Electronic Theory of Solids

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Lecture – 57
Type II superconductors

Hello and welcome. We have been discussing superconductivity and we will soon end this discussion. What we want to show you is, something extremely important that there is a famous work by a because of which are predicted even before any experimental evidence. That they are should be a state where magnetic flux, should penetrate a superconductor for certain kinds of superconductor. And in that case the flux will penetrate in flux tubes with quantized vortices.

And the flux, amount of flux will be like deduct flux quantum of flux for 2 e charges. At now, we know that it is a charge is 2 e and. At that time it was a revolutionary suggestion because there was no experimental evidence for it and it just came out of a calculation that he did using Ginzburg Landau theory. See Ginzburg Landau theory was already there in 1950. And it phenomenologically explained many things that happen inside the superconductor or for that matter in across any phase transition and that is what we want to discuss.

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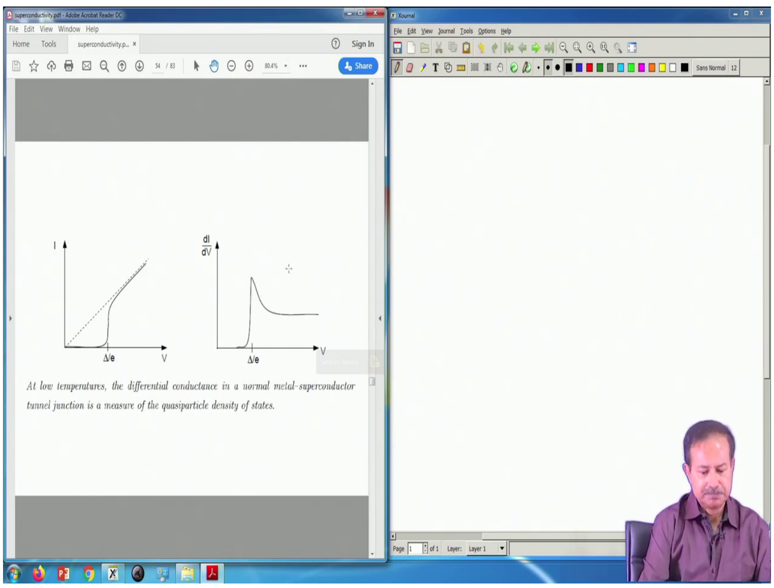

But just to recap that we discussed tunneling also. We found that tunneling is an extremely good experiment to measure the gap because this I verses V characteristic from a normal to superconductor tunneling gives the gap measure of the gap at low temperatures. And the differential conductance gives a direct probe, acts as a direct probe to the density of states of a superconductor and that is as you can see here, there is this features of pileup of density of states is to right after the gap and then, there is this tailing off and that was obtained by tunneling measurement.

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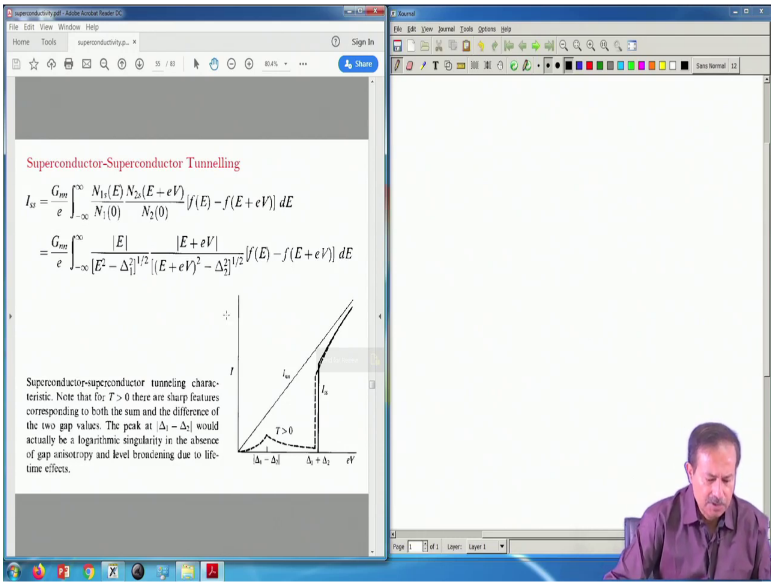

So, tunneling has come to be an extremely important measurement technique for superconducting density of states. The nature of the gap and if the gap if there is a full gap or there is a pseudo gap and all kinds of things can be can be obtained from tunneling measurements ok.

