Electronic Theory of Solids
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Lecture – 56
Electrodynamics of Superconductivity

So, we are discussing tunneling spectrum in a normal to superconductor or superconductor to superconductor junction. And first, what we did was to look at the tunneling between a normal and a normal state and then we found out that it is ohmic; that junction is ohmic. Now, what we are going to do is to look at the tunneling between normal to superconductor.

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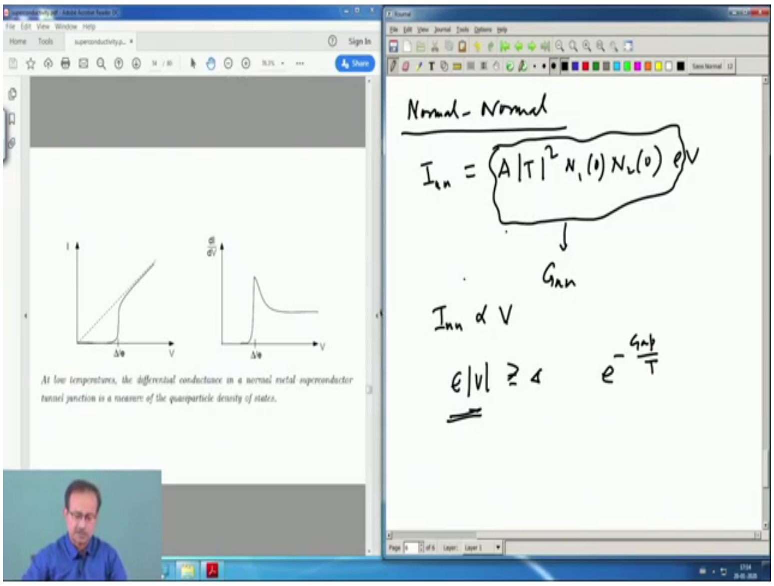

And we have just seen that, you cannot have the tunneling unless your E V; the voltage times charge E V matches the energy of the of delta. So, delta is the distance of the excited level from the putative Fermi level and then that is, so pic in terms of pictures.

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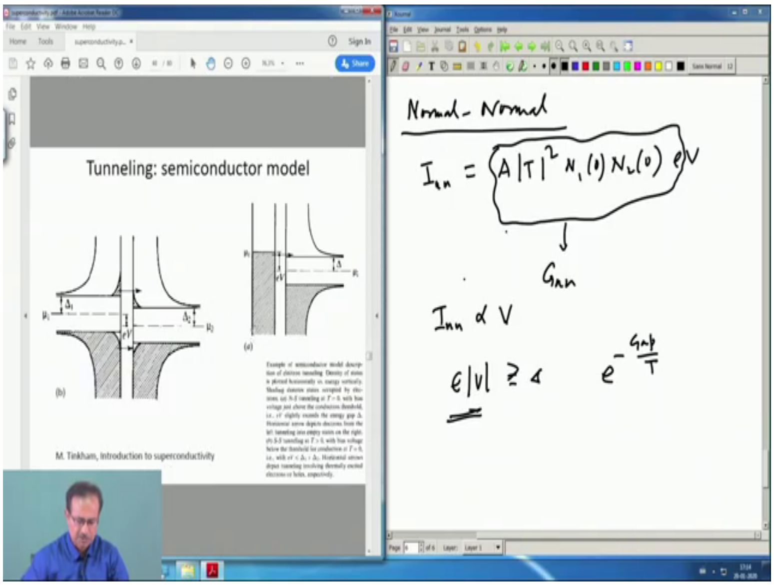

So, this is the picture, so your delta is here, and you have to supply your e V has to supply this kind of energy, ok. And so, this is normal to superconductor tunneling and this one is the superconductor to superconductor tunneling.

So, you look at both sides, you have this density, superconducting density of states and your tunneling. Here is on the one side you have continuous states which are normal metal and on this other side you have this gap and there is a large state here, then say divergence on both sides, right. And these are occupied, these are unoccupied. So, from Fermi level or chemical potential, you have to supply this amount of energy that has to come from this e V, to give you the tunneling.

