

Electronic Theory of Solids
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Lecture – 55
Tunneling and Ginzberg Landau Theory

Welcome back to Superconductivity. In particular, we will we are doing a BCS theory of superconductivity. So, I will continue with that for you a few more slides and then, try to develop some experimental understanding of what one should see as a proof of or an experimental verification of BCS theory. So, let me just recap a little bit because it is an in so, very unique subject and concepts are very different from what we have been doing so far in free electron theories or electronic theories or solids.

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Here, what we have done is combined an electron and hole. So, an electron and a hole linear combination of these gives us a quasi-particle in BCS theory. So, these are the elementary excitations over the ground state. So, they are linear, they are combinations of electron and hole; an electron and a hole. So, these kind of quasi-particles the idea is that they the ground state consists of many such states. So, it we had written it as no pair into some u_k plus v_k into a pair; 1 pair product over all such k states.

So, each k state here means that it is a k up and minus k down; each k here represents this ok . So, this we said is that is the BCS ground state trial ground state and one minimizes with respect to u_k and v_k . Since, u_k and v_k have this relation in between them, we actually have to minimize with respect to 1 parameter which we defined as a θ_k . So, we have done all that and we have also shown you that we have also seen that this wave function has for example u_{k_1}, u_{k_2} into 0, 0. If there are only k_1 and k_2 states, plus all this plus $v_{k_1} v_{k_2}, 1, 1$.

So that means, this has this is this has two pairs in state k_1 and k_2 and this has no pairs in states 1 and 2. So, this is a 0 electron state; then there are these 2, 2 electron states and then there is this 4 electron states right. So, 0 pair, 1 pair, 1 pair, 2 pairs. So, it is a combination of all these states. So, it is not an Eigen function of the n operator ψ BCS will not give me some n times ψ BCS. So, this is not correct. So, that is one important thing that one has to remember. Two very important things here are this. This is a combination the quasi particle is a combination of; quasi particle means the excitation the particles that define the excitation, they are fermions and they are single particle excitations and they are a combination of electrons and holes.

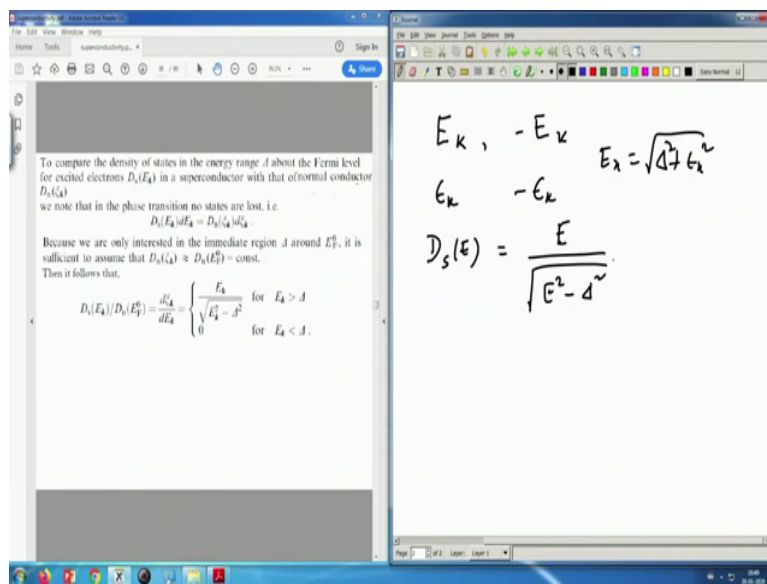
Then, these wave functions BCS wave function is not a not Eigen state of the number operator and the other things that one has to caution against is that often you will hear the term that these two bosons form a boson. These 2 electrons that that bind in cooper problem and then, form this pairs cooper pairs in BCS theory are not bosons. These are if you look at their commutation relations that operator, then you will find out that it is not a boson. In fact, in a clean superconductor these two particles are separated by these two members of the pair are separated by a distance of nearly 10000 angstroms or even more.

