Electronic Theory of Solids
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Lecture – 58
Josephson junction

We are discussing the Ginzburg Landau theory of superconductors; particularly we are interested in the electrodynamics of superconductor.

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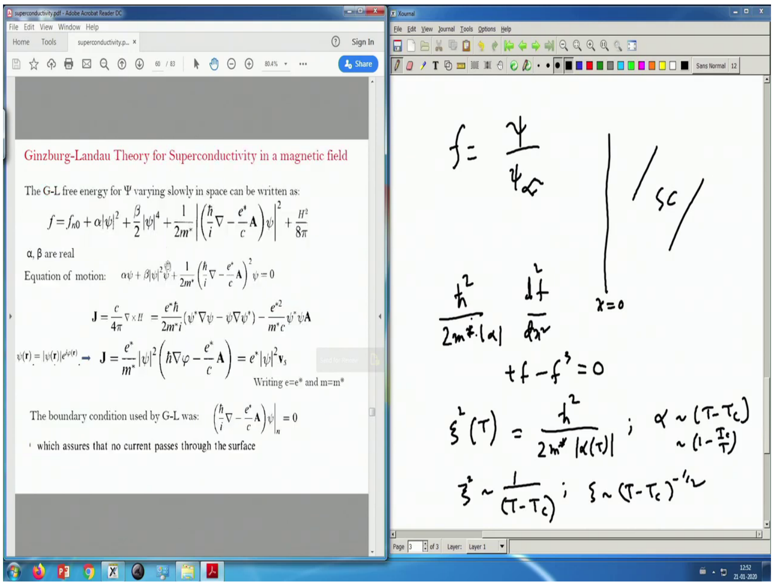

And, what I will show is finally, show you is that there is a state in which the magnetic field does penetrate superconductors and we will find out when and how does it happen. So, in that context we have written down the Ginzburg Landau free energies wrote down the equations of motion these equations; one gives me the equation for psi, another gives me the equation for the current ok.

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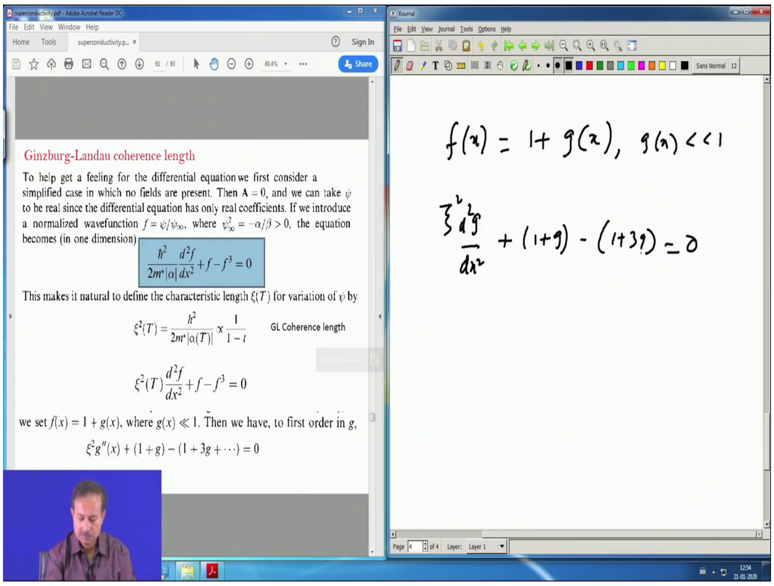

So, then we are we have taken the equation for psi and for situation where there is no field and solved it not yet solved, but wrote it down and immediately found out there is a length scale that appears in the problem. And, that length scale is this xi it is called Ginzburg Landau coherence length and that varies with temperature as T minus T c to the power minus half. So, let us go ahead and find out how psi behaves inside a superconductor.

So, f function as you can see is a ratio of two quantities one is psi and another is the bulk value of psi deep inside the superconductor this psi infinity. So, that means, it is I can write it as f x equal to 1 plus some g of x, where g of x is much much less than 1 because f x equal to 1 is the value deep inside the super conductor and we assume that the in homoginition is not in homogeneity is not so strong that it destroys superconductivity completely somewhere inside.

So, in that case g x will smoothly vary and vary in a within a small range and. So, therefore, g of x is much much less than 1 and they are both dimension less quantities, f x and g x are dimensionless. Then we can rewrite this equation by expanding this cubic term 1 plus g cube and this is what has been done xi square g d 2 g d x 2 this plus 1 1 plus g minus 1 plus 3 g equal to 0 and you can solve this equation straightforwardly. This is now a linear equation.