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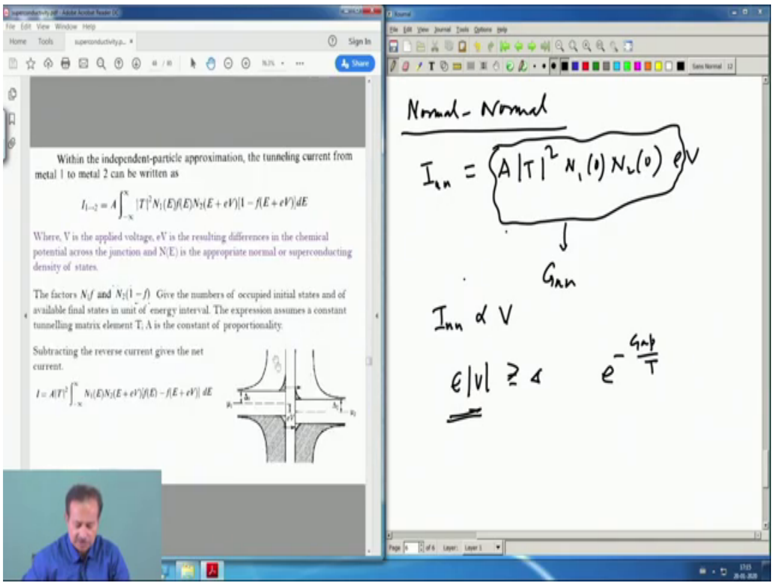

This is actually intuitively quite obvious and that is what your calculation also shows. The thing ok, so the let me show you some picture first. So, this is the current versus voltage for example, and as you can see that the normal one is this one dashed one; whereas, the this one has a gap. The normal to superconductor tunnel junction gives you a. So, there is a no current till your voltage hits delta by e ok; and then it starts picking up and then it becomes, after that it becomes linear. The other thing which will come to is something interesting, which is the differential conductance dI dV.

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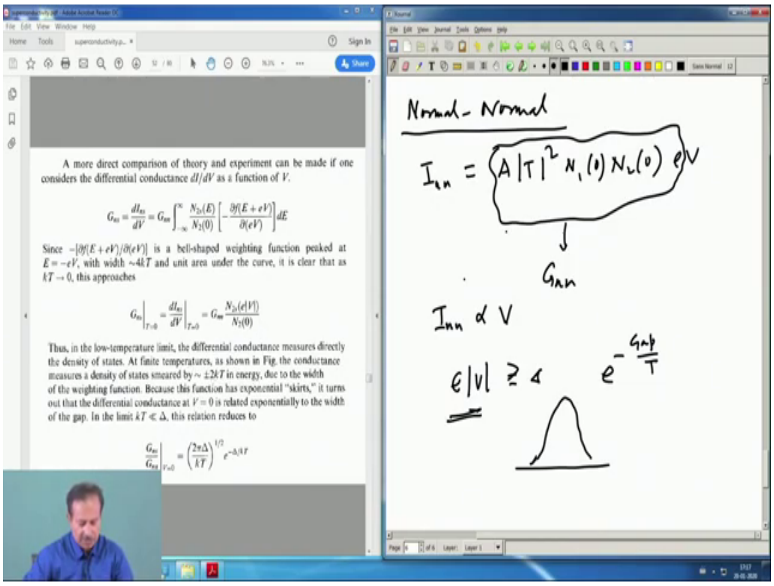

So, from that same expression, you can now just take a derivative; and the only place energy appears, voltage appears is here in inside the Fermi function, and so you just use this derivative. And again we know that the derivative of Fermi function is has a bell shaped structure right; negative derivative of Fermi function has a peak structure.

So, and the peak width is about 4 k T. So, in a region around 4 k T, you will get this smearing, so called smearing due to this peak and that is so, it is plus minus 2 k T. So, that the result, therefore is that, you will get a G. So, this is G n s equal to d I n s d V is 2 pi delta to the power half k T.

