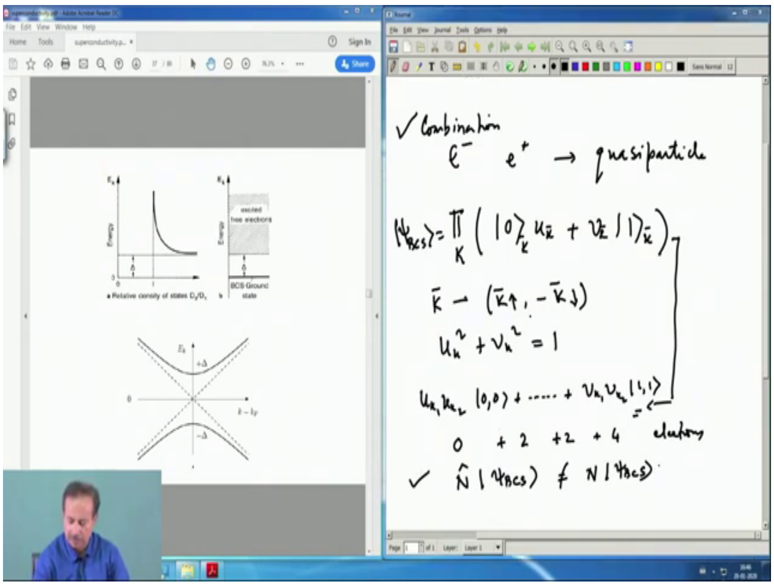
Electronic Theory of Solids
Prof. Arghya Taraphder
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 55
Tunneling and Ginzberg Landau Theory

Welcome back to Superconductivity. In particular, we will we are doing a BCS theory of superconductivity. So, I will continue with that for you a few more slides and then, try to develop some experimental understanding of what one should see as a proof of or an experimental verification of BCS theory. So, let me just recap a little bit because it is an in so, very unique subject and concepts are very different from what we have been doing so far in free electron theories or electronic theories or solids.

(Refer Slide Time: 00:55)



Here, what we have done is combined an electron and hole. So, an electron and a hole linear combination of these gives us a quasi-particle in BCS theory. So, these are the elementary excitations over the ground state. So, they are linear, they are combinations of electron and hole; an electron and a hole. So, these kind of quasi-particles the idea is that they the ground state consists of many such states. So, it we had written it as no pair into some u k plus v k into a pair; 1 pair product over all such k states.

So, each k state here means that it is a k up and minus k down; each k here represents this ok. So, this we said is that is the BCS ground state trial ground state and one minimizes with respect to u k and v k. Since, u k and v k have this relation in between them, we actually have to minimize with respect to 1 parameter which we defined as a theta k. So, we have done all that and we have also shown you that we have also seen that this wave function has for example u k 1, u k 2 into 0, 0. If there are only k 1 and k 2 states, plus all this plus v k 1 v k 2, 1, 1.

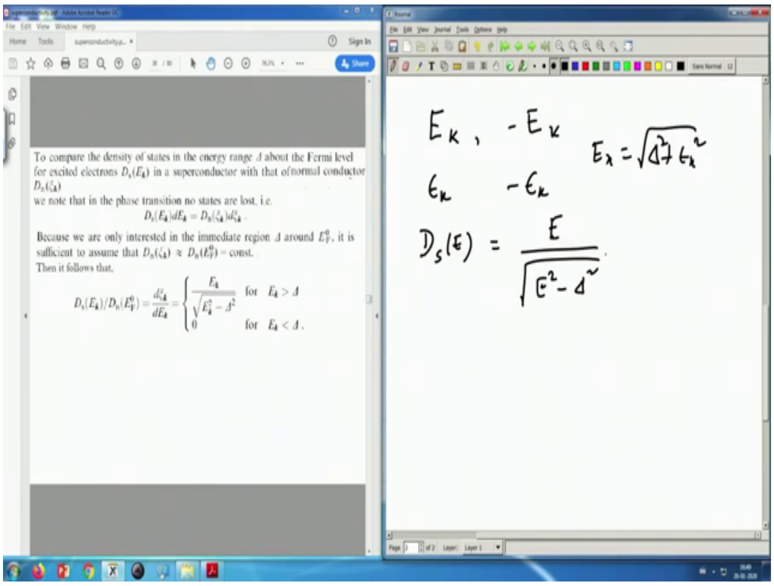
So that means, this has this is this has two pairs in state k 1 and k 2 and this has no pairs in states 1 and 2. So, this is a 0 electron state; then there are these 2, 2 electron states and then there is this 4 electron states right. So, 0 pair, 1 pair, 1 pair, 2 pairs. So, it is a combination of all these states. So, it is not an Eigen function of the n operator psi BCS will not give me some n times psi BCS. So, this is not correct. So, that is one important thing that one has to remember. Two very important things here are this. This is a combination the quasi particle is a combination of; quasi particle means the excitation the particles that define the excitation, they are fermions and they are single particle excitations and they are a combination of electrons and holes.

