Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 59
Vortices, SQUID, Quantum Supremacy & Qubits

Hello and welcome, let us continue our discussions on the superconductivity and towards the end we will discuss the topological states of matter that is something I alluded to in the beginning. These are extremely new topics of almost at the level of research, but one should have some idea of what it is and I will also give references from which you can pick up the basics of it.

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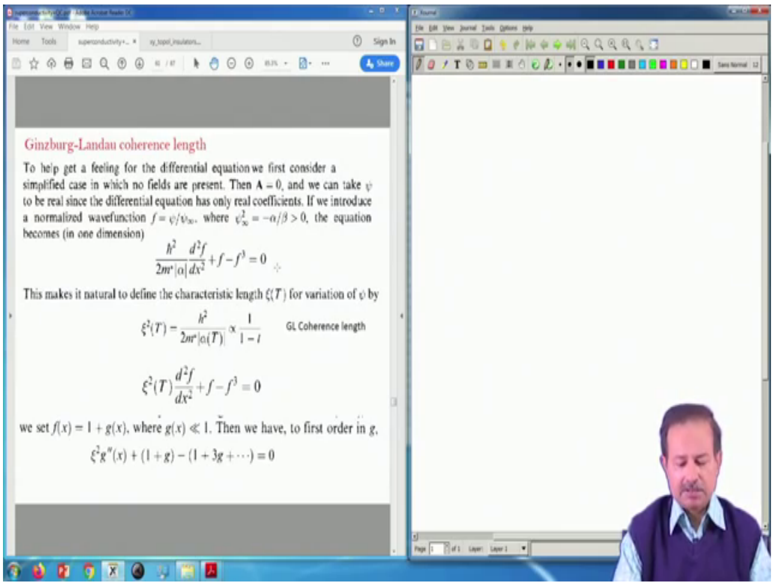

So, we were continuing with Ginzburg - Landau theory of superconductor, in particular we were interested in finding a problem the solution of a problem which was undertaken by Abrikosov in former USSR using Ginzburg - Landau theory. The problem is this that if you have strong magnetic field and if you have a superconductor typically these are called type - 2 superconductors.

And in these superconductors is it possible that you have a state where the magnetic flux penetrates the superconducting system and by the by definition at the core of those fluxes the magnetic order the superconducting order parameter goes to 0 so, it becomes normal. Now if one wants to work that out one starts from this phenomenological theory called Ginzburg-Landau theory which we outlined how does it work.

And it works by a purely phenomenological idea that close to the critical point the transition the free energy can be expanded around the normal state value. That means, the state in which there is no order the free energy of that plus a polynomial and some gradients and suitable terms that account for all the energies that are there in the system due to the formation of the order. And if the ordered state has lower free energy then the transition takes place and that is how it is set up and that is what gave us this equation in the absence of fields.

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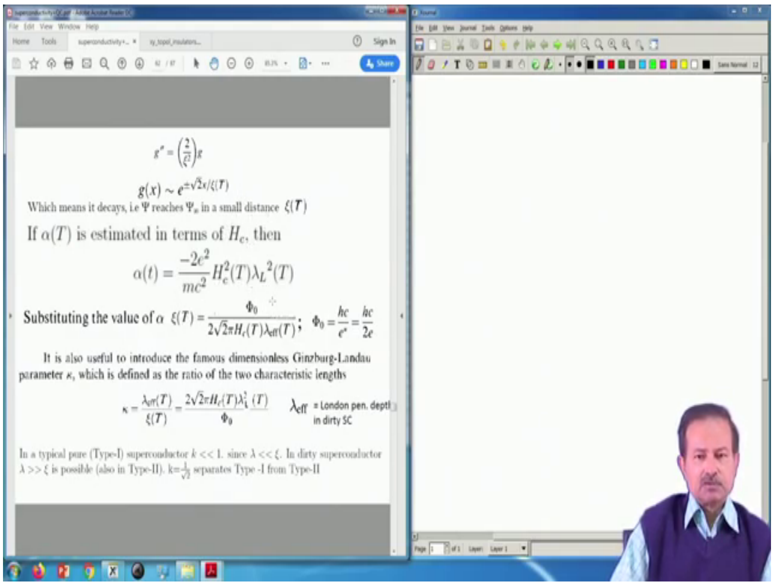

And from which we actually found out another length scale which is Ginzburg - Landau coherence length. And then of course, we already had the effective London penetration depth which in a disordered system for example, gets modified considerably from the original lambda sub L that we got from London equation London penetration depth.

So, this effective lambda and the ratio of this Ginzburg-Landau coherence length decides which is the way the superconductor will behave whether it will be type - I or type - II.

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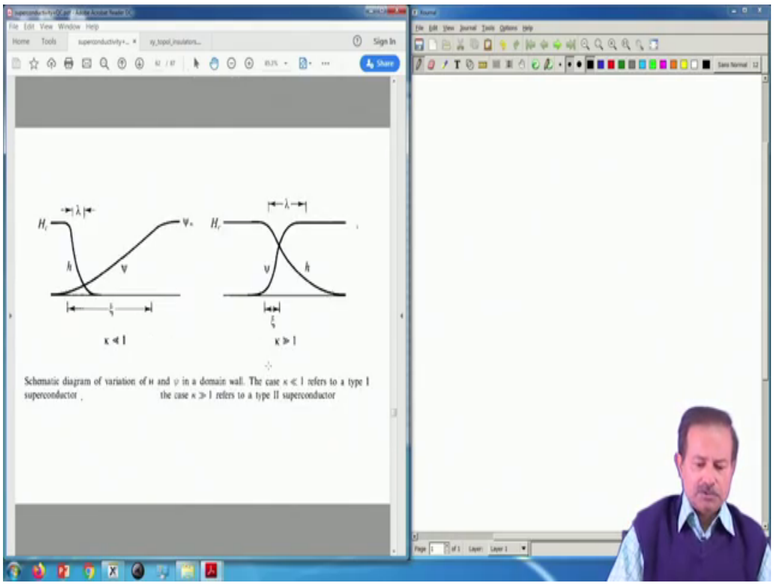

So, then these are the length scale in 2 different cases in type - I the c is large and in type - II c is small compared to lambda.