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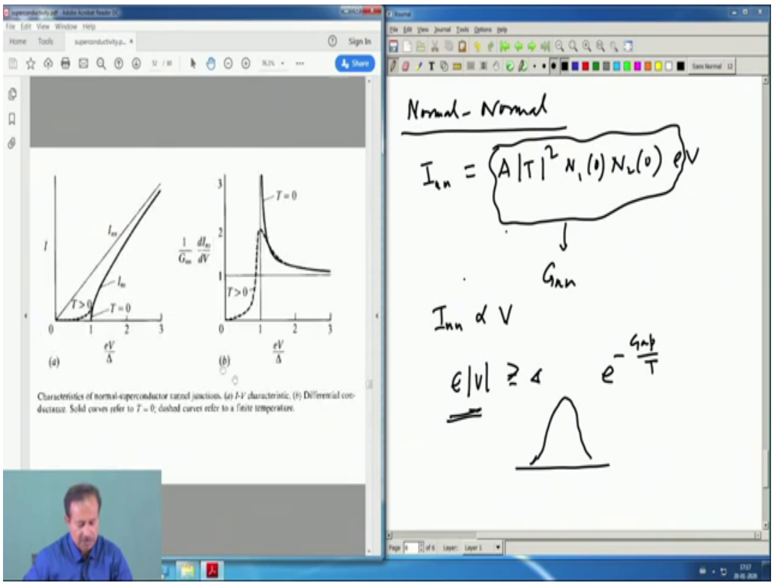

And it actually follows the at T equal to 0, this has this is just divergent this goes up and at just look at the structure; it is exactly like the density of states at T equal to 0. At finite temperature of course, there is a smearing and that smearing follows this curve; but this also tells you that there is a large pile up of density of states at e V equal to delta.

This is the other curve that I just showed, this is the I versus V and at T 0, then it starts from the solid line which starts from e V equal to delta and then it goes up. At T greater than 0, there is again these small carriers which are thermally activated; they are they contribute even below delta. So, that is something that is expected and that is exactly what you see. If of course, BCS theory is correct and these are experiments that actually sees that, BCS theory, these experiments see that the BCS theory indeed is correct, it gives right results.

So, this is a simplified picture and d I d V basically follows the density of states on the on this side and as a function of V. And this is actually true, I mean this experiment is a very important experiment, tunneling experiment; and by looking at the differential conductance to directly look at the density of states, it is a major experiment that one does in superconductivity.

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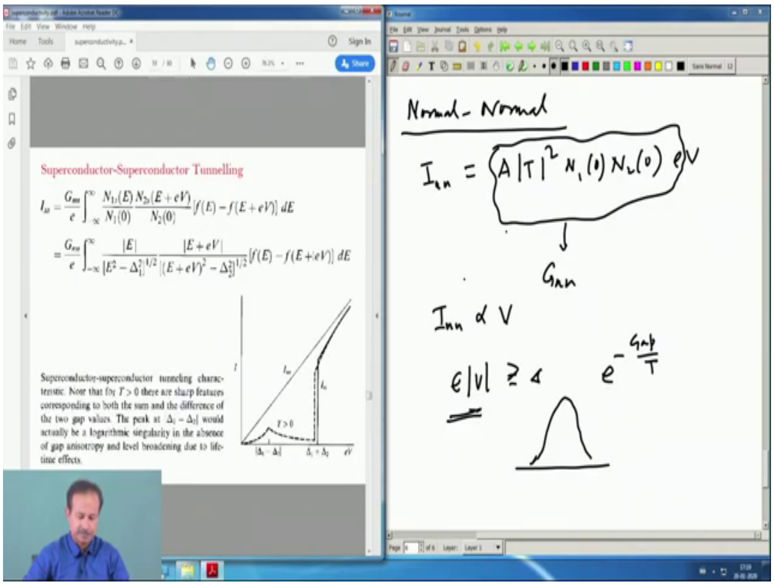

This is superconductor-superconductor tunneling and it has again a characteristic, which is a bit different in the sense that there are these two gaps. So, you have to consider the two density of states at two energies. And then the density of states forms you put in and then see where the conduction starts.

And this is the picture; see the I nn is ohmic and this one has this kind of structure, this is the I ss, ok. And there for T greater than 0 of course, there are these excitations available and you can actually check that there is this two sort of kink type structures at; there is a kink types peak here, at delta 1 minus delta 2 mod. And it starts picking up from delta 1 plus delta 2. So, your two gaps are there. So, your e V has to cross these two gaps and it starts conducting at that point.