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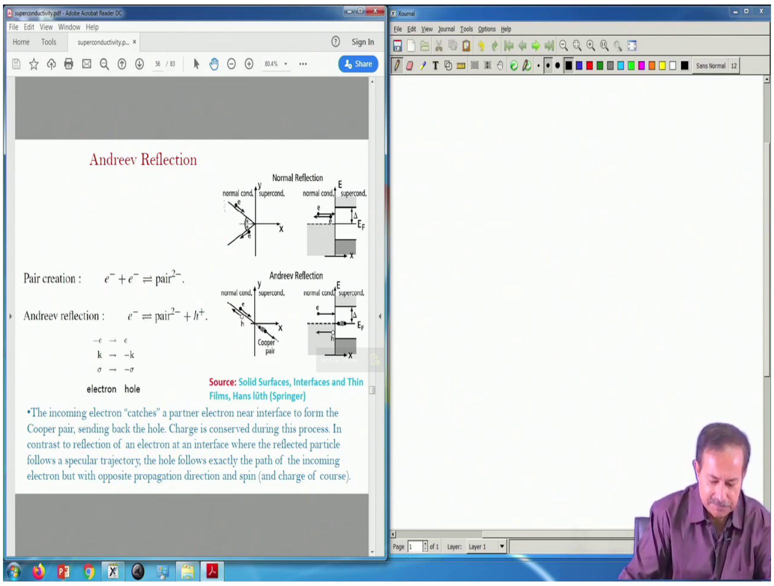

Then, we will discuss something extraordinary which is that there is this famous tunneling situation which is called the Andreev of reflection, in which the an electron from a metal is injected into the superconductor and then, in that case apart from the specular reflection that can happen at any boundary.

There was this remarkable phenomenon that instead of the electron coming back a hole comes back and not only does it come back, it just retraces the path of the original electron backwards. So, this was a this was actually a it cannot happen unless there is a superconductor on the other side. So, this is a for example, gaps are there in semiconductors.

So, I can think of normal to semiconductor tunneling, but we cannot get this Andreev of reflection; for example, at a normal to semiconductor junction. So, anything which does not produce this cooper pairs on the right hand side will not produce this Andreev reflection. So, that is actually a telltale signature that indeed cooper pairs are produced, there are two particles bound states on the a right hand side; particles pair up and this hole, then has to come back on the left hand side. And just for from conservation rules, it has to retrace the path of the original electron backwards.

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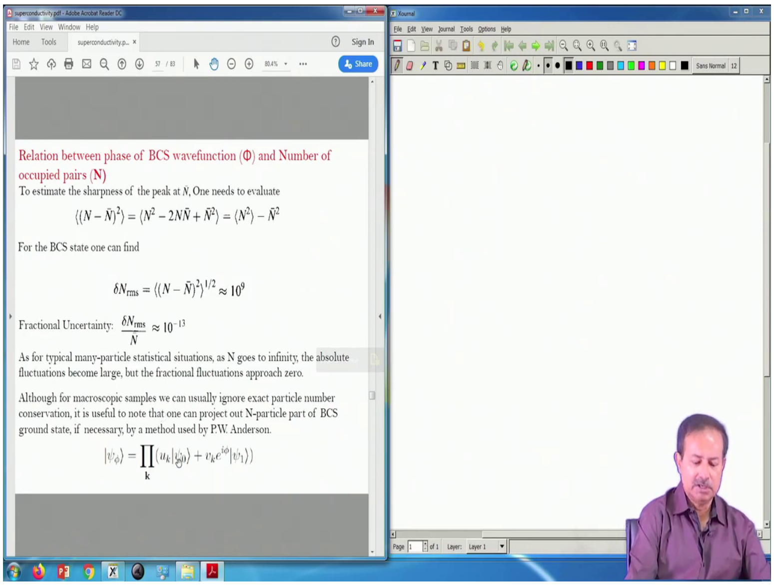

So, that was a that was a remarkable experiment, we discussed and then, we will discussed how to project out the N particle states or N by 2 pair state particles there are N particles; that means, N by 2 pairs out of the BCS ground state. Look at the BCS ground state, it has enormous fluctuations as it is not a number eigen state. Although the fractional fluctuate fraction delta N rms by N is still very low and that is why you can do a theory of superconductivity in a macroscopic system, without conserving the number you can work on it. But still the with the fact that you should be able to show that there is a way to get the finite number wave function.