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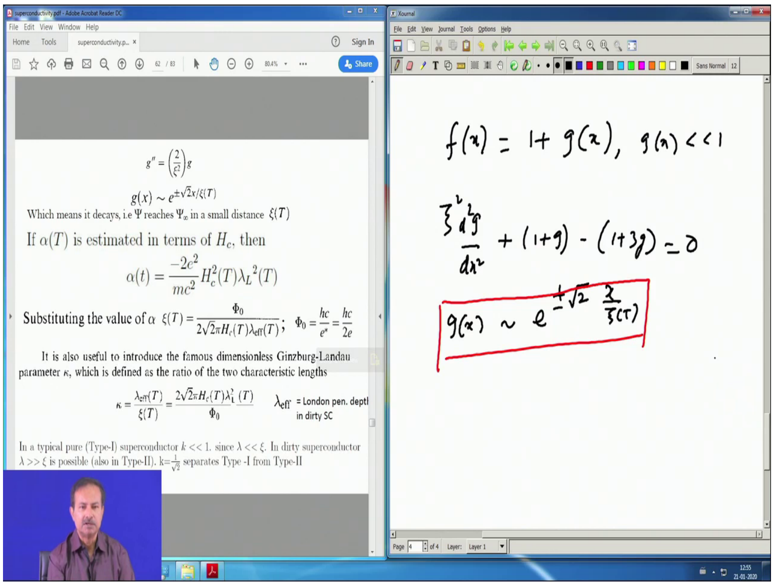

And, you can solve it giving g of x going as g of x going as e to the power plus minus root 2 x by c of T that is the solution that is for g. As I said g is the division from the infinite values so, that means, the over a length scale of g the function f basically restores its value of this i infinity.

So, if you are coming from deep inside the superconductor the xi x see if it is you are going inside the superconductor g this minus sign has to be taken and then it decays within a g decays within a distance of c typical decay I mean it is exponential decay. So, nearly one third is gone by the time you reach x equal to xi and that is; that means, that it attains its full value psi infinity within a distance somewhat below somewhat beyond xi of T.

So, that is xi of T is a measure of the distance by which the superconductor attains superconducting work function attains it is full bulk value ok. One can define these parameters in terms of other quantities and those are not getting into these are for experiments these are extremely important because they there are measurable quantities like H c and lambda effective in terms of with xi of T can be defined and so, this definitions are given in books. So, it is not important to realize to go through this algebra in the sense that this is you have to know what lambda effective is and so on.

This is a Landaun penetration depth in a superconductor where there are disorders and so, these are somewhat more phenomenological parameters, but these are things that experimentally actually need and they can find out so, so defining xi of T in terms of those quantities. The interesting thing is that there is a phi naught; phi naught is a universal quantity. It is an universal quantity in the sense that xi it is hc by e star; e star was just e this would be the Dirac quantum flux quantum whereas, here of course, each star is hc by 2e, but it is still a fundamental quantity is just the half of direct flux.

So, there is a fundamental physical quantity that is now appearing here and there is another quantity that is extremely important and that is why one puts in this xi of T here. One defines a kappa which is which is the ratio between these two and the lambda effective and xi of T and this actually defines the border between it is value defines the border between a type I and a type II superconductor.

So, for a type I superconductor kappa is much less than 1 and in lambda much less than xi whereas, is dirty superconductor lambda can be greater than and actually much greater than xi and in type II also that is what happens and kappa equal to 1 one actually in actual practice kappa equal to 1 by root 2 separates these type I from type II.

So, type I and type II are basically determined by the value of kappa and if kappa is much less than 1 you have type surely type I kappa much greater than 1, you have type II. The border is at the separation happens at 1 by root 2.

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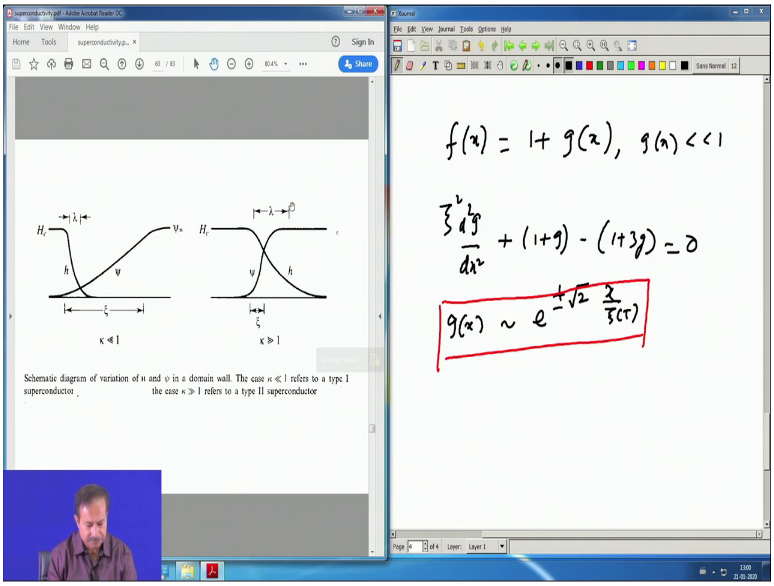

So, this is a picture of how lambda and xi behave in a type I superconductor and in a type II superconductor. As you can see the xi is large in a type I superconductor and lambda is small it is somewhat reversed in type II superconductor where the full value is attend quite quickly for the given function and h decays in a very long distance. Here it is just the opposite. It takes long it takes some distance xi which is large we for psi to attend its value at infinity now value deep inside the bulk.

So, this is how it is typical values in a normal superconductor which is not very dirty are like few 100 for lambda few 100 Angstroms for lambda and few 1000 Angstroms for xi in a type II superconductor. Of course, in the high T xi superconductor xi is very small. So, they are inherently mostly type II superconductors ok.