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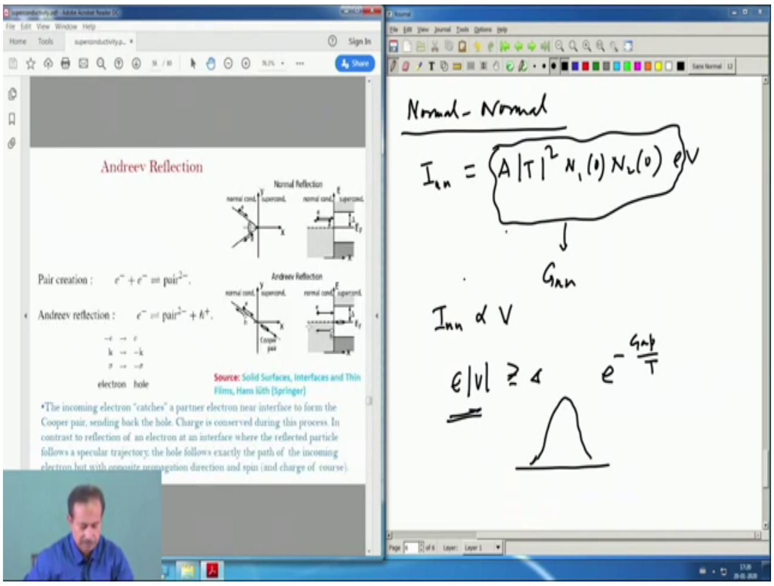

But there is an interesting tunneling situation that is very specific to superconductivity and that was noted by noticed by Andreev; he actually showed that it is not, of course those tunnelings do happen, but then there is another kind of a situation that can arise and it is very specific to superconductor. So, for example, tunneling from a normal state to a superconductors conducting state in which such a situation will arise and this is a brilliant intuitive idea and let me just explain you in simple terms and it is actually very simple.

So, now suppose you had a, you have normally looking at a reflection of a an electron from; when it comes from left strikes this interface and goes back to left. It is a generally is called a specular reflection, it follows the more or less it will be at these two angles are the same and it goes back. So, it comes, hits it and goes back at with the equal angle. That is the usual story or it can enter inside with the if your if you as you saw in this other Gaver's experiment. It can also go into the right with exponential tail, till nothing will go in until at zero temperature until your e V is equal to delta, then it will pick up. So, that is the standard scenario and the other scenario is that it will just go back with a theta 1 equal to theta 2 angle, so this is fine.

But then Andreev aggrieved and he showed that indeed if an electron comes here, there is another scenario that is possible; that it suppose it does not have sufficient energy to enter that e V energy to enter on the right, it can catch an electron at the close to the boundary and form a pair and that pair can enter on the superconducting side; its a cooper pair, so it goes in and becomes part of the condensate.

But then since it caught an electron and then entered, you have to balance the charge; so what happens is that, a hole goes back to the left. So, the charge is balanced. Look at this the; see electron plus electron is a pair, you can think of a pair as an electron is equal to a pair plus a hole; so an electron comes, becomes a pair, part of a pair and sends back a hole, so that is everything is then satisfied. The thing that is different here from the above picture, above picture of course, the electron went back; but here not only does the hole go back, it goes back exactly at the in the same direction and path to conserve the energy momentum everything right and of course, the charge.

So, it just retraces the path that the electron came from came through and it goes back as a hole. And this is, so this is what is shown here; an electron becomes a hole. So, and on the electron side an electron came with charge minus e with momentum k and spin sigma; and on the reverse direction a hole goes back with charge plus e, momentum minus k and spin minus sigma. So, that everything is balanced.

So, this is called an Andreev reflection and this is seen; and this is actually a terrific signature that you really have a superconductor with a gap on the other side. And it is a remarkable situation, it happens only from normal to superconductor transition; I mean this scenario that I showed is normal to superconductor and that is why it was seen. And this is an example, why superconductivity is, another example why superconductivity so exotic; because you send an electron at the interface and you get back a hole which is absolutely retracing the same path on the left hand side.