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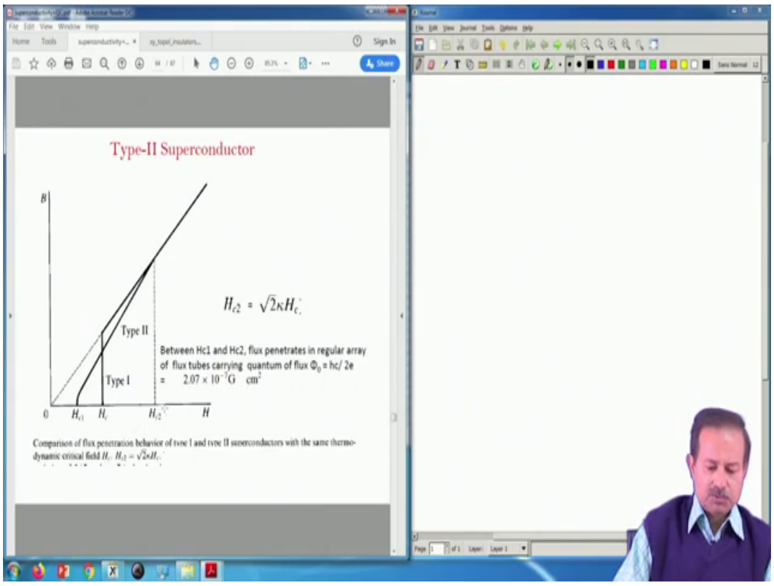

And this is how the B versus H curve will behave in type-II superconductor for example, and in type-I type-I of course, you have only 1 H c at which superconductivity is destroyed and type-II has two H c values H c 2 and H c 1.

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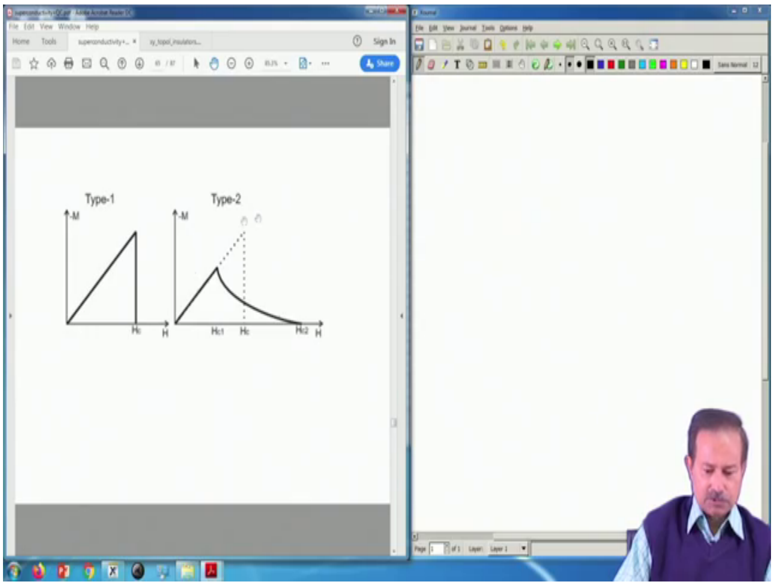

Next figure will show it better here it is. So, the M is proportional to H is inside a superconductor minus M is proportional to H and that is what is plotted. So, it increases and then at H c it drops to 0 and the super conductivity vanishes so that is what type - 1 it does. And in type- 2 it linearly goes up of course, but then there is a flux penetration field penetrates the superconductor. So, there is a slow variation and then finally, at H c 2 the superconductivity vanishes finally, H c somewhere between H c 1 and H c 2.

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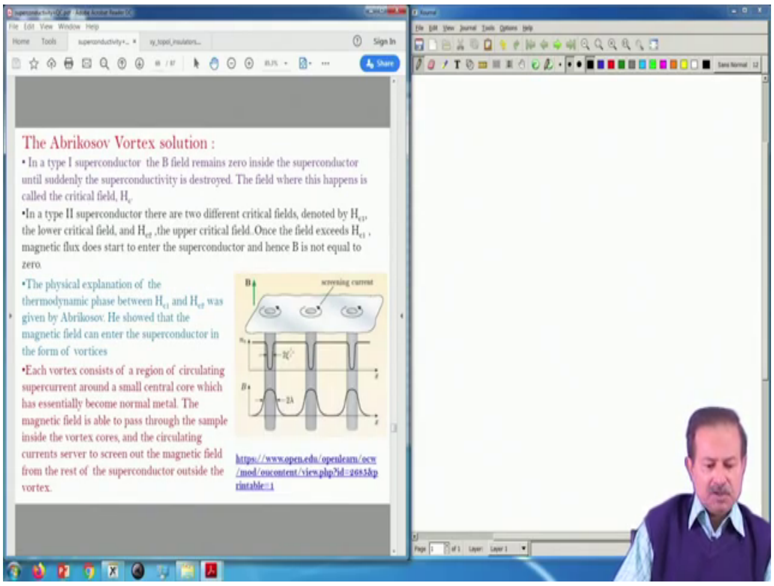

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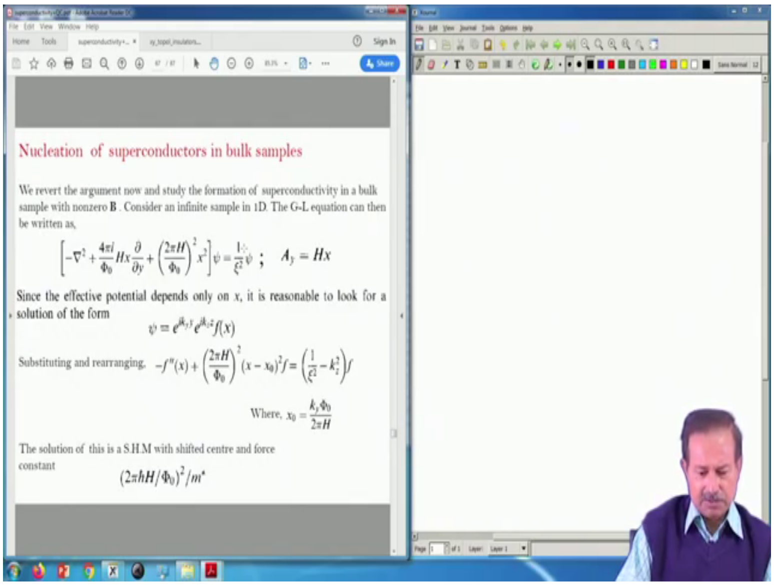

And the vortex solution we outlined as to how Abrikosov got it is very much like the principles the mathematics is almost the algebra is more or less similar to what one did in quantum hall effect integer quantum Hall effect and this is the Landau famous Landau problem that you put a good electrons in a perpendicular magnetic field strong magnetic field.