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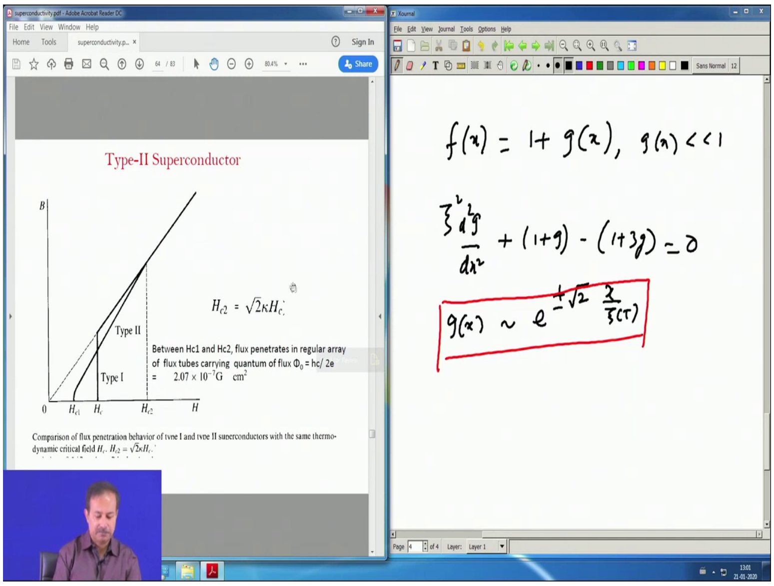

The other interesting thing is we can discuss type II superconductivity and there are there is some quantity called H c 2 appears which I will discuss briefly quickly and. So, this is how it behaves between H c 1 and H c 2 the flux penetrates there is a field called H c 1 there is a field which is H c 2 between these the flux penetrates. So, if you are coming from large fields you are reducing fields. So, at H c 2 you will first penetrate the field and then it will carry on till H c 1.

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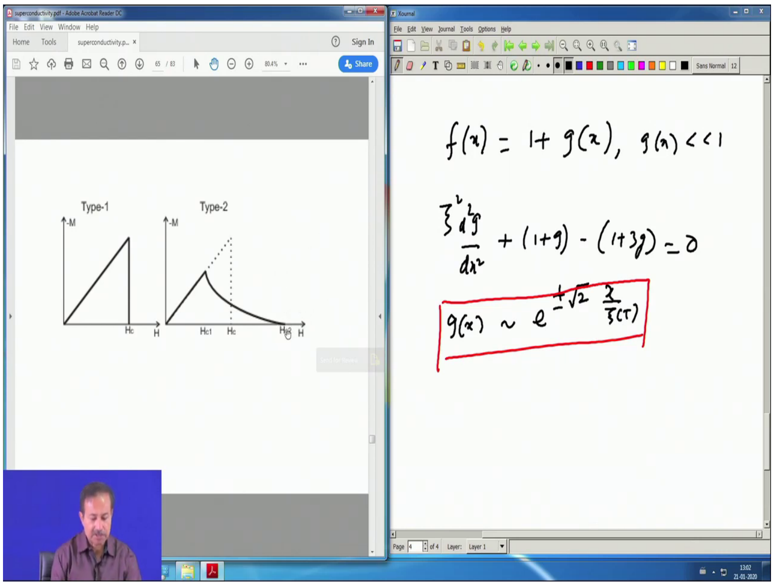

So, let me just show you the other picture which is much more much easier to comprehend and this is how a of type I superconductor here M versus H curve it goes up to minus 4 pi M and then that is minus 4 pi M is the value in the magnetization. As you increase H there is a H c at which the magnetization drops to 0 ok. So, this is negative because in a superconductor you know it is diamagnetic and the. So, it rises with the magnetic field and then it just it rises means it goes negative and then at H c 2 it is just drops to vertically drops to 0.

Whereas in a type II superconductor M versus H curve is like this it goes up then it starts dropping from H c 1 and it takes all the way up to H c 2 where H c is between H c 1 and H c 2 and it goes to 0 in a type II superconductor at H c 2. So, there is a large region between H c 1 H c 2 where flux has field has penetrated the system ok.

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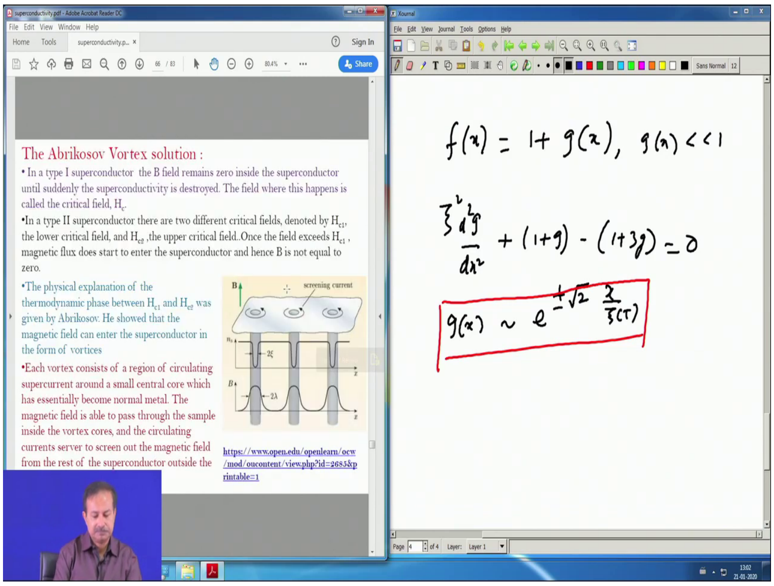

So, let us just look at that as I mentioned that Abrikosov was the in his remarkable theoretical paper who predicted that the magnetic fields can enter a superconductor particularly in inhomogeneous superconductor and in a type II which is now called type II superconductor and as we just discussed these are like tubes that penetrate the super conductor.