So, you send a current and you get back a current; because hole moving to the to backwards is the same as current, so the current remains conserved. So, these experiment is has been done and this is named after Doctor Mister Andreev, he who first did it.

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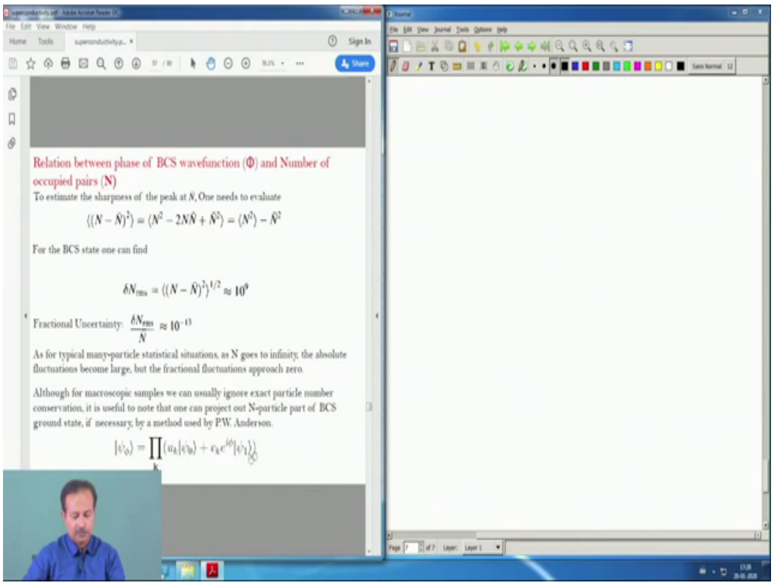

There is an issue that I mentioned right in the beginning and there is at least twice is that the superconducting wave function is not BCS wave function, is not an Eigen state of the number. Now given that, one can actually calculate the fluctuations in number and one can actually show that the fluctuations for a macroscopic thermodynamic system can be negligible. And that is a very assuring thing, because otherwise you would not be able to do, able to form superconductors; because the superconductivity will just fluctuate in number, the state , the that state will have such huge fluctuations in number that it will be difficult to do anything with it.

So, the , so what one can calculate; I am not showing the calculation, because that is not necessary, it is just an information you can keep that, you can calculate the mean square fluctuation from. So, this is from an deviation from the mean and square and average of that and that gives you a number for about a typical system which is a Avogadro number of particles, this delta r m s which is square root of that, turns out to be above a billion, ok.

And then you divide this by N, then you will have 10 minus 13. So, this is like a statistical situation in a many particle system; when n goes to infinity for example, extremely large and this will go to 0 and 10 minus 13 is already for all practical purposes 0 and the fluctuation, the fractional fluctuation goes to 0.

So, you can actually work with BCS wave function and this is in a macroscopic system, then you can ignore exact particle number conservation; but one can actually go beyond this and that is what one can do. From the BCS wave function, you can actually project out the fixed number part of fixed number wave function also the. So, suppose you want to find out the wave function; the what is the wave function corresponding from here , which is the term that gives me say N by two pairs or N particles, ok.

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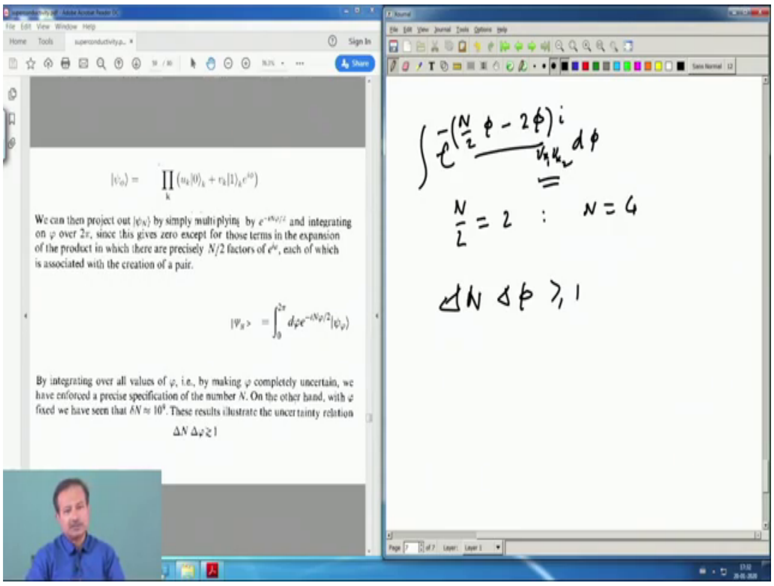