Then, these wave functions BCS wave function is not a not Eigen state of the number operator and the other things that one has to caution against is that often you will hear the term that these two bosons form a boson. These 2 electrons that that bind in cooper problem and then, form this pairs cooper pairs in BCS theory are not bosons. These are if you look at their commutation relations that operator, then you will find out that it is not a boson. In fact, in a clean superconductor these two particles are separated by these two members of the pair are separated by a distance of nearly 10000 angstroms or even more.

So, there are many electrons in between and so, there is no way this can be considered treated as a boson. So, to say that this is a boson and then, following that there is a Bose condensation into superconductivity is not a right way to go about. So, that is something I would like to caution you against. There are beautiful discussions on this on many books. For example, in Schrieffer superconductivity book also has a nice discussion on it, you can annex superconductivity and superfluidity book also has a discussion. Please go through it, if you want.

But that is just a cautionary statement. There are of course, in modern days there are with the new superconductors. There are the considerations of BCS to BEC Bose Einstein Condensation scenarios. But that is not what we are discussing here, we are discussing the old superconductors pre 1986 days and there the superconductivity is described by a BCS theory ok.

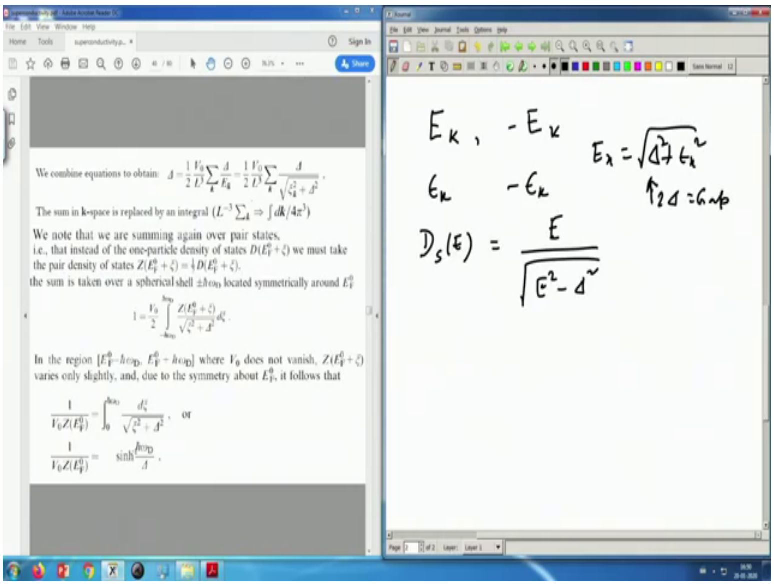
(Refer Slide Time: 06:31)



So, then of course, we found that the excitation spectrum has energy to add an electron is plus Ek to remove an electron remove a quasi-particle is minus Ek; just us to add an electron to a non-interacting fermi c is Ek and to add a hole or remove an electron is minus Ek. So, just the correspondence between these two except that these Ek's are not such simple form and they are Ek equal to delta square plus epsilon k square ok.

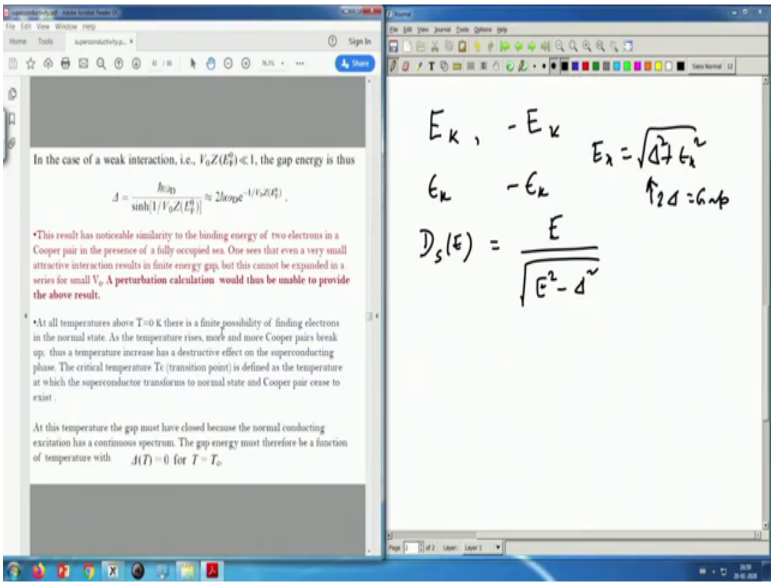
So, that is the that is a very different thing and these are the corresponding density of states. For example, D s of superconductor is plotted here. It has a large singularity. It has singularity at delta. So, it blows up when if you are at E equal to delta ok. So, that has consequences, we will come to that. So, this is what it this is how it is defined. It is a Ek by E square minus delta square ok. And this is of course, not defined for in a region mod E less than delta ok. So, all that we have done.