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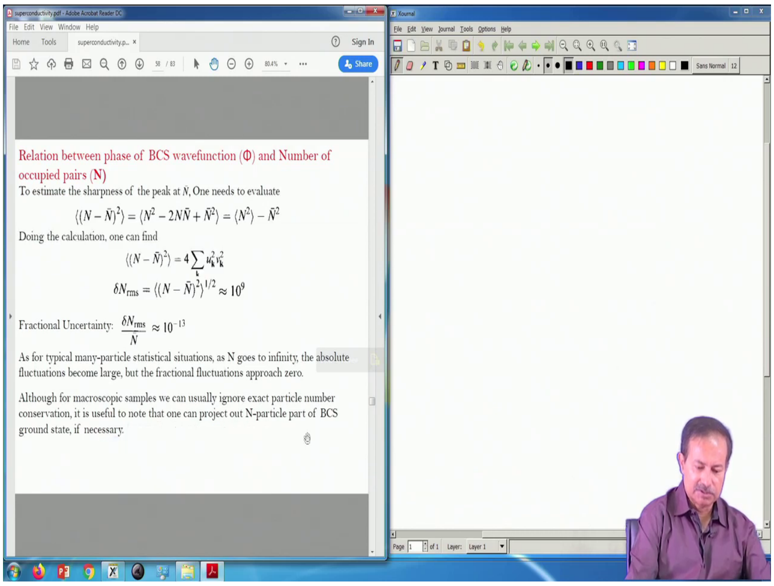

and that is a from here one just goes one more step and integrates out all the phase and then, when N by 2 is equal to this see this v k is contain the phase.

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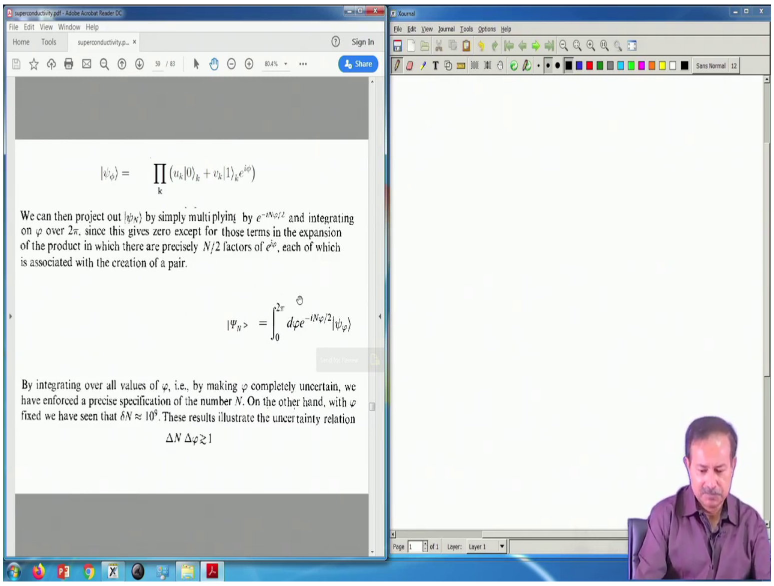

So, the suppose there are n number of N by 2 number of vk's in this wave function and when and that particular one will be picked up by this N by 2 because there is there will be a delta function right. So N by 2 minus say alpha times phi; if alpha is not equal to N by 2 then this will be 0. So, only the alpha equal to N by 2 term will be picked up from this product and ah; that means, that you have taken alpha times vk so, twice alpha number of electrons.

So, that is the way to pick out a finite number of a finite number eigen state from this and then, that that is another corroboration that the there is this uncertainty the delta N delta phi must be greater than 1. And that is born out by this, this calculation, this famous very intuitive very nice and very simple calculation.

And this also predicted the way forward towards something called Josephson Junction which has come to be known as Josephson Junction. And it just shows that if there are two superconductor, the same superconductor for example, take niobium for example, two niobium superconductors are joined together and may be any which are insulating very thin insulating barrier. Then, these two superconductors conductors are identical; they would like to become one superconductor.