So, there are many electrons in between and so, there is no way this can be considered treated as a boson. So, to say that this is a boson and then, following that there is a Bose condensation into superconductivity is not a right way to go about. So, that is something I would like to caution you against. There are beautiful discussions on this on many books. For example, in Schrieffer superconductivity book also has a nice discussion on it, you can annex

superconductivity and superfluidity book also has a discussion. Please go through it, if you want.

But that is just a cautionary statement. There are of course, in modern days there are with the new superconductors. There are the considerations of BCS to BEC Bose Einstein Condensation scenarios. But that is not what we are discussing here, we are discussing the old superconductors pre 1986 days and there the superconductivity is described by a BCS theory ok.

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So, then of course, we found that the excitation spectrum has energy to add an electron is plus E_k to remove an electron remove a quasi-particle is minus E_k ; just us to add an electron to a non-interacting fermi c is E_k and to add a hole or remove an electron is minus E_k . So, just the correspondence between these two except that these E_k 's are not such simple form and they are E_k equal to Δ^2 plus ϵ_k square ok.

So, that is the that is a very different thing and these are the corresponding density of states. For example, D_s of superconductor is plotted here. It has a large singularity. It has singularity at Δ . So, it blows up when if you are at E equal to Δ ok. So, that has consequences, we will come to that. So, this is what it this is how it is defined. It is a E_k by E square minus

delta square ok. And this is of course, not defined in a region mod E less than delta ok. So, all that we have done.

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The slide content includes:

We combine equations to obtain:
$$\Delta = \frac{1}{2} \frac{V_0}{D} \sum_{\mathbf{k}} \frac{\Delta}{E_{\mathbf{k}}} = \frac{1}{2} \frac{V_0}{D} \sum_{\mathbf{k}} \frac{\Delta}{\sqrt{\epsilon_{\mathbf{k}}^2 + \Delta^2}}$$

The sum in k-space is replaced by an integral ($L^{-3} \sum_{\mathbf{k}} \Rightarrow \int d\mathbf{k}/(4\pi^3)$)

We note that we are summing again over pair states, i.e., that instead of the one-particle density of states $D(E_{\mathbf{k}} + \zeta)$ we must take the pair density of states $Z(E_{\mathbf{k}}^{\pm} + \zeta) = \frac{1}{2} D(E_{\mathbf{k}}^{\pm} + \zeta)$, the sum is taken over a spherical shell $\pm \hbar \omega_D$ located symmetrically around $E_{\mathbf{k}}^{\pm}$

$$1 = \frac{1}{2} \int_{-\hbar \omega_D}^{\hbar \omega_D} \frac{Z(E_{\mathbf{k}}^{\pm} + \zeta)}{\sqrt{\zeta^2 + \Delta^2}} d\zeta$$

In the region $[E_{\mathbf{k}}^{\pm} - \hbar \omega_D, E_{\mathbf{k}}^{\pm} + \hbar \omega_D]$ where V_0 does not vanish, $Z(E_{\mathbf{k}}^{\pm} + \zeta)$ varies only slightly, and, due to the symmetry about $E_{\mathbf{k}}^{\pm}$, it follows that

$$\frac{1}{V_0 Z(E_{\mathbf{k}}^{\pm})} = \int_{-\hbar \omega_D}^{\hbar \omega_D} \frac{d\zeta}{\sqrt{\zeta^2 + \Delta^2}}, \quad \text{or}$$

$$\frac{1}{V_0 Z(E_{\mathbf{k}}^{\pm})} = \sinh^{-1} \frac{\hbar \omega_D}{\Delta}$$

The whiteboard content includes:

$E_{\mathbf{k}}, -E_{\mathbf{k}}$
 $\epsilon_{\mathbf{k}}, -\epsilon_{\mathbf{k}}$
 $E_{\mathbf{k}} = \sqrt{\Delta^2 + \epsilon_{\mathbf{k}}^2}$
 $\uparrow 2\Delta = \text{gap}$
 $D_S(E) = \frac{E}{\sqrt{E^2 - \Delta^2}}$

And then, what we did was that we calculated this gap. This is the gap in the spectrum because the lowest value epsilon k can take is 0. So, this becomes the gap plus and minus delta. So, 2 delta is the gap. So, this we found we found out.