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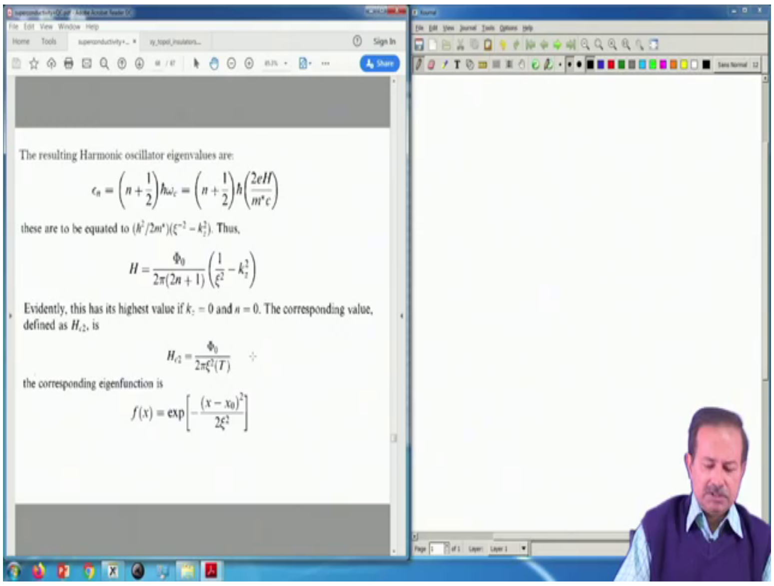

And then of course, you find the solution and the idea is that you from that you find out what is the first value of magnetic field at which superconducting solution becomes non zero. So, it is called nucleation of superconducting conductivity and it starts at a particular value of H which we called H c 2 if you remember that picture this is where the superconductivity starts.

So, psi becomes nonzero here ok. So, that is how one finds out the coexistence coexisting state, but that just tells you that there is a coexistence of both H and B and psi mod psi square in this formulation inside the superconductor ok.

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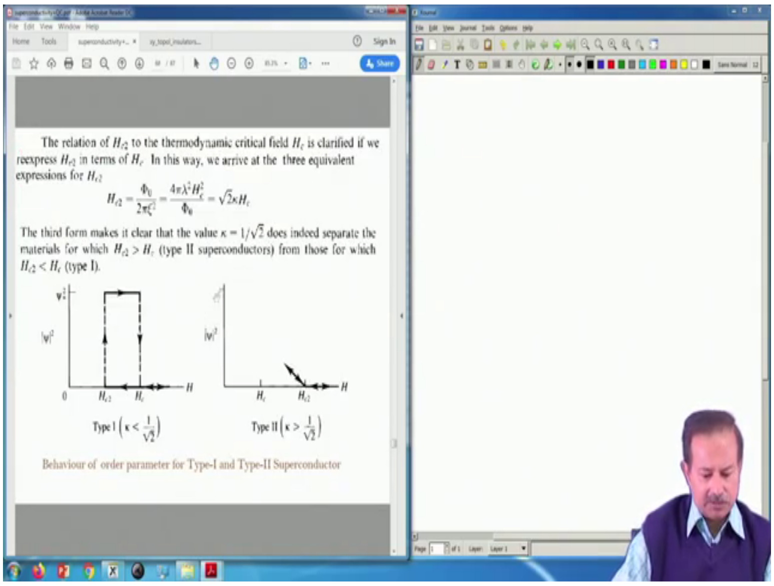

That is that was the theory and then therefore, of course, you can estimate the H c 2 in terms of H c and find out it find that one finds that it is root 2 kappa times H c. So, kappa equal to 1 by root 2 as we showed earlier is the point at which the two kinds of superconductors are distinguished. So, that is the distinction point between two superconductors type -I and type – II. So, all these came out of this calculation of Abrikosov, but he went further.

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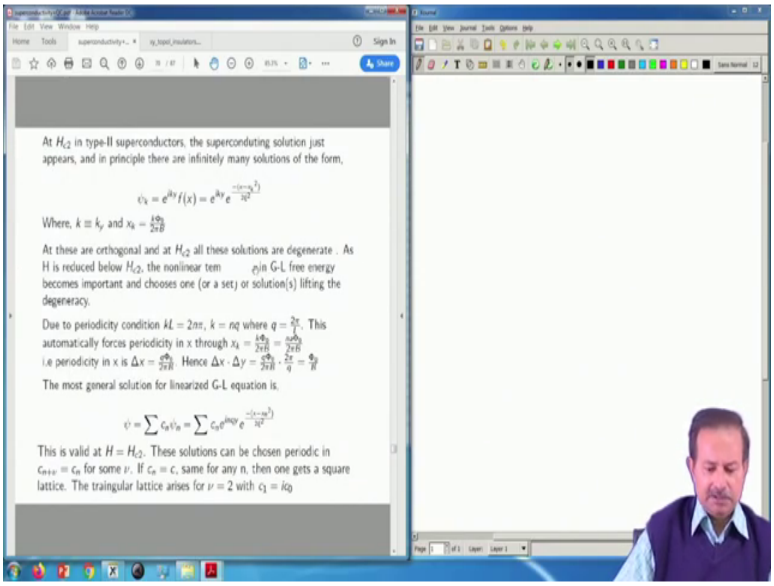

And then he of course, found out something remarkable that he showed that these superconduct this the flux that goes inside the superconductor goes in quantized vortices and these quantized vortices are like flux tubes and they are not random inside the superconductor, they have a lattice structure; that means, they are like crystalline lattice.

So, if you look at from the top of the superconductor where the flux tubes are coming out you will see that they have a regular pattern and what is what was found that these pattern is a triangular lattice and that is what he actually showed. So, this calculation is a bit involved I am not getting into the details he did fantastically intuitive calculation.

And found out that of course, he is there was slight numerical mistake he first found a square lattice structure, but later on it was corrected to a triangular lattice. These this is a remarkable calculation and if you want you can look up in the literature , but that the facts is that the flux does not penetrate randomly it penetrates in tubes the magnetic field and in the flux goes through tubes in quantized vortices and. So, the flux value is quantized and the structure of these tubes is not random it is a triangular lattice ok.

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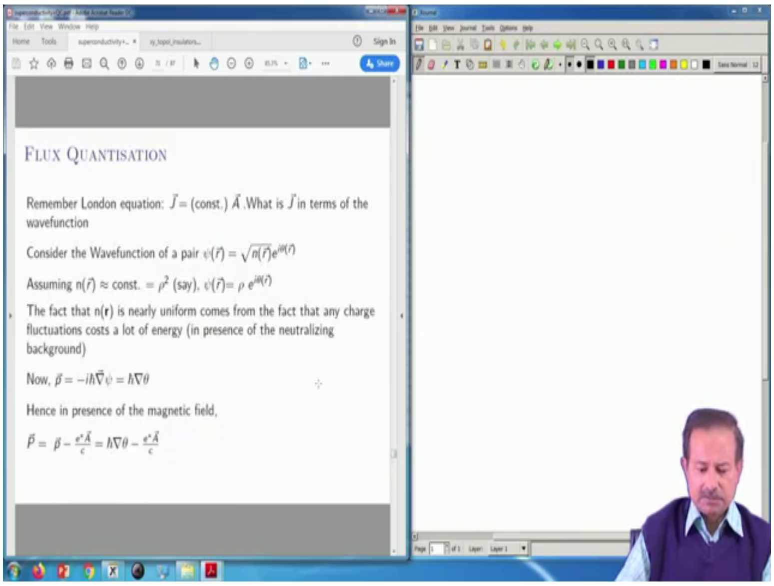

So, flux quantization is a bit I mean this is almost straightforward to see why it happens and if you remember your London equation you can you see that it is proportional to A and you can write down these wave function the superconducting wave function for the pair for pair wave function as root over n r e to the power i theta r.