And then, the phases must it could be equated the phases must become equal and in order to change phase, you have to change the number that means, electrons or pairs have to flow from one side to the other that means, there is a current per without any field without any potential. So,that is actually what is this famous josephson current is and that is and that happens only if you have superconductors, where the good quantum number is the phase and not the number. Numbers can fluctuate. Once you try to fix the phase, numbers become uncertain and that is the ramification of that is this flow of current without a voltage. We will come to that towards the end.

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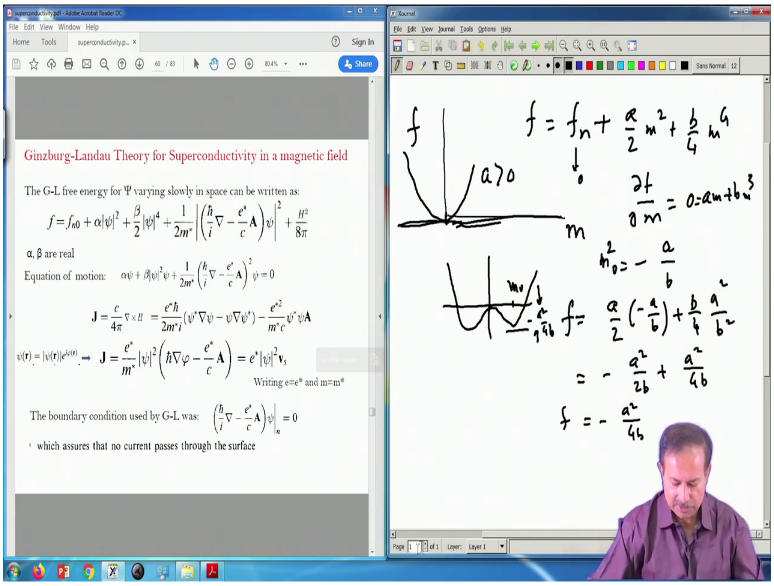

So, let me just now go ahead and write down the Ginzburg Landau Theory, which I had introduced the other day and the idea is that you have to write the free energy in a way that, the free energy as a function of some order parameters say m. And that is a that is what is done here and then, what we showed was that from this kind of a free energy with say free energy as a function of so free f. So, let me put the normal state free energy equal to 0. This I said to be 0 and so, this free energy is an expansion close to t c about the normal state free energy.

So, we can write it as a 2 a by 2 m square plus b by 4 m to the power 4; this by a by 2 and b by 4 are chosen in such a way that once you take the derivatives this 2 will cancel with this 2 the 4 will cancel with this 4. That is about it, there is nothing; no mystery about it. So, this is the situation when a is greater than 0. Remember b is always greater than 0; otherwise, you cannot have a you will have a m going to infinity solution.

So, the situation changes when a becomes less than 0 and that is what we showed. That this will become m naught and m naught square equal to minus alpha by b beta minus a by b sorry. And a being negative. This is perfectly alright. This is positive. So, you can actually put it back in and then, the f minus fn; fn is said to 0. So, let me just again not use fn and then, f equal to in the ordered phase, where you have a finite m naught is the solution. See this, I obtained by taking a derivative of del f del m equal to 0; equal to 0, the solution of that is this and then, I put that solution back there.

So, a by 2 into minus a by a by b in b plus b by 4 into a square by b square ok. So, this is minus a square by 2b plus a square by 4b. So, it is minus a square by 4b ok. So, this gives me m a m plus b m cube. So, that solution is m square equal to minus a by b. So, this is the free energy. So, that is the value. So, that is our minus a square by 4b. So, that is a stabilization that comes is a negative energy and that means, that your energy has come down below the original m equal to 0 state, which has which is at 0 energy here. At m equal to 0, it is 0.

So, that is how Ginzburg Landau of free energy is done. So, let me just tell you a bit more formally, what is done for superconductors. For superconductors same thing is done, but the choice of order parameter which is non zero below the Tc and zero above the Tc, is a bit it requires a bit of intuition and that is what these people did. One could for example, choose gap. Gap is non zero below T c and 0 above T c. But gap is not the right quantity to choose you can show. Of course, later on Gorkov had shown that this psi that is chosen here has some relation with the gap, but let us not get into that detail let us just follow what Ginzburg Landau did.