So, that can be done and that was done by this simple trick that. So, what you do is that, you add a phase to it this u k v k are; they are not real , they may not have to be real, they are complex and you can write a relative phase here e to the power I phi. And then this is the BCS wave function and then what you do is that, you just multiply it by e to the power minus I and phi by 2 and then integrate over all phi.

Now, see what happens; for example, suppose your N is. So, N by 2 is equal to say 4. So, as we remember, there is just two k states, k 1 and k 2 from which I found out that, the state there is one part of the wave function is one part, it was a linear combination of zero, one and two pairs; that means, four particle state was there with a coefficient v k 1 into v k 2.

So, that part of this wave function, we will have a phase which is e to the power twice phi, right. So, 2 into phi means, 4 into phi by 2. So, this will give me e to the power N by 2 into phi minus this one which is a 2 into phi into i, right. And then and we are integrating with respect to, so this is minus; we are integrating with respect to phi d phi, and that will be what; because this will be just a delta function and then that part it will be N by 2, it will be non zero when N by 2 equal to 2; that means, N equal to 4.

So, by just writing it, choosing the corresponding N, I can find out that part of the wave function; where there was which is, so there was a v k square here, v k 1 into v k 2 remember. And e 2 into e to the power twice minus 2 phi and I just pick out that part of this wave; out of the entire set which is a linear combination of all N by 2 possible pairs, I can just pick out the one I want by choosing the corresponding N. So, by integrating over all values of phi; that means, when you make phi now uncertain, is completely uncertain by, because we are integrating over all phi between 0 to 2 pi and; that means, your you are picking out a particular N.

That means this conjugation exists that, when you; when phi was fixed then delta one was of the order of a billion. Now, I am making phi unfixed, integrating over all phi's uncertain, then my N gets fixed so; that means, there is this uncertainty, it follows the uncertainty delta N delta phi greater than equal to 1. So, this is an another example as that that BCS wave function is a beautiful new kind of a phase actually a coherent state, and it is a wave function that you have not seen, we have not seen before in our study of electronic theory of solids.

This is probably the first instant when where a number non conserving wave function was used to study a problem and it shows that the BCS wave function is actually a wave function with the fixed phi phase; whereas you can pick out the particular number wave function, part of the, of the out of the wave function you can pick out the particular part which has a certain number, definite number and that is how it is done. And that is a beautiful aside I thought I should show you; it was first shown by Andersen.

And its consequences are terrific; because if phi is fixed, then the numbers can fluctuate. For example if you took take two superconductors; one has a phi 1, another has a phi 2, then they will try to become try to have the same phase, because BCS theory, BCS wave function has the same, has a fixed phase. So, the entire system will try to become a superconductor with the same phase right; both are superconductors, so they will try to become the same superconductor and they will match their phases. By changing phase, that means one of the phase or both the phases will adjust, they will change; that means numbers will fluctuate, that means number of pairs will move from left to right or right to left.

That means by just putting two superconductors together; you can have a flow of current and that is actually something that happens and that is shown to be a beautiful, that has tremendous applications and that we will come towards the end of the this lecture.