And, therefore, there is a screening current across around this tube and typical distances between these tubes is twice the Ginzburg Landau coherence length and the tubes size is typically lambda the Landaun penetration depth lambda effective actually twice of that.

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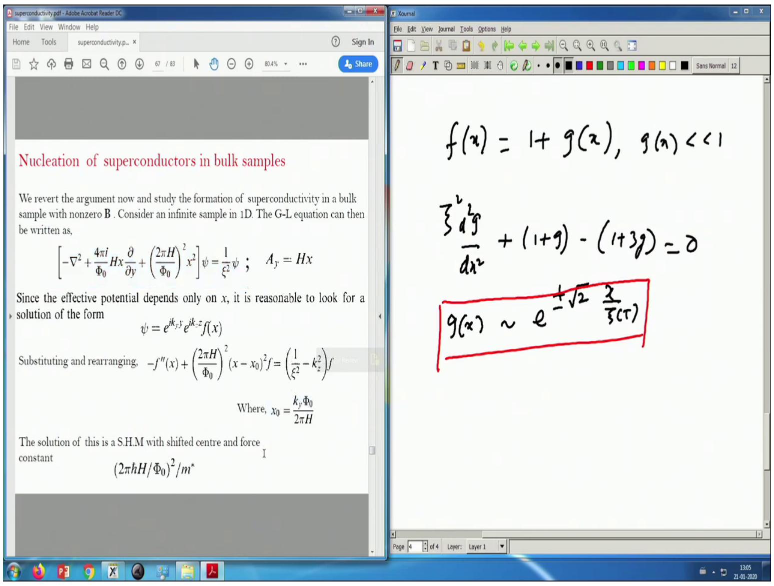

So, let us just do the calculation very quickly because this is in presence of a Landau gauge and remember A y equal to Hx. So, you now have a magnetic field and you are trying to solve that equation on motion again and the equation is very similar to what we did for quantum Hall with integer quantum Hall effect and that is exactly what it is except that this side has 1 by xi square. This equation had this 1 by xi square into on this side and so, xi square delta square was there.

So, then xi square has been taken to the right hand side and this equation then is exactly what you get in integer quantum hall effect case also. A y is chosen as H x this is the gauge that is taken. So, then you can write down the same solution again when y and z directions are plane waves whereas, direction will turn out to be a displaced harmonic oscillator whose centre is at x naught where x naught is this k y phi 0 by 2 pi H.

These are very similar to that calculation. So, I am not repeating it you can look up. What is interesting is that you do not have to solve this you know the solution. So, you do not have to do anything more. What is interesting is this right hand side this is 1 by xi square minus k z square into f and this is the eigenvalue. And, we know what the value is eigenvalue is n plus of h cross omega c.

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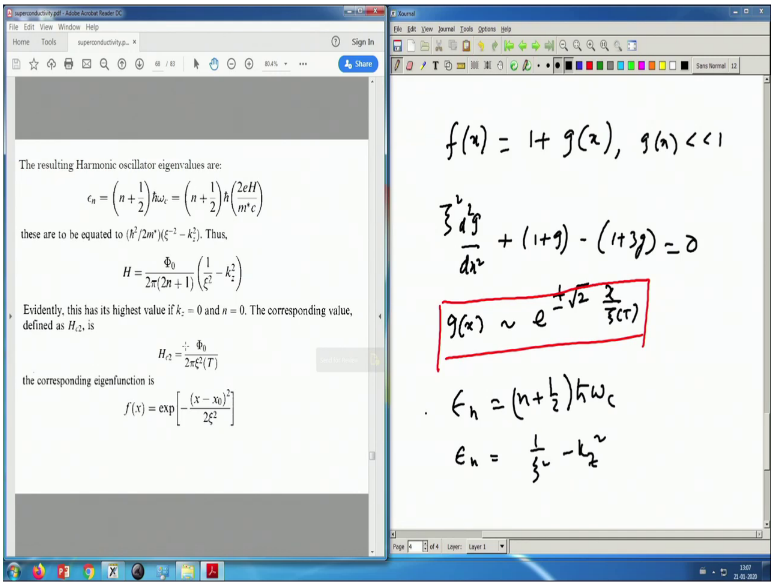

So, you just equate this n plus of h cross omega c to that 1 by xi square minus k z square that was there. Therefore, you can immediately pull out an H because h cross is the c contained at h. So, you equate it with this quantity with n plus half h with 1 by xi square minus k z square and you will end up for this equation. So, these two equations you are using n E n equal to n plus half h cross omega c and E n is also 1 by xi square minus k z square from the previous page.

This one. So, these you equate you will get the equation that is written down here. So, this gives the value of the magnetic field so, H. So, the now what we try to find out is whether magnetic field and the superconductivity co-exists or not. So, let us see the highest value H can have is when k z equal to 0 obviously, and when n is also 0 right.