(Refer Slide Time: 08:04)



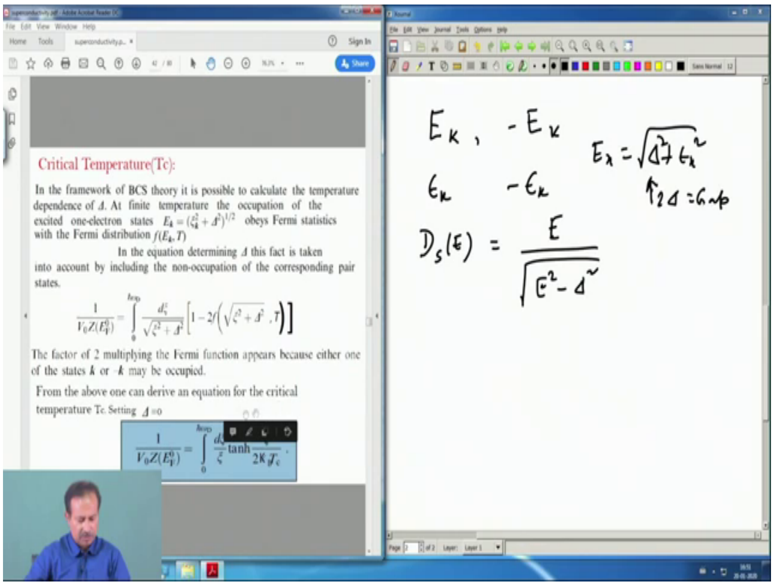
And then, what we did was that we calculated this gap. This is the gap in the spectrum because the lowest value epsilon k can take is 0. So, this becomes the gap plus and minus delta. So, 2 delta is the gap. So, this we found we found out.

(Refer Slide Time: 08:26)



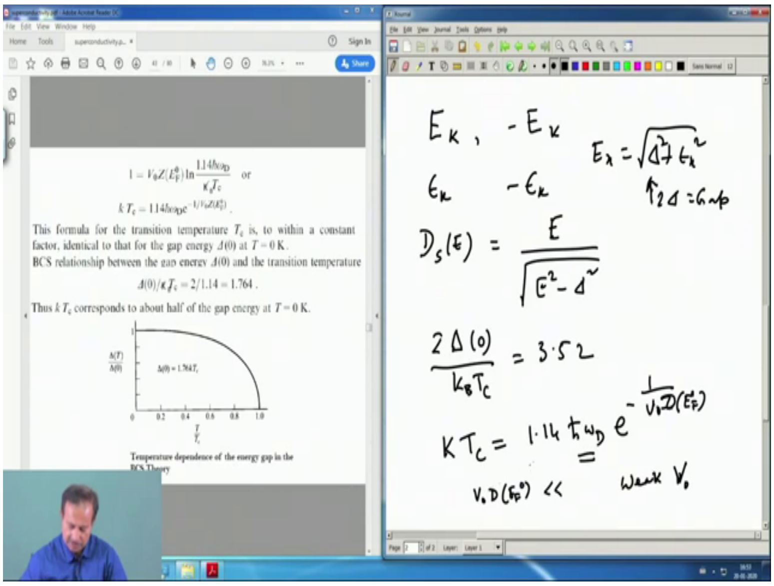
Now, we calculated the gap equation and. So, the for the gap, what we found out was that it is a it has this kind of a structure delta ok. Then, one can go ahead and so, this delta has 2 h cross omega D by into E to the power minus 1 by V 0 times the normal density of states at fermi level ok.

(Refer Slide Time: 08:51)



Then, one can calculate it at finite temperature the only difference that comes is this 1 minus 2 F factor. This basically tells you that two electrons at k and minus k have to be free to do not have to those states do not have to be occupied to because they can so that they can form pairs. So, that is all there is to it. So, that is the only place through which the temperature enters in the problem and then, you can work through it right this is the same equation and then, just invert the equation after integration to get your kB Tc.

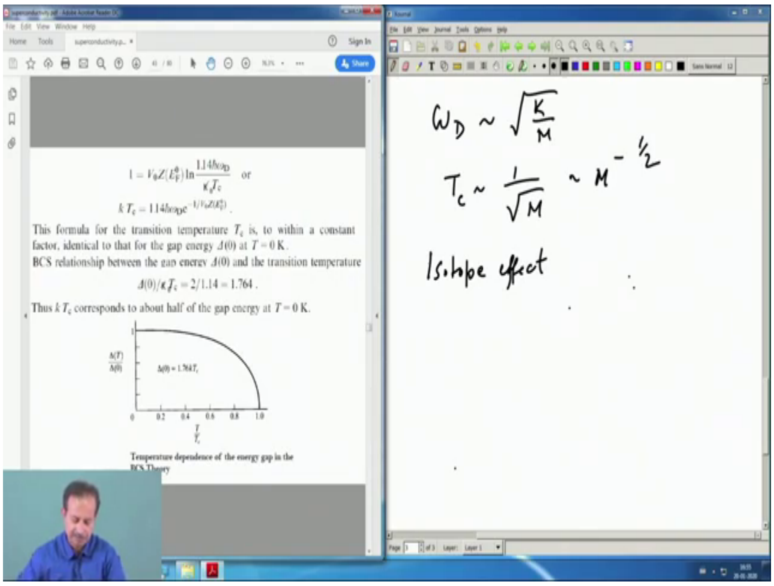
(Refer Slide Time: 09:38)