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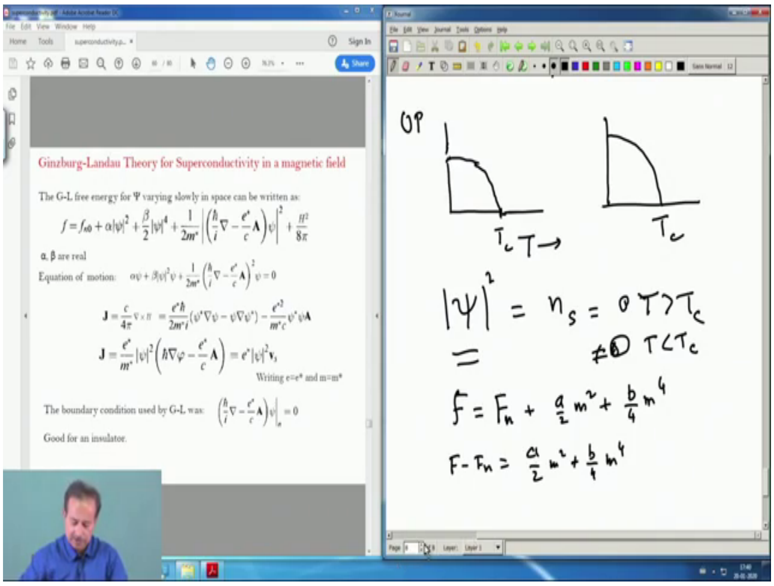

So, let me just go to a topic which is required when we do electro dynamics of a superconductor. And because you want to understand what happens when a superconductor is placed in a magnetic field and, so that requires a treatment which is a bit different, we do not approach that problem from a microscopic theory. And for that, an approach based on Ginzburg Landau theory is the best approach to follow and that is what we will start now.

So, our aim now is a bit different; what we are trying to work out now is to see what happens if a superconductor is placed in a magnetic field. Now of course, we know that there is Meisner effect; so all the fields will become expelled. But in a remarkable paper a, because of showed in late 50s around 57 that, that is not the case always. There is a state possible, which is where the magnetic field can penetrate the superconductor depending on the conditions that we will outline; and that in that state superconductivity and magnetic field, magnetic flux inside the superconductor are co-exists.

In the sense that the flux actually penetrates in some tubes, tube like structures, which are called vortices and the superconducting order parameter goes down to 0 at the core of these vortices; and the rest of the situation, outside this vortex the superconductor remains a normal superconductor, just like a the superconductor it was. But this is a unique new situation where Meissner effect is in a sense not followed; but the super the vortices, the magnetic field enters the superconductor in vortices. So, and that vortex that vortices, these tubes are not random, what because of showed, is that they form a lattice of their own. So, that is like a crystalline object of vortices of these tubes, ok.

So, let me start from Ginsburg Landau theory instead of writing this long beast on the right. Let me just give you an example of Ginsburg Landau theory starting from absolutely simple arguments. As I said, suppose there is a phase transition as a function of temperature, we know that the there is something called an order parameter. I mentioned that something which is non-zero below T c. So, suppose this is T c, so this I called order parameter; it can be magnetization for magnets, it can be the density for a liquid to gas transition and so on and so forth.

So, this we have seen that these order parameter which is nonzero in the ordered state and like in spins case we did it was a magnetization or staggered magnetization for antiferromagnet; this will, this generally follows a structure like this, a behavior is like this sorry. So, it follows a behavior which is that; that it goes down to 0 at T equal to T c, ok. So, it continuously goes to 0. So, in superconducting case also we, as we showed that this super conductivity; for example, goes to 0 at T equal to T c.

So; that means, we have to write, we should be able to write down an order parameter for the super conductivity which is 0 beyond T c and nonzero below T c, ok. Above T c it is 0; below T c it is nonzero. And Ginsberg and Landau we are able to write down a an order parameter which is like this. So, they said that, let us think of a superconductor wave function, which is like it is a pseudo wave function you can call it; whose squares gives me the superconducting density, ok. So, and because this psi can be complex, so I put a mod there and mod psi square are gives me n s, the superconducting density.

Now, superconducting density, super this is called the super fluid density or superconductor densities, so whatever. So, this n sub s , the superconducting component of the density is 0 above T c. So, this is 0 for T greater than T c and finite nonzero for T less than T c, so this is a good choice for an order parameter. So, then some mod psi square is the order parameter that they chose.

Now, you can easily see that, suppose I have a free energy for example, F equal to the, so close to this is done very close to T c. So, that I can expand this the around the normal state to free energy plus some a by 2 m square; m is, suppose a m is my order parameter plus b by 4 m to the power 4, ok.