So, these are the these are of course, n is the n is 0 is the lowest Landau available and we can also set k z is equal to 0 is very large wavelength for example, along the that is maximum that you can have in this z direction. It is like a free particle to fairly very large wavelength.

So, the corresponding H H value is called H c 2 and what does it mean? It means that at this value both f which is the f is the solution remember f is the f is what you are looking for I mean the solution for f is already written down psi is something is this. So, as long as f is non-zero psi is also non-zero ok. So, that means, in this when this happens this is the largest magnetic field for which f is non-zero actually that means, psi is non-zero. That means, at this magnetic field the beyond this magnetic field your f will become 0 that will be means that this superconducting wave function will vanish after this H c 2.

Below this H c 2 the equations admit of both h and f; that means, psi non-zero and that is what means that this is called the nucleation of superconductivity. So, as if you have a very large magnetic field you are. So, you are now reducing the magnetic field and you are looking at what magnetic field suddenly the superconducting wave function becomes non-zero and this is the way one you can think of it.

Or the other way is you are increasing H and then. So, you have a superconductor you are bring you have a very large H and then you are bring the H down at one particular point suddenly there is a superconductivity appears and at that value is H c 2. So, this is the this below this value superconductivity and magnetic field coincide. They co-exist inside this system how do they coincide that we will see.

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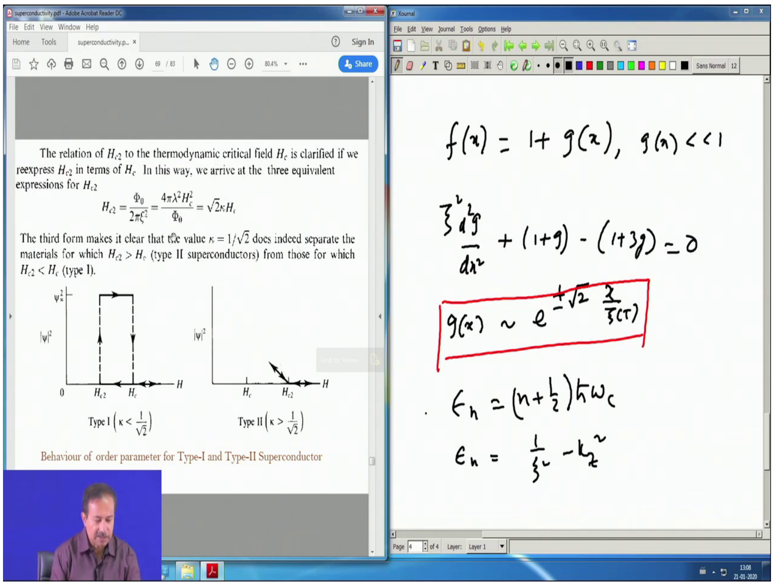

So, this one can actually calculate the relation of H c 2 with the thermodynamic critical field H c and that relation gives you this 2 kappa into root of 2 kappa into H and that is why the kappa equal to 1 by root 2 separates the two types of superconductors type I and type II because when H c 2 becomes when kappa is 1 by 2 H c 2 will be 1 when kappa equal to 1 by root 2 then H c 2 will become H c. So, it will then move here and we will get type I superconductor.

So, that is at 1 by root 2 H c 2 becomes H c and at for H c 2 less than that you are in a type I superconductor and this is how the psi behaves with respect to H as you bring it down in a type I at H c it starts moving up and becomes infinite value of superconductor and then. So, actually at H c 2 you go up and then if you are decreasing if you are increasing the field at H c 2 you will. So, H c H c 2 is less than H H c this is how it will behave.

So, you are coming from high magnetic field decreasing and only at H c 2 you can nucleate superconductivity and the moment you do that it will immediately go to psi infinity value the bulk value that is because you are already way below H c. And, then of course, if you are increasing the field then you have to go up to H c to destroy it. So, this that is how the H c is defined the field at which superconductivity is destroyed.

So, there is a hysteretic region. Whereas in type II the H H c 2 is greater than H c. So, if you are reducing H you will start having superconductivity here psi will become finite at H c 2 and go up and go up and till H c 2 it will keep going up till H c it will keep going up, but till H c 1 it will go up. But, interesting thing is that it retraces the path there is no hysteresis because it H c 2 is much larger than H c.

So, that is the; that is the distinction between. So, that is what that is what that picture showed us. So, that picture is recovered from this calculation.

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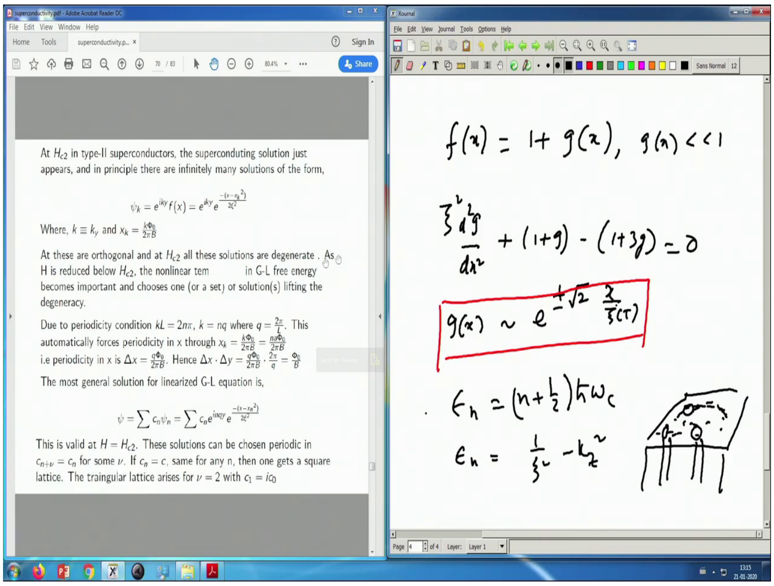

Now, Abrikosov went even further he tried to figure out how these fluxes penetrate the superconductor how the magnetic field penetrates the superconductor and the he actually worked it out. I am not going to give to work it out in detail and that is a separate calculation and that is something that is a knowledge that you should keep, the calculation is somewhat complicated and very very intuitive.