And then you can that is the. So, n r is kind of rho square and then you can write psi r is rho e to the power i theta and then of course, just use London equation and assume that the. So, amplitude of the superconductive wave function is more or less well formed and it does not vary strongly. So, grad write psi as a grad of psi as h cross grad theta so, that is how the p operator will be.

So, p minus e A e star A e star is the charge of the pair by c is h cross grad theta my the canonical momentum in presence of a magnetic field will just become this h cross grad theta minus e A by c. This we have also seen in the current expression from the Ginzburg - Landau equation and of course, London equation says that this must be 0 alright.

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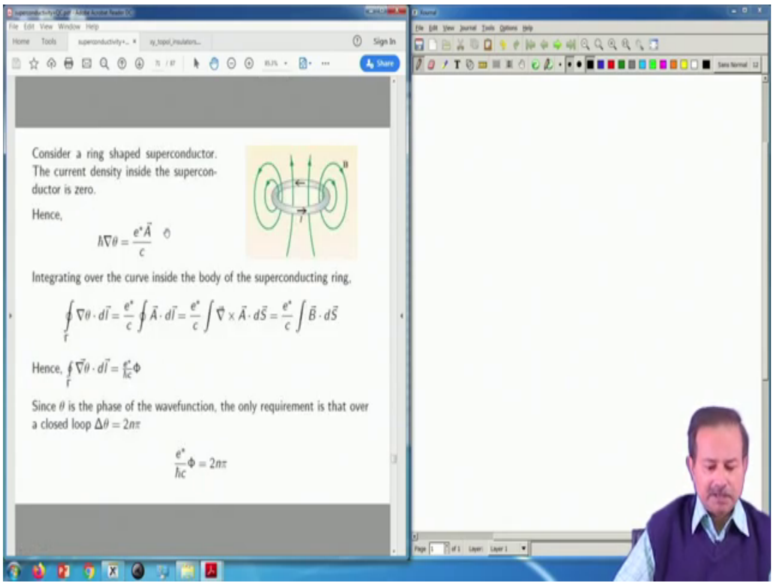

And that is exactly what happens and then you can just integrate over a curve inside the body of superconducting ring and then you will get this expression A dot dl which is B curl A dot dS is e star by c into B dot Ds. Therefore, grad theta dot dl is equal to e star by h cross c into phi the flux. Now, theta being the phase of the wave function it has to be single valued over a closed loop.

So, it has to come back to the to either change will be either 0 or multiple of 2 pi and that is what is used here and that immediately gives you the quantization.

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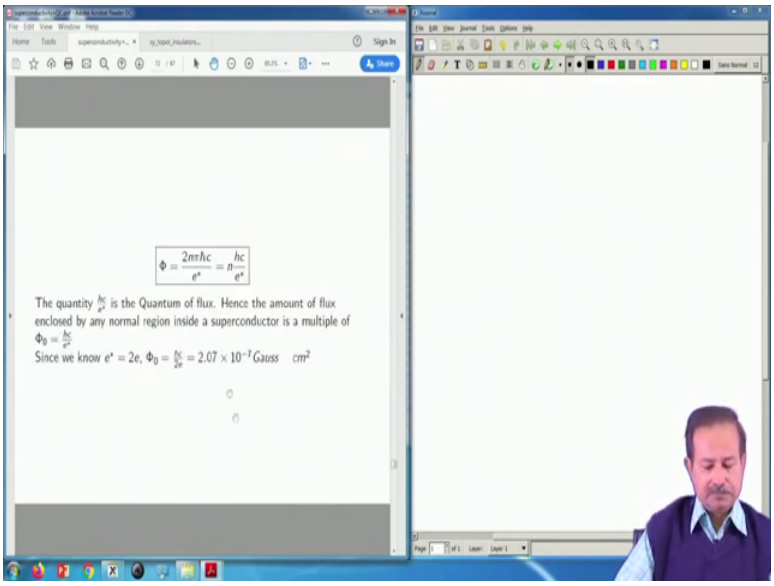

So, these fluxes are quantized and the quantum is n times this is the number of that number n 2 pi n into hc by e see hc by e is we know that quantum of flux of course, here it is e star which will turn out to be 2 e that we know. So, this is the value.

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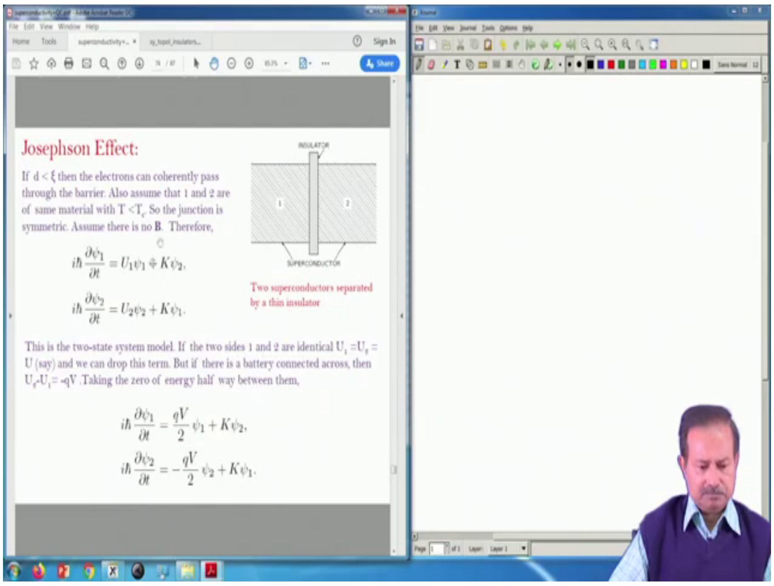

Then we did the Josephson Effect which is basically coupled super conductors with a thin insulating layer.