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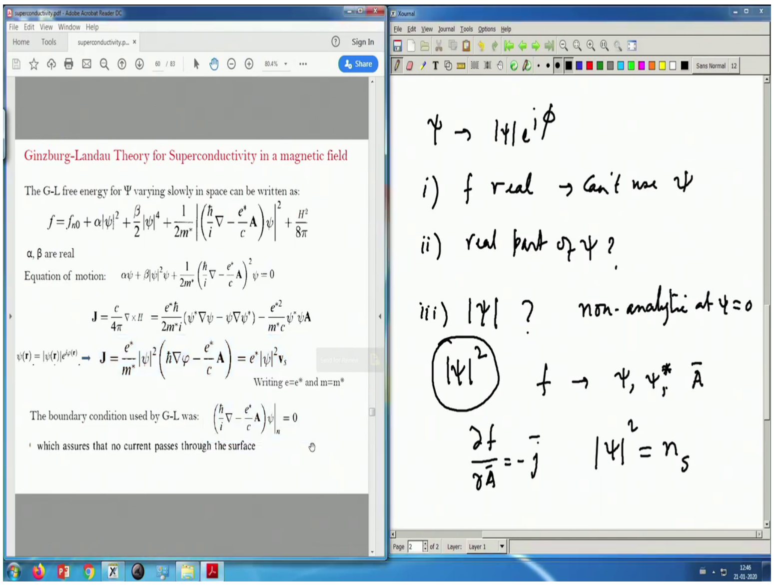

So, what they did was that the chose issued kind of a pseudo wave functions psi and in terms of that they wanted to write down the free energy and psi's in general complex. So, it has a it is some psi e to the power i phi say. So, it has both magnitude and phase. Now, three things one should note is that the free energy is a real quantity. So, you cannot expand in terms of psi. So, free f is real. So, cannot use just psi right. So, this is kind of a polynomial expansion, we are doing around the free energy of the normal state at and this is so valid very close to T c because otherwise psi will be large and you cannot do that.

So, mod psi has to be small in that region if this is valid. So, and then problem is that psi cannot be used because it is a complex quantity. Then, one could think of psi real part; real part of psi well that also cannot be used because then that will depend on the phase, absolute phase of the wave function and the free energy should not depend on the phase of the wave function of this wave function psi. I mean the free energy should not depend on the choice of your phase. And three is can we choose mod psi? Sorry, mod psi the problem with mod psi is that every odd power of mod psi is non analytic at psi is equal to 0.

So, so mod psi, but this is non analytic at psi equal to 0. So, that you cannot choose because free energy is after all a analytical function. You have to take derivatives and all that. So, so then the natural choice that is left be left for them was take mod psi square. So, this is the order parameter, they chose and that is how the whole thing is done. So, that is an expansion using mod psi square. So, once one does this, this algebra you can you if you like you can repeat, it is not difficult at all. Those who have done classical mechanics, have used these classical field theory, have used this. But in few if you do not want to do the algebra, it is worked out at places you can try to take a look at it; its absolutely straightforward.

It is not difficult at all the notation, these complications come from this term the canonical momentum anyway. So, once so, the minimization has to be done with respect to mod psi with respect to psi and psi star and you can so minimize f, you have to minimize with respect to psi and psi star and you can get this equation. The h square by 8 pi is added because there is a magnetic field, because there is A sitting here curl A is A. So, that has to be incorporated there, because the magnetic field has its energy density right which is h square by 8 pi so, ok. So, the equation of motions, there are two there is another one through here another variable which is A, the vector potential you have to extremize the free energy with respect to A as well ok.

So, then what you get these two equations. This is the first one; this from minimization with respect to psi star and this one is minimize by minimizing with respect to A. Remember A is also contains A through curl of A. So, that has to be that has to be realized. So, and the this is of course, that gives the Maxwell equation that will show that that J normally a del f del A is equal to j the minimal coupling is j dot A. So, it gives you j. So, that is so that gives me this J which is equal to c by 4 pi curl of h from Maxwell equation and that is equal to this ok. So, this is done then and so, minus sign.