So, just that he found out that there is a you know the oscillators the fact that you can put the oscillators centre the simplomatic motion centre of the simplomatic oscillator solution that you get within a range I mean that range he found out and that range give him delta x into delta y equal to phi 0 by B.

So, this is inverse of the flux I mean number of flux quantum basically see a into area into B is equal to B is the flux total flux and phi 0 by B. So, inverse of that is the number of flux quantum and each of these fluxes carry a flux quantum which is H c by 2 e here and from there he did a beautiful analysis it is just a remarkable calculation and he found out that this flux tubes do not go uniformly inside.

They do not form a liquid like structure they can or a random structure, they go in regular arrangements and these regular arrays are they form a lattice and that lattice is called the Abrikosov flux lattice. The turns out that for the this kind of superconductor the structure that is more stable is a triangular lattice structure and that is slightly lower energy than the square lattice structure and so, he predicted that there will be lattice structure and now we know that this triangular lattice structure is available, this is experimentally measured and the measurement is very simple actually.

What you do is that you just put some iron files on the superconductor. So, take a superconductor put some iron files randomly all over small iron particles and then take it to a type suppose a type II superconductor then thread the magnetic field.

So, put a magnetic field below H c 2 and then the magnetic fluxes penetrate and they penetrate in a lattice which is triangular and then you just check the super conductor and iron files will basically go towards the magnetic fields where the magnetic field has penetrated and then you will see a pattern of this iron powders exactly as the pattern of the magnetic fluxes below the arrangement of the flux tubes below and that is how it is figured out easily. You can actually do it in your MSc lab or BSc lab and if you have a low temperature say liquid nitrogen for example, and you can find this lattice structure.

So, that is all we wanted to discuss about type I type II superconductivity and Abrikosov vortex lattice. This is again a remarkable piece of intuition and to confidence in your theoretical calculations that you predicted it way ahead of the experimental even people even did not even think about that kind of a situation. So, Abrikosov later got Nobel Prize and this is really a remarkable piece of intuitive calculation. So, I stop my discussion of this Ginzburg Landau theory here.

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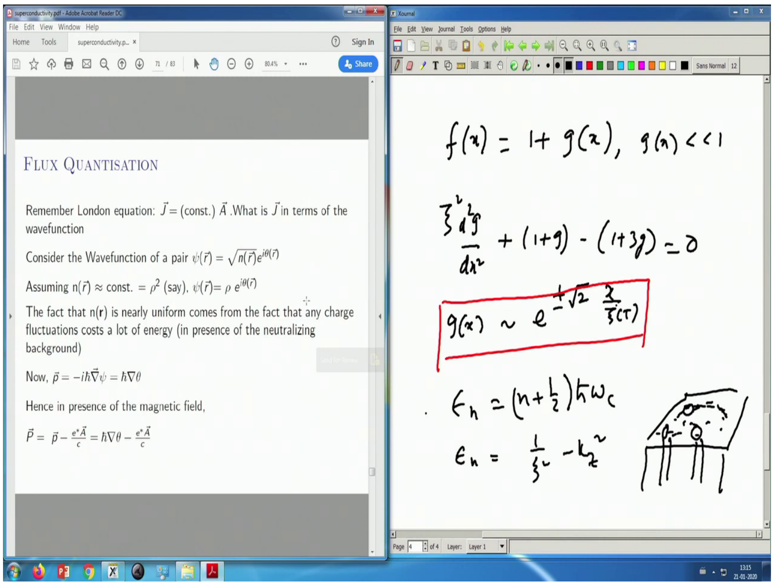

And, let us just go over to another very very important applications of superconductor that I alluded to when I discuss this delta n delta phi uncertainty. So, let me just go through quickly what it is. It is called flux quantization it is.