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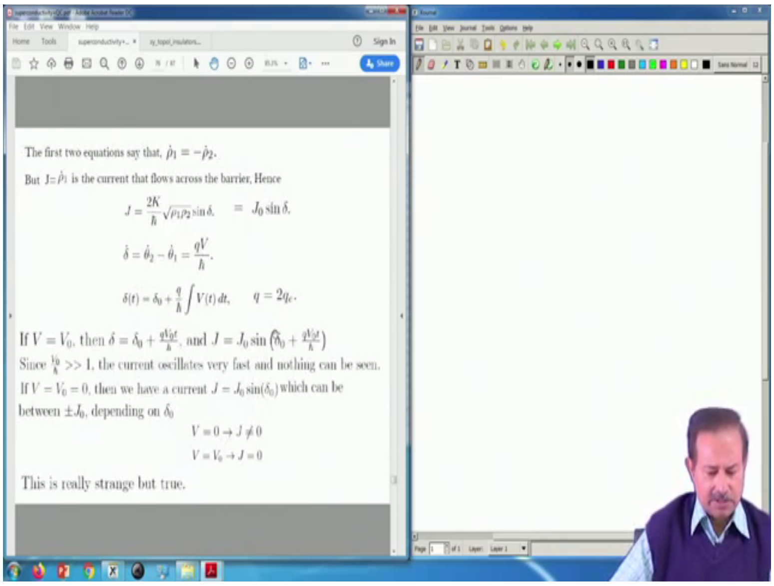

And we found out that in even without a potential between the two junctions across the junction you can get a current and this is remarkable when you bring two superconductors in proximity.

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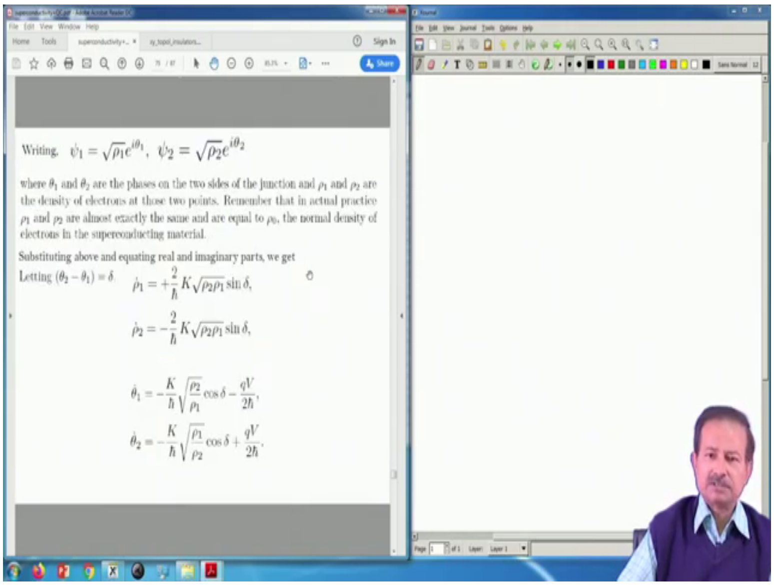

And so, that just comes out of the fact that these two the super conductivity conductor has a uniform has one wave function uniform wave function throughout and the two theta has tried to become the same at the across the junction and that means, they will exchange cooper pairs and that is exactly what this calculation shows.

And it shows interestingly that if V equal to 0 the voltage across this junction is 0 then you have a non zero current whereas, if the voltage is finite then of course, the current fluctuates so fast that you will effectively get average current 0 ok.

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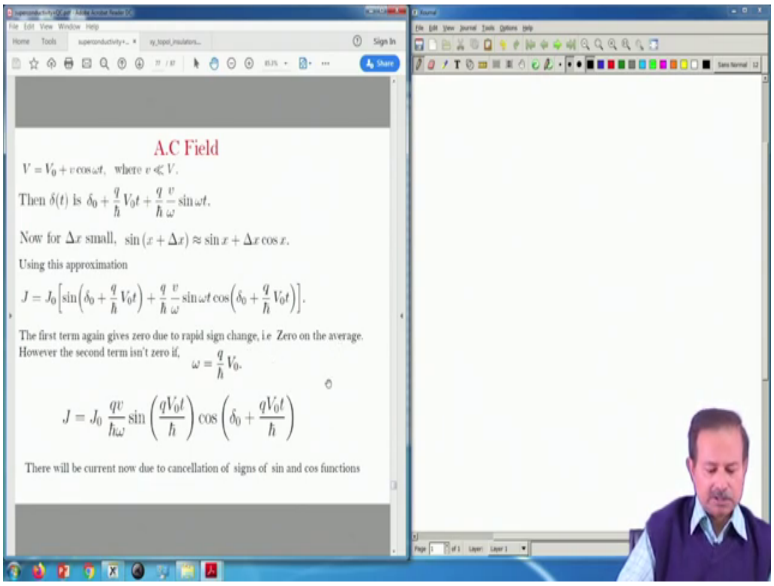

So, then you can extend it to AC fields and you can choose a particular frequency at which you will get currents. So, there is called AC Josephson effect.

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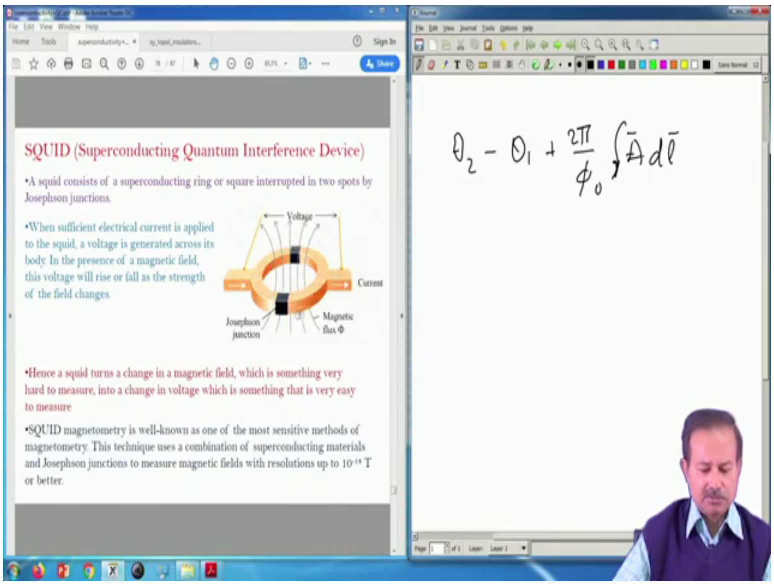

There is a an instrument that we I mentioned briefly where you have this the flux due to a magnetic field can be measured of course, flux can be measured in many other ways even in MSc labs you measure magnetic field and flux and so on, but the precision of this instrument is extraordinarily high and that is why this is a an instrument of choice for anybody doing research in basic sciences or material sciences or even in cases where you want to find out extremely low magnetic fields somewhere.