And which is this famous formula. There are 2 delta by 2 delta at 0 temperature divided by kB Tc equal to 3.52. So, this is a formula that is checked that is numerically checked and this is a telltale signature that you are dealing with a weak coupling, by weak coupling one means the V 0 is small and that kind of a superconductor. The other interesting thing that I would like you to remember is this formula kB Tc equal to 1.14 h cross omega D; do not remember the formula, but remember that there is an h cross omega D sitting here, even in delta there was this term; minus 1 by V 0 into some the original density of states. So, D of E F; the normal state density of states at E F 0.

Now, this of course, is a small quantity because V 0 delta is a V 0 D E F is very very small. So, that is on that that is used here and ok. So, that is what we that is now that is of course, that gives me the weak this is called the weak coupling, very weak V 0 and that is what happens in old BCS superconductors. The coupling is extremely weak and it is retarded over this frequency range h cross omega D. Defined only there across the fermi surface, above the fermi surface. So, but the thing that is interesting is this h cross omega D by in front and times there is a numerical factor that; if you remember your omega D by what is it?

(Refer Slide Time: 11:42)

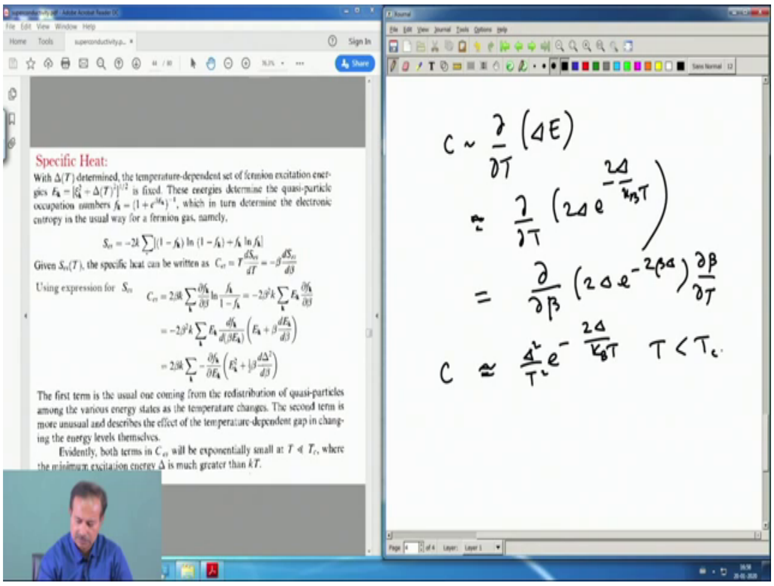


It is the frequency of the phonons typical Debye frequency. So, so frequency of phonons and that we know is generally its root over K by M right. The spring constant divided by the mass; that means, if you change the mass of the atoms that are responsible in forming the super conductivity, I mean the material atoms of the material where super conductivity is happening. Then of course, you will find that if you change the atoms by a different mass, its Isotope for example, then your superconducting Tc should be proportional to also root over 1 by M and that is something. So, that is m to the power minus half. That is something called the Isotope effect.

So, as you change your mass of the isotope change your atom to its isotope, then the Tc also changes as M to the power of minus half and this is called the famous Isotope effect. This was checked of course, this was this was already known and people found this out from BCS theory easily. You can see that how it see how it appears ok. So, these are some issues that I would have liked to would like to mention that remember that there are strong experimental verifications that exist. We will show you further experimental verification of BCS theory.

So, there are there are extremely important issues in super conductivity like for example, this isotope effect whenever a superconductor appears, people try to check if there is an Isotope effect; why? Because if there is an isotope effect, then you know that the lattice vibrations are involved in the superconductivity somehow and so, that is why isotope effect is an extremely important experimental fact that one checks ok.

(Refer Slide Time: 13:57)