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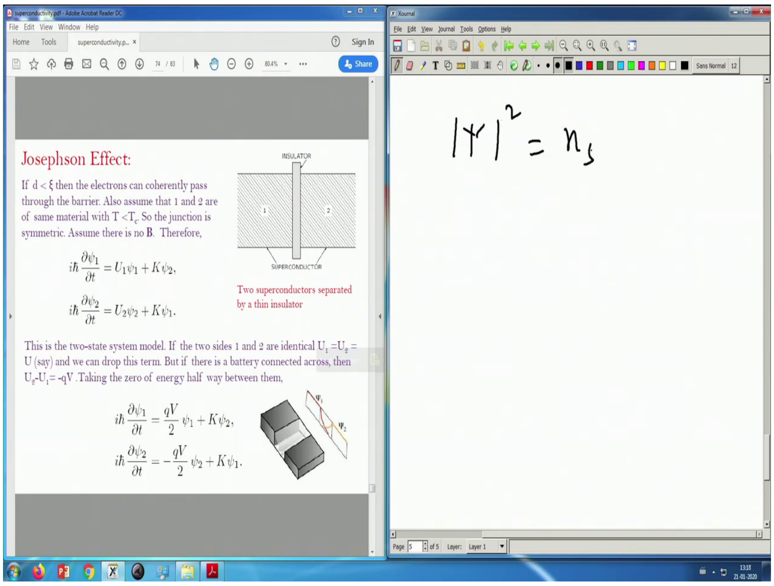

So, let me just do the one that is important which is what I was alluded to which is Josephson effect. So, in Josephson effect what one does is you just as I said you put two superconductors separated by very thin very very thin insulating region and then you basically it is so small that it is less than the length is less than the coherence length. So, that this super conductor and this superconductor can talk to each other.

And, then you can simply write a Schrodinger equation for the superconducting wave functions and is it is like a two state model again. And, this is the actual geometric physically this is how it is done there is a small insulating layer. You can basically ace out the superconducting regions and that gives you a insulating region ok. So, let us just quickly go through the calculation.

If you put a if you connect a battery then this the two potentials U 1 and U 2 that are arbitrary here that the difference is at least qV and one can just then set U 1 and U 2 to midpoint I mean the one is at minus qV by 2 another is at plus qV by 2 qV is the charge corresponding to this functions the ok. So, we are discussing we have been discussing this for long ok.

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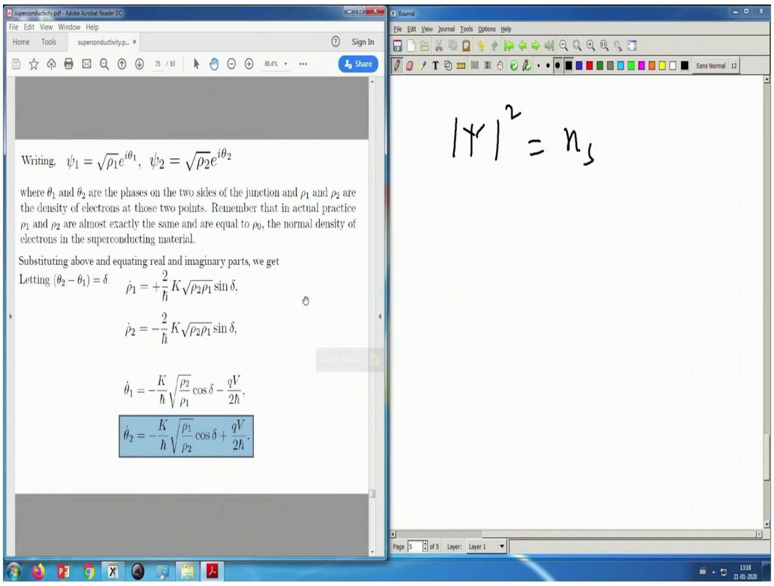

So, then the calculation is goes as follows you just you can write this psi 1 as root over n s e to the power i theta 1. Remember that mod psi that we were using mod psi square was n s. So, mod psi square we wrote down as n s. So, psi can be then written as rho 1 and rho 2 at the two densities on the two sides which are this n s 1 and n s 2 ok. So, once we do that these phases are all that we are interested in now.

So, then write down the equations and define theta 2 minus theta 1 equal to delta, go ahead and you can calculate you can equate the imaginary and real parts to get these two equations real part is this and imaginary part will give you this. So, these equations are absolutely simple algebra you can work it out.

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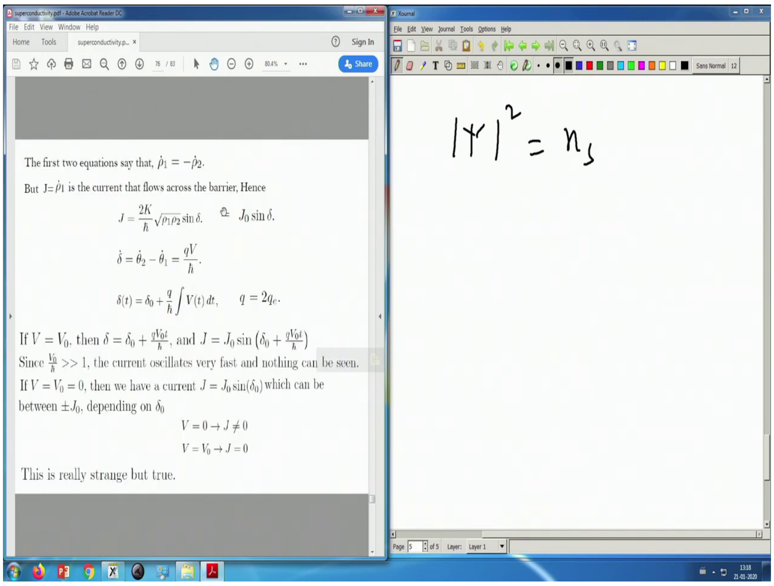

Now, the first two equations as you can see so, basically rho 1 dot equal to minus rho 2 dot, but current is rho 1 dot that flows across the barrier; that means, the reverse current you can think of is rho 2 dot flowing in the other direction. So, J of the current is then 2K by h cross root over rho 1 rho 2 into sine delta because that is what you will get from these equations right rho 1 dot is this and that is the current.