So, this , even powers are taken; because I assume that there is an m minus m symmetry which is for example in a ferromagnet we have seen that, without a magnetic field that symmetry exists. So, in that case F minus F n close to the transition; because close to transition I can expand in this polynomial, because m is extremely small at T c m goes to0. So, this is the result right, a by 2 m square plus b by 4 m to the power 4; I still do not know what a and b are.

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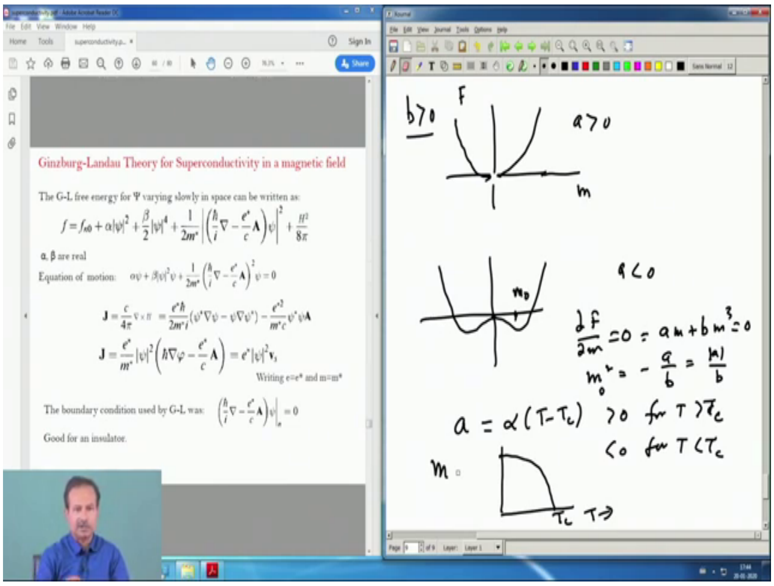

Now, what let us just draw it for b has to be greater than 0 you see; if b is not greater than 0, then m will become infinity. Because then the free energy will be, you will gain a lot of free energy by making m going to large, because b is negative; that is the highest coefficient of the highest power, you should not do that. That is clear from this graph that I am drawing.

So, F minus F n s, let me set F n to be 0; I mean I am starting from F n equal to 0 you set my free energy scale there. And verses m, if you look at it for a equal to 0, b equal to 0; sorry a greater than 0 and, so this is a greater than 0, remember b is always 0. Otherwise if a and b were both negative, I would get a curve in this direction right; and then the you can see that the m will choose values at infinity, so that this energy goes to minus infinity. So, that is not allowed, that is never happens. So, I have to have a positive b, which is this. Now, in positive a this is what. So, the solution is minimum is that m equal to 0, right. So, that is the solution.

Now, I will make a less than 0 , then what will happen is that; you will see that the curve now looks like this. So, the m equal to 0 solution which was minimum, now is no longer the minimum; the minimum has shifted to m naught. How much is m naught? I can easily find out, I can just do del F del m equal to 0. So, that gives me a m plus b m cube equal to 0. So, m equal to 0 solution is not what I am looking for and a is negative, so m square equal to minus a by b; a being negative, minus a is positive. So, this is basically mod a by b. So, that is the solution m naught.

This is of course, symmetry exist plus m naught and minus m naught a degenerate. So, this is what Landau and Ginsberg started using that, depending on the sign of a; you can go from a m equal to 0, order parameter 0 state which means you are above T c disorder state to a state which is below T c, which is a finite m, which is a finite value of the order parameter. And what they did is that; simplest choice is that take a to be such that it is some alpha times T minus T c.

So, it is greater than 0 for T greater than 0, T greater than T c; and less than 0 for T less than T c. So, it mimics this situation right; that a will become negative as your T goes below T c and then immediately your you will find that, your you have a finite m solution and this gives, you can solve for m as you as I have just done. And this will give my T minus T c to the power half, so that is. So, m verses T now we will m. So, m versus T will have a solution of this kind; again T equal to T c it will go to 0 this.

So, that is the that is what Landau and Ginsberg did and that is what we are going to follow. So, we will set mod psi square equal to 0 above T c, mod psi square less than 0, mod psi square finite below T equal to T c. So, that sets of the theory for Ginsberg and Landau and that is how they did it and that is how superconductivity in a magnetic field is done and that is what we will follow from the next class.