The next step is this that you can write psi as mod psi e to the power i phi and assume that mod psi is a much weaker variable in terms of r. So, mod psi does not vary too strongly with respect to r. See all these terms that the gradient term that has been taken here is because the superconductor now is in homogeneous. So, there may be regions which are superconducting, strongly superconducting, then may be regions which are not do strongly superconducting. So, it has an r dependence. Earlier when we were doing, doing this is theory superconductivity was uniform. Here, of course, it is an inhomogeneous situation.

So, for that it is you cannot do BCS theory and this is probably the best shot that one can do and this grad term takes care of this in homogeneity and then so, basically a change in psi costs energy and that you would take care of that by this, this gradient term. So, both psi and grad psi call I mean having those costs energy and then, that energy is actually psi stabilizes and grad psi increases energy. So, we have to be careful and we have that is why these all these terms are taken care in the free energy. They are kept in the free energy.

Anyway so, I get these two equations. These two, these two are my equations of motion and then I can just a sorry these and these are my equation of motion and writing psi r as mod psi r e to the power i phi I, I can just convert this to a gradient with respect to phi, the phase assuming that the mod part the amplitude part varies very slowly and that is basically this equation that gives me the square times. So, if mod size square can be identified for example, I mean that is what in the back of their mind because it is a kind of a wave function they thought and this should be then connected to n sub s the super fluid super superconductor fractions, super conducting fraction.

So, and then of course, then you can see that this equation is what you expect from simple quantum mechanical arguments ok. The boundary condition where they used was this that there is no current passes through the surface ok.

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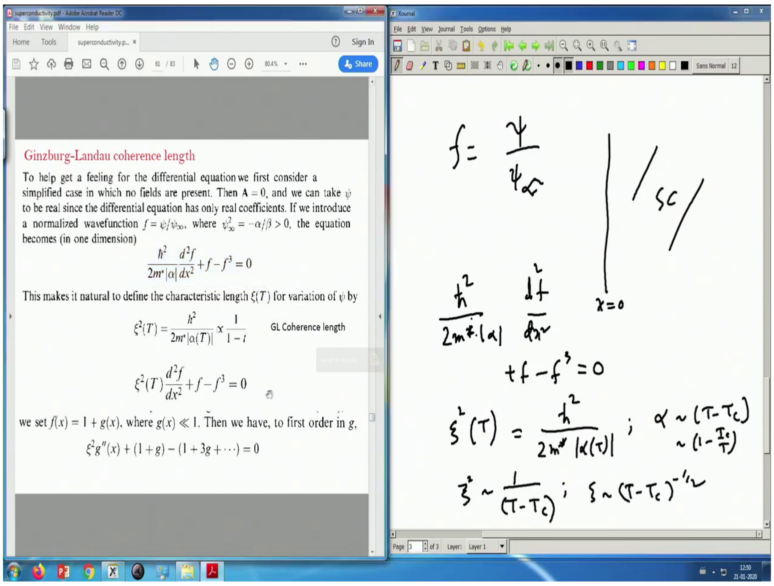

Now, let us just try to solve this equation and first thing is that we define a variable which is f equal to psi by psi infinity by psi infinity one means that the value deep inside the superconducting region. So, it is in the bulk value, way far away far away from the surfaces or defects or impurities or if there are magnetic fields around inside which we will come back to.

So, all that is no disturbance, undisturbed bulk value of superconductor superconducting wave function. So, this is the. So, psi may vary within a in a certain region in inhomogeneous a superconductor. For example, it will vary and that is what we are after. So, we will see how this quantity f behaves from these equations and that will tell us how psi b goes to its bulk value which is the psi infinity.

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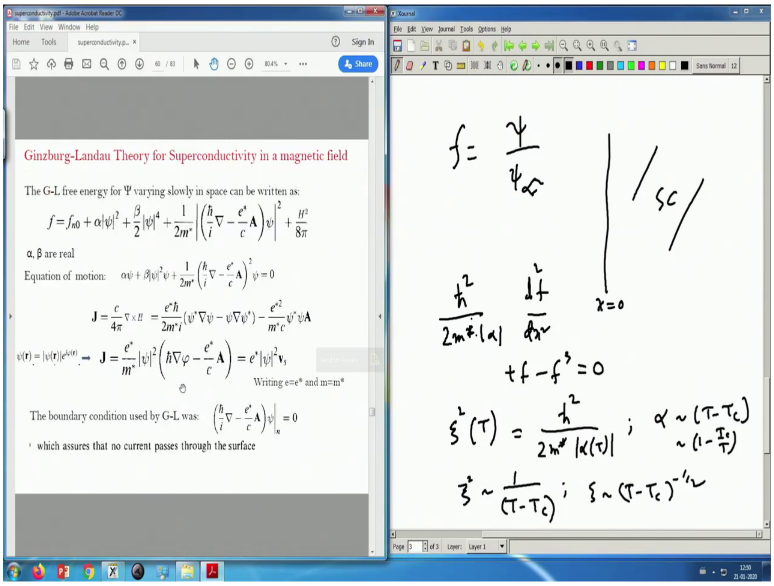