And, the delta dot is theta 2 dot minus theta 1 dot and that turns out to be qV by h cross again from the second set of equations ok. And, then you can integrate and get some delta naught constant of integration plus integral of V t, where V equal to V 0 then delta is basically this quantity this constant potential. So, this is a DC basically DC voltage and you get this and you can write your J therefore, as you can put that delta in here and J is J sine delta naught plus q 0.

Now, the thing is that V 0 by h cross is large very very large of course, h cross is in extremely small quantity. So, the this part oscillates extremely fast and therefore, you cannot see this ok.

Whereas, if V 0 is equal to 0 so, this will average out to 0 if V 0 is finite whereas, if V 0 is 0 then you can have a current which is just J 0 sine delta naught and delta naught is basically the is this quantity it is the difference between the 2 theta 1 and theta 2 at V is 0 now.

So, it is just the difference between these 2 at t equal to 0. So, if the initially there is a difference between the two phases that is that will give you a current which can be between. So, the current can be now be plus and minus this oscillates between plus 0 and minus 0 between depending on delta naught. So, it is a very contrary situation; if you have no voltage you have a current, if you have a voltage you have no current. So, this is what happens in the DC Josephson junction this is really strange, but this is true.

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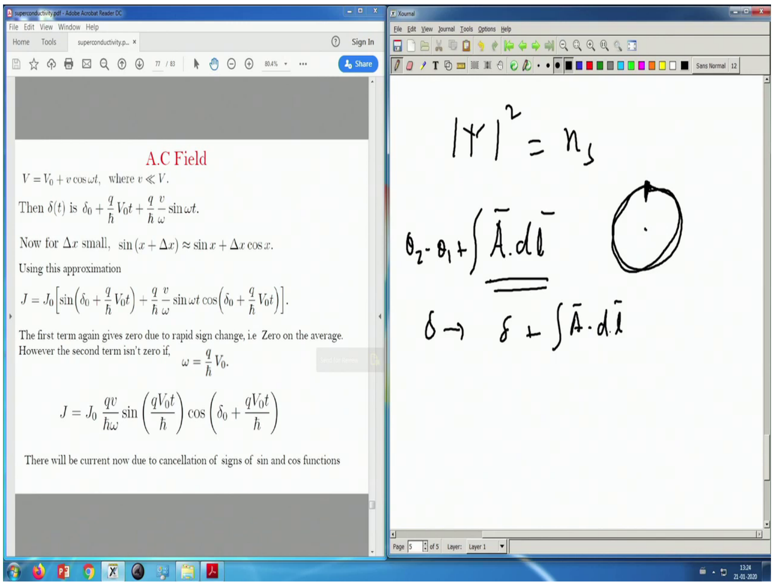

In AC field you can just go ahead and do this calculation, it is again the simple calculation another term will appear and then you just for delta x small you expand and what you get is finally, this quantity. Now, again in this quantity this will be oscillating very fast this will also oscillate very fast the last q V 0 t by h cross q V 0 t by h cross they will oscillate very fast and they will wash out.

But, if somehow the what happens is that you can choose your delta 0 in such a way then you can actually so, this is on an average this is 0 because of this is qV 0 t by h cross being very large rapid sign change will make it 0. But, you can choose your omega to be this q by h cross into V naught because the omega is the frequency you are applying for outside, the AC field the AC voltage and then of course, you can get these quantities where for example, look at it if delta 0 is to pi by 2 this is sine square. So, this will be this will then show up in a oscillatory, but it is always positive.

So, this will give you a current. There will be no cancellation due to change of signs and you can have an AC Josephson effect with a suitable periodic periodicity coming from here ok. So, this is the remarkable situation. Again, because this as I said the DC one is really amazing that if you have no voltage you have a current you have a voltage you do not have a current and of course, there is also an AC Josephson effect and Josephson junctions are extremely important because these are the ones that are used to measure the magnetic field.

Because, if you have a magnetic field then there will be additional phase coming from say suppose you have a ring Josephson ring with a this is a Josephson ring and you are threading a magnetic field through it that then; that means, there will be a A dot dl due to the magnetic field and that phase changes the overall phase of this psi theta 1 and theta 2 that we started with and that therefore, you can measure this flux that flows through this ring.

And, it is extremely sensitive that one can measure very very small fields due to this theta 1 theta 2 minus theta 1 minus or plus this additional field. So, the your delta will change to delta minus A dot dl and so, make it plus. So, delta plus A dot dl and that A dot dl can be picked up because that is that is in the phase and the current that the phase shifts means current the profile of the current where it goes to the oscillation points where it becomes minimum and maximum those shift and from that shift you can immediately find out what this A dot dl is and therefore, what the flux is that means.

So, this is called A dot ds right and therefore, it is a flux and then this is measurable. You beautifully one can measure the flux and you can actually measure even extremely tiny fields by this kind of measurement. So, the application of Josephson junction is tremendous and Josephson rings, junctions and so on and that actually led to the person Josephson getting a Nobel Prize along with Esaki and Giaever for this. This very simple, but extremely interesting and important calculation, and suggestion which was borne out by experiment.