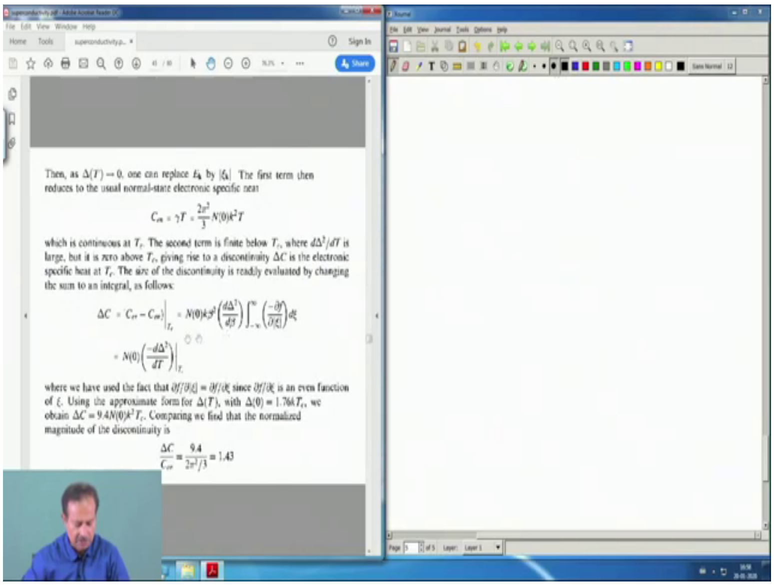


So, let us go ahead and then of course, we did this specific heat and as we showed that the specific heat for example, has this behavior right the see the at low temperatures, there has to be a there is a gap in the spectrum, the excitations. After all you have to excite these quasi particles to get a specific heat right. You are supplying some energy thermal energy and then, these quasi particles are excited. But then of course, we know that these quasi particles are gapped right.

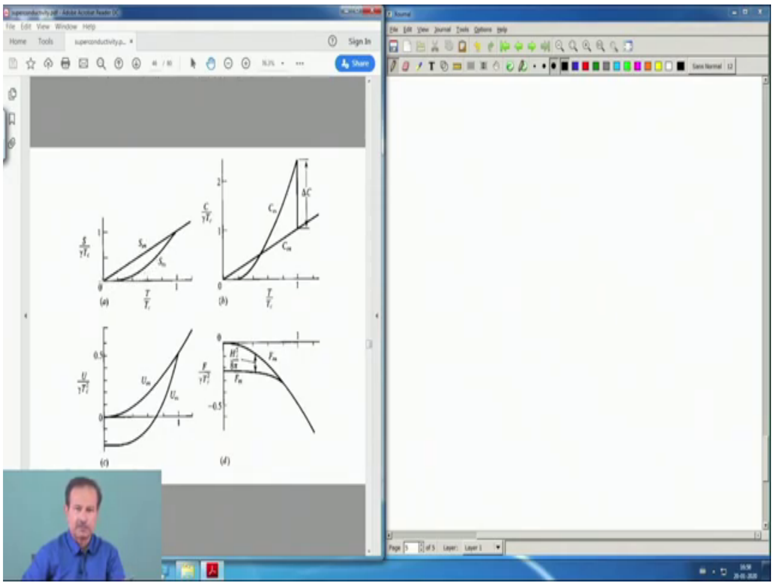
So, then, then the specific heat must be equal to the change in energy for example, del T of the superconducting energy change. I mean this we are already calculated and the delta E is this is written as del del T of 2 delta E to the power minus delta 2 delta by k B T right. And then, you can immediately see that you can convert it into del del beta 2 delta E to the power minus 2 beta delta; beta is 1 by k B T del beta del T and that gives you some term which is delta square by T square E to the power minus 2 delta by k B T.

So, that is the form that this specific heat should take at temperatures below T c and above T c of course, it becomes like a normal metal gamma T. So, T less than Tc and detailed calculation is doable and done and this calculation we had done.

(Refer Slide Time: 16:05)



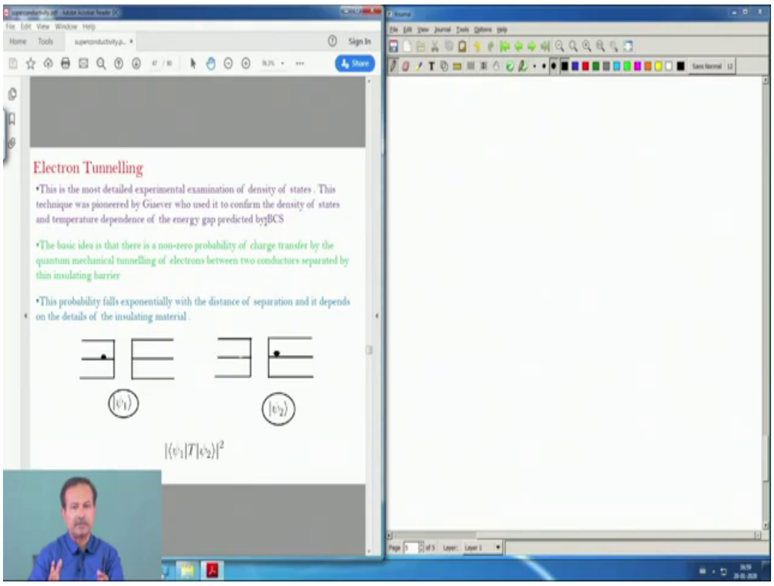
(Refer Slide Time: 16:08)



And we what we found was indeed a specific heat which is exponentially down at low temperatures and then of course, we also calculated this jump, this discontinuity at Tc which we called delta C and that turns out to be delta C by C en is 1.43 and these are also checked experimentally.

So, the entropy also gets depressed before below Tc because now you are in the in the condensate and there is a gap in the excitation spectrum. So, entropy goes down and finally, of course, it reaches 0 as does this one, the normal one, so that is reflected in this specific heat ok.