So, then one can just read just from this first equation, from this equation if you have A equal to 0, then you can just write this equation in one dimension for example. Suppose, you are you have a semi infinite superconducting this side is superconductor. And then, you can just the only variation is along the x direction; because this side is non superconducting, this is superconducting starting at x equal to 0 say and then, all that varies is along x equal to 0; y and z are perfectly symmetric. So, you only look at the x variation and write down this equation.

Then, look at this equation, there is this h cross square by twice m star mod alpha d to f dx 2 plus f minus f cube equal to 0. This is what you will get. I mean this is absolutely simple algebra. You just substitute this, divide the things by psi infinity; every side divided by psi infinity and you will get this equation from the previous one, from the equation of motion ok. So, what does it give me? It gives me this h cross square by twice m star alpha times this. Now, f is dimensionless because it is a ratio of 2 psi's. So, the only dimension comes from x that means, this h cross square by 2 m star alpha should also have a dimension of length square, just to cancel out this x.

So that means, you have you can define a length scale psi square which is of course a function of T. As I said the other day that the coefficient alpha is generally chosen at T minus

T c. So, that it is positive above T c and negative below T c, as we as I just showed in the first discussion the a coefficient which is the alpha coefficient here.

So, psi square T is h cross square by twice m star alpha. So, alpha is a function of T explicitly put in here. And then of course, as I showed that alpha should be proportional to T minus T c which is 1 minus T c by T which is 1 minus small t; small t is T by T c. So, that is put in here. So, psi square is proportional to 1 by t minus T c; that means, psi is proportional to T. So, psi goes as T minus T c to the power minus half.

Now, there is one thing you must have noticed that in these discussion, we are using this e star and m star and that is what is generally used because at that time, no one knew what is the charge of these objects that we are dealing with and what is the mass of these objects, we are dealing with. Then and now, we know of course these are 2 e and 2 m. So, that is what m star finally, one puts into to connect to experiments, but generally you can keep e star and m star ok.

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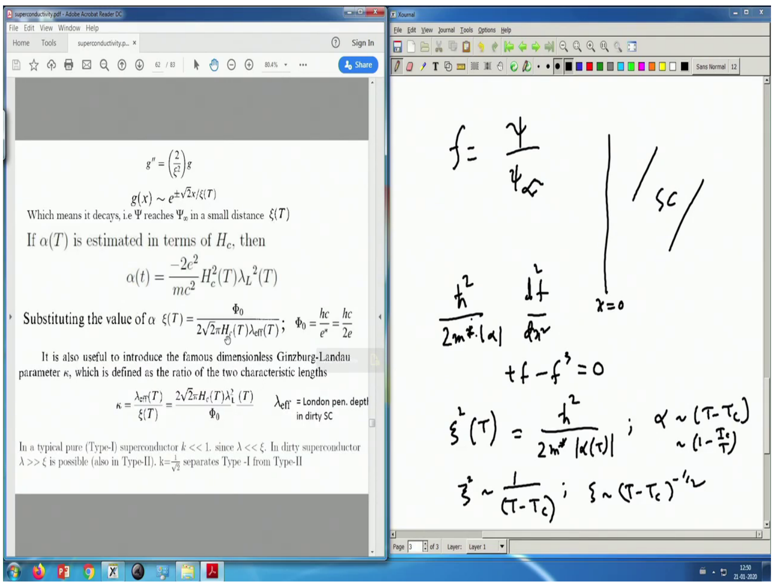

So, this is read down the equation has now become this that size square d 2 f dx 2 plus f minus f cube equal to 0. So, this is the equation that we will start now, and work through it and show that how this equation gives me the psi deviating from psi infinity and how that deviation behaves with as you go inside the superconductor.