So, the accuracy the resolution is almost like 10 to the power minus 14 Tesla and this is just remarkable and how does it work. It works very simply just as we said that the Josephson effect it depends on the difference of the two phases right. And when there is a magnetic field of course, there is an additional phase that comes in which is just 1 by phi not A dot dl right, A is the vector potential. So or 2 pi by phi not the way you write typically is sorry. So, this is the magnetic additional theta change in theta that creeps in because of the magnetic field.

And the instrument can sense this at this change in phase because the current depends on the of the phase and that is now, that is converted into a voltage here in this instrument called a superconducting quantum interference device or SQUID. This device is shown here, there is two junctions it is a two junction SQUID. So, these are two Josephson junctions and the current gets divided into 2 and then you can just work out.

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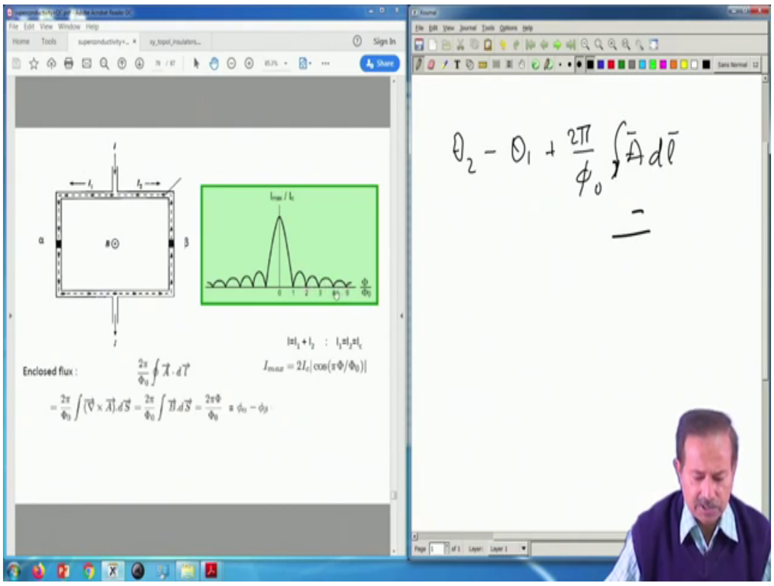

And you will see that the enclosed flux if you have a magnetic field B that is perpendicular to this plane or I mean the if it is not perpendicular you just take the perpendicular component to calculate the flux times the area and then you will get this gives you the value of the phase difference between the two leads. Suppose initially they were there were no phase differences and once you put this magnetic field due to this vector potential the flux threading through the ring you will get a change in this and change in the phase is just 2 pi phi by phi naught which is what is I have written here 2. So, this is the change.

So, that is what one basically measures and then you just see that there is a kind of an interference pattern because of the because; obviously, the once the flux goes through this integer multiples of 1 2 phi by phi naught is 1 2 3 4 at these points there are minima, you can look at this expression this is the I max is twice I c cos of pi phi by phi naught. So, as a function of phi which means the magnetic field times the area you will get a get this oscillatory pattern and from that you can figure out what phi is.

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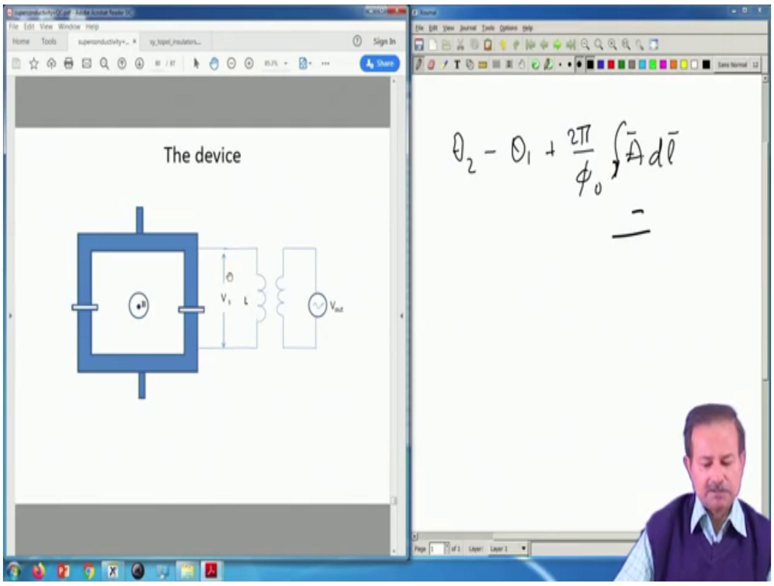

It is actually done this way the device you converts this into a voltage. So, it is the magnetic field to voltage transit transducer in some sense. So, flux to voltage transducer. And you measure the voltage across this inductively inductance and this basically between these two points. And then it induces another voltage here you can multiply amplify it and this output voltage is what is measured and from that one can read of the magnetic field or change in field.

So, that is the device and this device is so important and the theory is fairly simple that this is one of the most used magnetic measurement technique in all of all over the world and you can go to any lab in a so, we seen more or less a research institute any research institute or universities or and then or IITs or somewhere and you will find this instrument and see how it works ah.

It has actually revolutionized the way we do magnetism in these days it of course, got a Nobel Prize also Brian Josephson got Nobel Prize with two others.

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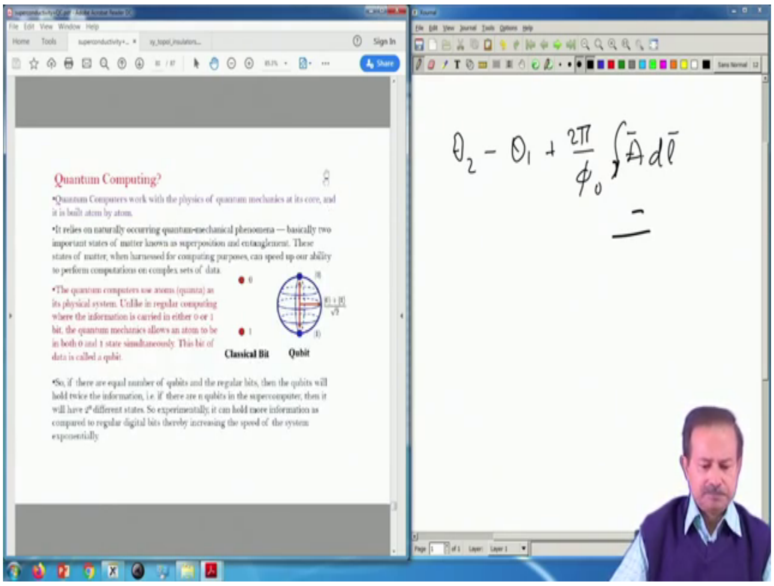

Now, the very recently you must have heard this quantum supremacy and all that there is a lot of noise there is a paper in nature. Then of course, question is what is quantum computing I will not get into that is not a subject which we are studying here, but I will just outline what it is because it uses this one uses solid state techniques to generate this so, called qubits. So, and the technique one uses in one of the techniques one uses is using Josephson junctions.