(Refer Slide Time: 16:54)



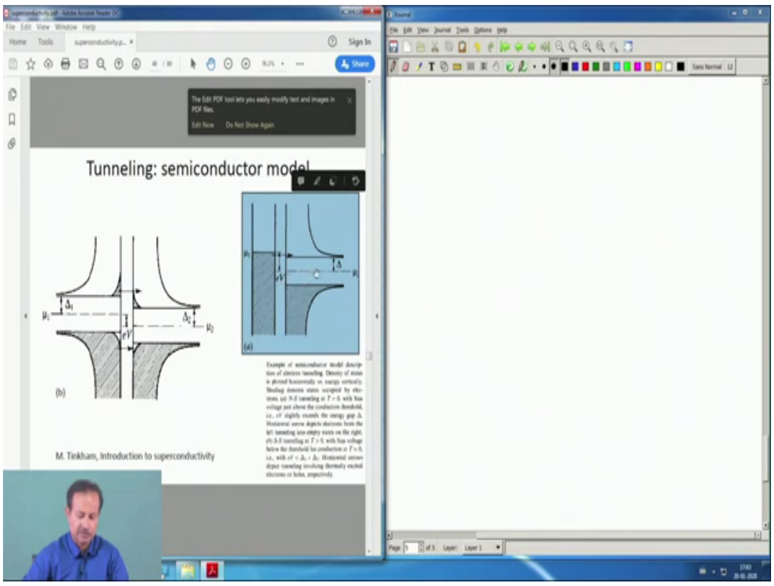
Now, there is another experimental technique that was extremely useful and that was done to directly look at the gap; whether there is a gap in the spectrum and how much is the gap, how does the gap behave and so on. So, that is kind of a direct experimental verification of BCS theory and that is the electron tunneling. Obviously, in a semiconductor for example, how do we know something is semiconductor, we can do an optical experiment and shine a light and try to excite an electron from the valence to the conduction band and see if it is there is a gap to it. So, up to a certain energy of the photon, there will be nothing and then act this twice the gap, the photon will start getting absorbed.

So,. So, similar things can be done here, but here what is done is one looks at the tunneling of an electron from a normal state to a superconductor, from a superconductor to another superconductor and so on and that will actually see as I will show that c is the gap, that kind of processes actually samples the not only the gap the density of states after that also. So, here also you will expect that an electron when it goes from a metal to a superconductor, superconductor has a gap. So, when it enters into the gap region; then, it will not enter right. It you have to supply an energy sufficient to go over the gap to form from part of the excitation spectrum. So, that gap will come up here also, in some way or other and that is what we are going to show.

So, the way this experiment is done is it was pioneered by Giaever, who also got a Nobel Prize for this experiments temperature. So, he looked at the gap the density of states and the temperature dependence of it. So, the basic idea is that a non-zero probability of charge transfer by the quantum mechanical tunneling of electrons between two conductors separated by thin insulating barrier. This was the original idea. This was being done in metals for some time. Now, this was brought into a superconducting situation. So, what one does is that to a metal and a superconductor brought into proximity for example, or a metal and a metal or a metal or two superconductors, they are brought in to their proximity, separated by a very thin layer of insulator.

So, let us assume that the electron was in states psi 1, it was here. This psi states psi 2 is where, it has tunneled into the other material on the right hand material. So, that matrix element is this thing psi 1 T psi 2 and (Refer Time: 20:09) and generally, that is taken to be a temperature independent k independent and so on and so forth. Just treated as a constant; corrections can be done and those are details but for the time being we will just follow this procedure. So, that matrix element will be taken out from the integrals that come ok.

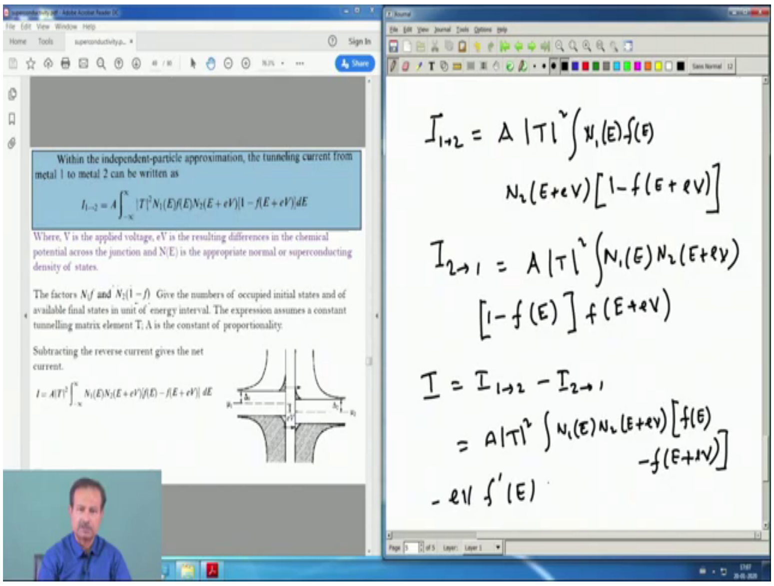
(Refer Slide Time: 20:29)



Now, the model that one follows is a typical model of a semi for example, semiconductor from a semiconductor to a semiconductor or a metal to a semiconductor. So, for example, in this right hand picture, if you see that the chemical potential in an insulator is on the and in the middle and that is what is. So, that is the amount of energy you have to supply from the left hand side to get into a tunnel into the tunnel.

