then try to understand what is this, what kind of solution one can expect? Obviously, it should be a wave kind of solution which we call plane wave.

So, with this note let me conclude thank you student for your attention. So, let us meet in the next class and try to understand more about the Maxwell's equation.

Thank you.