So, you see the use of Josephson junction it can it is basically a quantum interference device and so it can be used to generate qubits to two different states which between which you can you have a superposition. So, I will not get into the details the quantum computers is you must have heard it so many times, it is a buzz word and people are pursuing it and they use basic quant like atoms, photons, spins to generate these states quantum mechanical states.

And then unlike in the classical case where 0 and 1 are the only two states you need in a in a bit classical qubit a classical bit you here you can actually have a superposition between the these two. So, it is a some if alpha 0 plus beta 1 kind of divided by root over alpha square plus beta square divided by these so, you normalize it by these alpha square plus beta square.

So, that kind of so, here for example, the 1 the 2 coefficients are 1 and 1. So, the normalization is 1 by root 2 this is simple superposition of 2 states 0 and 1. So; that means, you have now if you have n such qubits then you can have 2 to the power n such states to play with and that is those states have to be generated they have to be stabilized, they have they should not become decoherent. So, that is where the trouble is and of course, then you have to manipulate them also read and write and so on.

So, those I will not get into, but the only reason we are discussing this is that one of the methods of generating this is using superconducting qubits micrometer or less size Josephson junctions as qubits. So, the it is just as he says if there are equal number of qubits and regular bits which means classical bits then the qubits will hold twice the information. That is n qubits in a superconductor in a superconductor will have 2 to the power n different states.

So, experimentally it can hold much more information compared to a regular digital qubit that we nowadays use and so, the speed of the system increases exponentially because it is a 2 to the power to the power something. So, it is like an exponent exponential increase.

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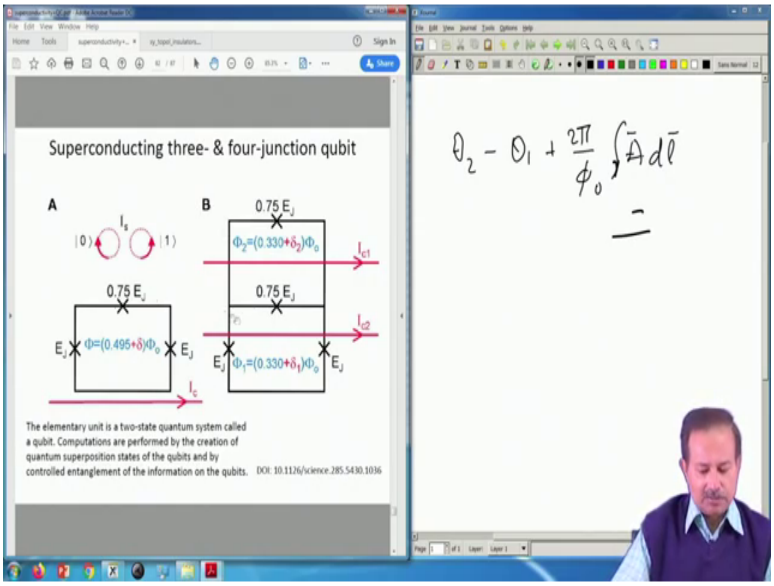

So, the these the ones which are suggested long back not very long, but in the late 1990s and early 2000 was this from (Refer Time: 22:11) and this is came out in science and this is people are trying to use it. And so, quantum supremacy claims that they have this they refer to this paper as source as how they generate the qubit the. So, I mean as a possible route to generate the qubit.

So, this is simply three junction three Josephson superconducting three and four junction qubits. So, in as you can see here you have two directions of current and you can linearly super superpose them. So, this superposition is at the heart of quantum computing and the elementary unit is a two state quantum system called a qubit which I am mentioning so far so, long.

Computations are performed by the creation of quantum superposition states of these qubits and by controlled entanglement of the information on the qubits. So, that control is a bit non trivial and the whole system has to be kept in very low temperatures so, few milli Kelvin's apparently. And so, this is how the basic architecture of the qubit works in a quantum computer of course, the algorithm and other things are very different ball game and that we will not discuss here right.

This is a now a separate course to discuss quantum computing and it is algorithms and so on the mathematics of it. But at the core of it we are these Josephson junctions as a possible qubit. This is a 4 junction qubit on the right and. So, you can create more states and entangle them.

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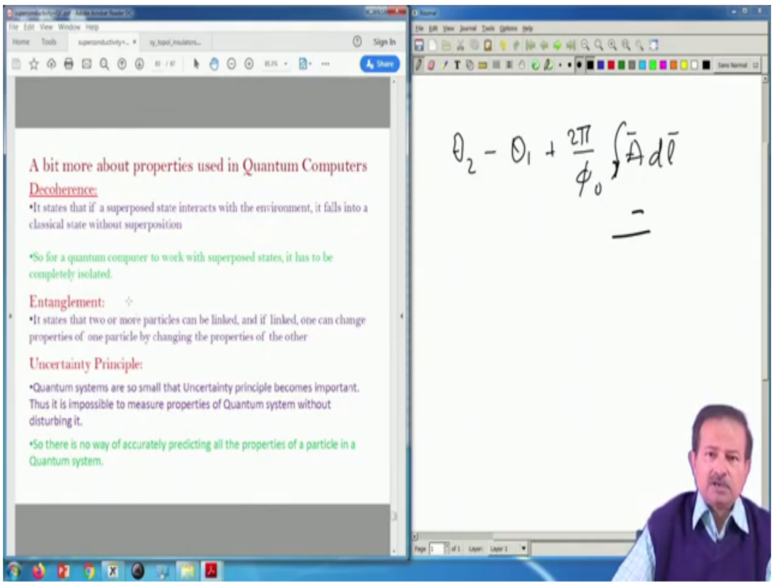

So, of course, you have to prevent decoherence as I said. So, you they typically work at extremely low temperatures like milli Kelvin. So, it has to be completely isolated from the from fields and other things from outside. So, these are things that you have to go you have to take care if you are doing a quantum computation at the level of devices, because uncertainty principle will become important at that level.

And so, you cannot simultaneously measure conjugate properties besides to try to measure you know that there is collapse that can happens so, the system can go to a state collapse to a state. So you have to have a measure properties without disturbing the system much and so on ok.