So, mu 1 has to be sufficient that it has this eV energy, it can supply this eV energy and only then, can you go to the top band. Because this band, the lower band is completely filled up. These lower states are filled up you have to go here. So, the energy required from the fermi level or the chemical potential is eV. So, that is a typical model from metal to semiconductor for example.

(Refer Slide Time: 21:34)



So, ok; let us go ahead and do it. So, this independent particle; this is basically independent particles. This left hand and right hand particles do not see each other and do not interfere with each other. Besides there is no interaction when it is it enters the second system except for whatever is already. Suppose, the second one is superconductor; then of course, there will be a superconducting pairing interaction and all that.

But so, the in this kind of tunneling picture like a single electron tunneling, there is this is trivial. This result is straightforward because what does it say that it is says what is the current from 1 to 2 and that current is some constant times this mod T square, which I will take out, times the density of states in material 1. So, there has to be states there. So, that at that energy to tunnel into where the electron is and then, tunnel to the right.

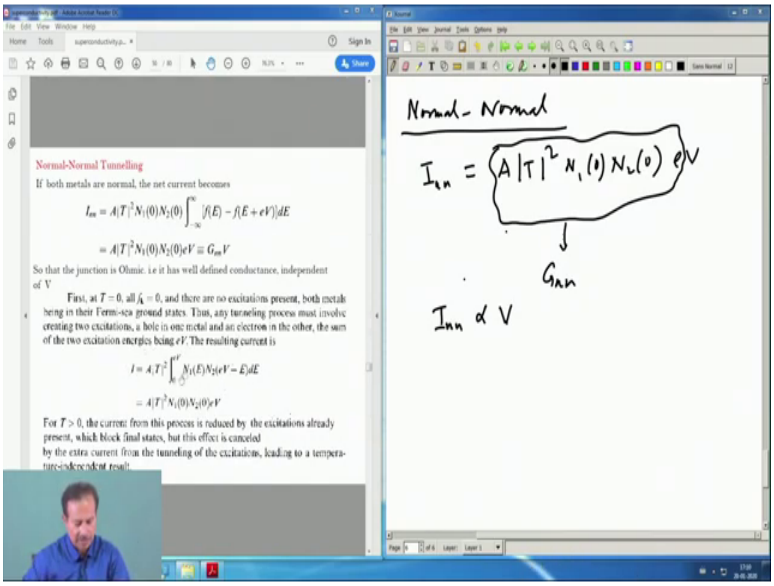
And of course, the electron has to be there that probability is f E into you have to have again states on the right hand side at an energy E plus e V if the potential you are maintaining is V. So, the potential difference being V so, the energy of the electron that enters is E plus e V. And then therefore, and there has to be no electron already present at that energy. So, this is f E plus e V.

So, that is the basic equation that you have. Of course, there is also a current that is going from 2 to 1 right and that will be again this a times T square into N 1 E N 2 E plus e V. There is a reverse current, back current also. But now, you have to have a state absent at 1 minus f E and there has to be a state at f E plus e V right. So, this so, the net current I is I 1 to 2 minus I 2 to 1 ok.

Now, you can just do the algebra straightforward, what you will find is that this is A times this mod T square into N 1 E N 2 E plus e V into f E minus f E plus small e into capital V that is it. And now if of course, these E is that we are talking about are closed at the fermi level at the chemical potential. So, compared to that this e V could be smaller and one can just make an expansion and again this will give me e V times if I just subtract f E from this thing, then this will be minus f time e. Just do a Taylor expansion and this will be the term here ok.

Now, minus f prime of E, we know is a delta function at low temperatures. So, we can replace it by that and then, the result is what is shown on the on here. This is the result.

(Refer Slide Time: 25:36)



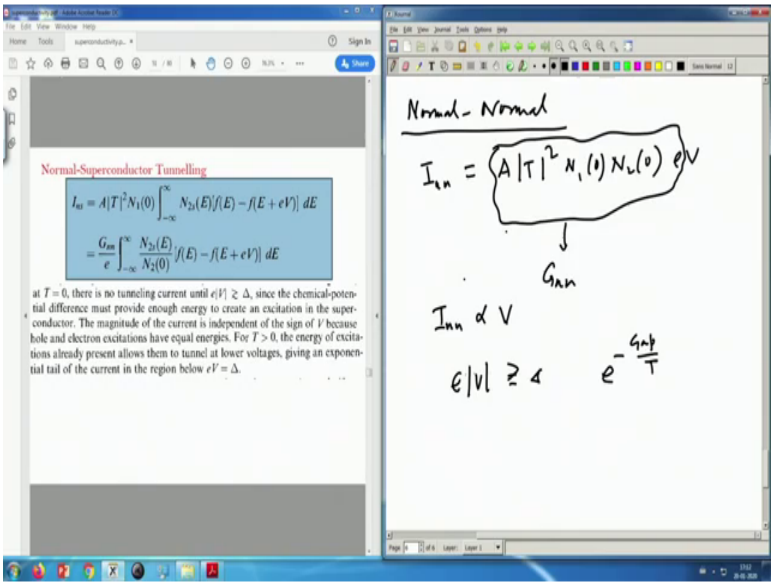
So, for example, for electron so, this is the main result and then, for a metal-metal where this result doing a tunneling between metal to metal. So, that is called normal to normal.

Student: (Refer Time: 26:03) so that we are (Refer Time: 26:05).

So, there see this remember when I use this f prime E as a delta function, remember I am using it for the normal state that is where the delta function the fermi function of course, appears here and then, I will replace this by the their densities of states at the; fermi function derivative is a delta function at low temperatures and that will convert all both this E's at the fermi level. So, that is what I am going to do for normal-normal tunneling. So, normal-normal will then be just a T square N 1 0; 0 means the fermi level N 2 0 into e V.

Now, this whole thing up to this is called G normal-normal and this is then, you can see that this thing is the junction he is now an Ohmic junction because the current is proportional to the voltage. So, this is this relation tells you that Inn is proportional to voltage and all these quantities here are more or less temperature independent, voltage independent and all that. So, Gnn is not temperature dependent, not voltage dependent right. To first approximation that is correct ok. So, this can also be derived from a normal, from another argument which is shown which gives the same argument; so, the same result.

(Refer Slide Time: 28:09)



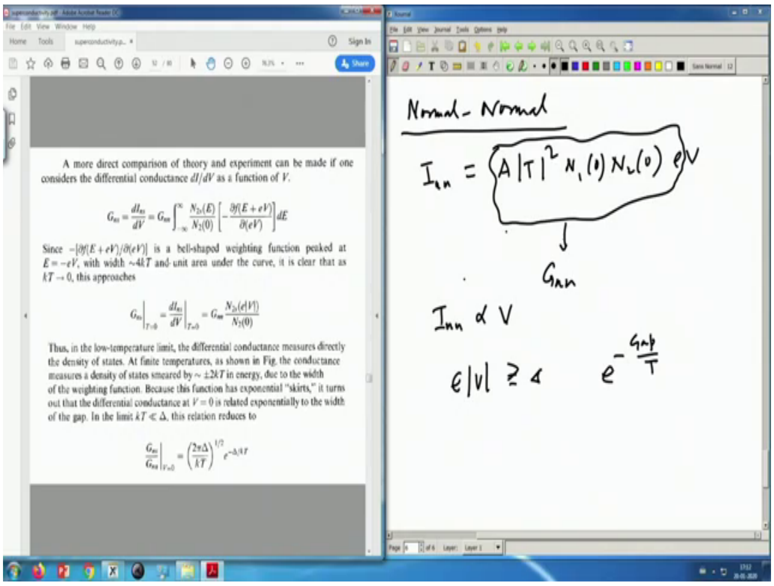
Then, we can go for a normal to superconductor tunneling. So, in normal superconducting case of course, N 2 has to be replaced by the superconducting density of states and that is exactly what is done and so, then it is just this relation f E minus f E plus V and N 2 s by N 2 0. See this, N 2 0 is divided just to convert it into Gnn. So, that you get another N 2 0 at here in front. So, that that is what is done and there is E additional E that comes in because you had e V and here you do not have it. So, that is this relation. There is no direct proportionality to D to V.

So, at T equal to 0, there is no tunneling current until e V equal to delta of course. As you can see from here there is no tunneling current until you hit until your Ev hits delta becomes delta or more than equal to delta. So,. So, that is that is the result that we were and we were anticipating that is what will happen. Since, the chemical potential difference must provide this much of energy to create an excitation in the superconductor. So, the magnitude of the current is independent of the sign of V which is because hole and electron excitations have equal energies. The both are plus both are E of k; capital E k.

Of course, for T greater than 0, there are thermally excited quasi particles and these of course, will lead to. So, the electron can go to can tunnel into and thermally excited into this. There are already present thermally excitations at that energy. So, it can tunnel through and there is a tunneling which is of course, down by the factors thermally again by gap divided by T.

So, so that means, there is at fine at 0 temperature, tunneling starts only at eV equal to delta. At finite temperature, there is a very small tunneling that happens which is exponentially down and then finally, takes off at delta.

(Refer Slide Time: 30:38)



The. So, there is this other thing that one can see which will I will discuss in the next class where what one does is that what instead of looking at directly the tunneling spectrum which you can of course, one has actually looks at the derivative of the current with respect to the field and these are called differential conductance and this gives you a directive look at the density of states that you are seeing on the side into which you are tunneling. So, that is what I will do next.