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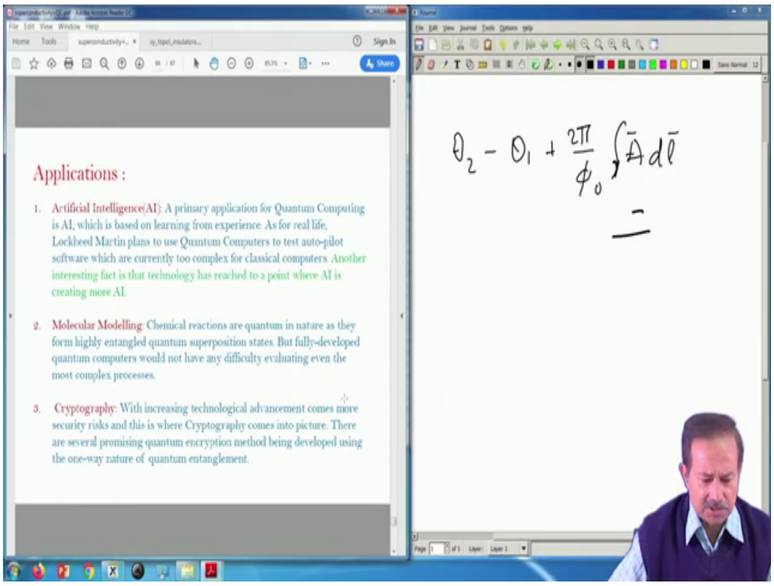

So, those are details I will not get into, these are applications where the latest one has a is a on 22 of October 2019.

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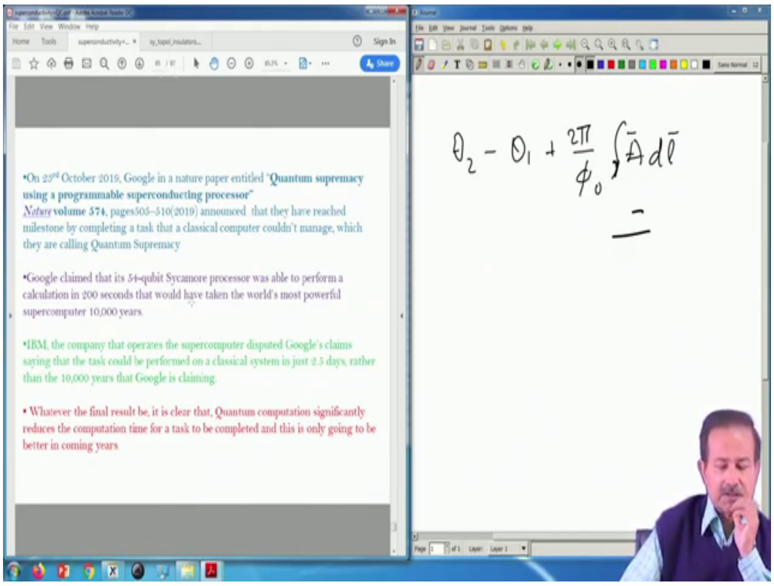

Google has announced in a nature paper they have announced also in a press conference there is a machine called quantum supremacy which is a programmable superconducting processor which uses programmable superconducting processor.

So, they have been able to entangle states, retrieve states and entangle them in a way they want I mean. So, that is remarkable achievement if it is a really achieved and of course, the at the heart maybe these superconducting Josephson junctions as the qubits. They have used 53 of them to do the computation so, 2 to the power 53 is the number of states there.

They claim that the sycamore processor which is which they called sycamore processor was able to perform a calculation in 200 seconds that would have taken the world's most powerful supercomputer 10000 years. This was immediately disputed within a few days and IBM claimed that it is the they can do the computation even plus even with a usual regular computer they are much much lesser time 2.5 days or so. So, no matter what it is it is still an achievement and it is a first step towards doing many other companies and research institutes are engaged in it.

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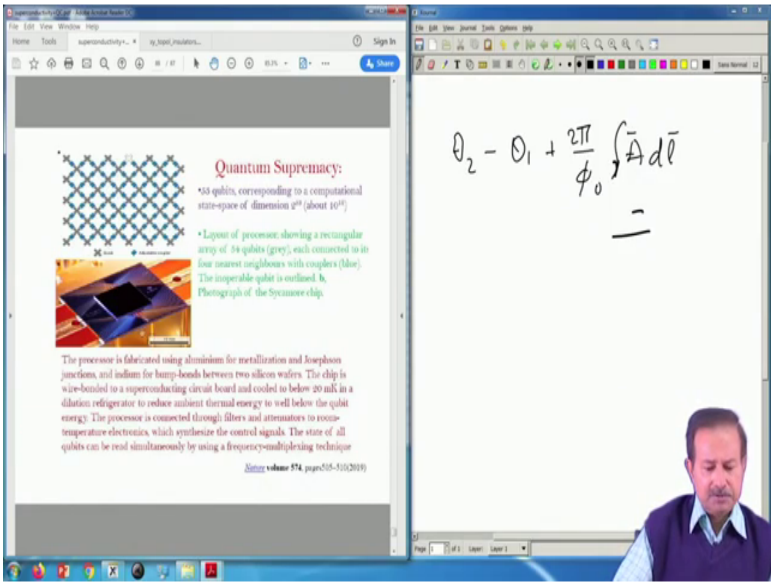

And this is the picture of the quantum supremacy basic architecture these 53 qubits. So, it has a dimension of 2 to the power 53 state space. So, that is where the those are the states that are manipulated, that is about 10 to the power 16 which is enormous I mean this is just really very very big. Of course, to have a real quantum computer one needs to go to much much larger state space typically 10000 qubits are a possibility that people are talking about and that is where the real difference will come up will show up.

So, these are these couple Josephson junctions as you can see these are problem these are the you needs. The processor is fabricated using aluminum for metallization and Josephson junctions and indium for bump bonds between two silicon wafers. The chip is wire bonded to a superconducting circuit board and cooled to below 20 milli Kelvin in a dilution refrigerator.

The processor is connected to filters and attenuators to room temperature electronics which synthesizes the control signals. So, that is how the states are manipulated the read and write and information. So, the qubits can be read simultaneously the state of all the qubits can be read simultaneously by using some multiplexing techniques which is a technical issue, but this is at the heart is are these Josephson junctions that we have just studied and that is the reason they are so important is because they are extraordinarily sensitive and they are quantum interference devices.

So, they are natural choice for qubits. So, this is all superconductivity that we will do there are lot more this is a fascinating subject, there is this new high DC superconductors which have dramatically changed our perception of super conductivity and they are the possibility of possibilities are endless in fact, every other day new materials are coming up and their order parameter is an interesting issue, because the psi in those new superconductors may have may not be isotropic that it is not a constant gap is not a constant.

And therefore, there are nodes in the gap, there are 0s in the gap, which are fascinating physics connected to them. And those who are interested can look up some of the literature in there are there are n number of literatures in large number of literatures available on high temperature superconductor and that is you are welcome to delve into it.