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The slide content includes:

In the case of a weak interaction, i.e., $V_0 Z(E_{\mathbf{k}}^{\pm}) \ll 1$, the gap energy is thus

$$\Delta = \frac{\hbar \omega_D}{\sinh(1/V_0 Z(E_{\mathbf{k}}^{\pm}))} \approx 2\hbar \omega_D e^{-1/V_0 Z(E_{\mathbf{k}}^{\pm})}$$

*This result has noticeable similarity to the binding energy of two electrons in a Cooper pair in the presence of a fully occupied sea. One sees that even a very small attractive interaction results in finite energy gap, but this cannot be expanded in a series for small V_0 . A perturbation calculation would thus be unable to provide the above result.

*At all temperatures above $T \Rightarrow K$ there is a finite possibility of finding electrons in the normal state. As the temperature rises, more and more Cooper pairs break up, thus a temperature increase has a destructive effect on the superconducting phase. The critical temperature T_c (transition point) is defined as the temperature at which the superconductor transforms to normal state and Cooper pairs cease to exist.

At this temperature the gap must have closed because the normal conducting excitation has a continuous spectrum. The gap energy must therefore be a function of temperature with $\Delta(T) = 0$ for $T = T_c$.

The whiteboard content is identical to the previous slide:

$E_{\mathbf{k}}, -E_{\mathbf{k}}$
 $\epsilon_{\mathbf{k}}, -\epsilon_{\mathbf{k}}$
 $E_{\mathbf{k}} = \sqrt{\Delta^2 + \epsilon_{\mathbf{k}}^2}$
 $\uparrow 2\Delta = \text{gap}$
 $D_S(E) = \frac{E}{\sqrt{E^2 - \Delta^2}}$

Now, we calculated the gap equation and. So, the for the gap, what we found out was that it is a it has this kind of a structure delta ok. Then, one can go ahead and so, this delta has 2 h cross omega D by into E to the power minus 1 by V 0 times the normal density of states at fermi level ok.

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The slide on the left is titled "Critical Temperature(Tc):". It states: "In the framework of BCS theory it is possible to calculate the temperature dependence of Δ . At finite temperature the occupation of the excited one-electron states $E_k = (\xi_k^2 + \Delta^2)^{1/2}$ obeys Fermi statistics with the Fermi distribution $f(E_k, T)$ ". It then says: "In the equation determining Δ this fact is taken into account by including the non-occupation of the corresponding pair states." The equation shown is:
$$\frac{1}{V_0 Z(E_F^0)} = \int_0^{\hbar\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta^2}} \left[1 - 2f(\sqrt{\xi^2 + \Delta^2}, T) \right]$$
 It notes: "The factor of 2 multiplying the Fermi function appears because either one of the states k or $-k$ may be occupied." and "From the above one can derive an equation for the critical temperature T_c . Setting $\Delta = 0$ ". A smaller version of the equation is shown below:
$$\frac{1}{V_0 Z(E_F^0)} = \int_0^{\hbar\omega_D} \frac{d\xi}{\xi} \tanh \frac{\xi}{2k_B T_c}$$

The whiteboard on the right has handwritten notes: $E_k, -E_k$, $E_k = -E_k$, $E_k = \sqrt{\Delta^2 + E_k^2}$ with an arrow pointing to Δ and the note $\uparrow 2\Delta = \hbar\omega_D$, and $D_s(E) = \frac{E}{\sqrt{E^2 - \Delta^2}}$.

Then, one can calculate it at finite temperature the only difference that comes is this 1 minus 2 F factor. This basically tells you that two electrons at k and minus k have to be free to do not have to those states do not have to be occupied to because they can so that they can form pairs. So, that is all there is to it. So, that is the only place through which the temperature enters in the problem and then, you can work through it right this is the same equation and then, just invert the equation after integration to get your $k_B T_c$.

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And which is this famous formula. There are 2Δ by 2Δ at 0 temperature divided by $k_B T_c$ equal to 3.52. So, this is a formula that is checked that is numerically checked and this is a telltale signature that you are dealing with a weak coupling, by weak coupling one means the V_0 is small and that kind of a superconductor. The other interesting thing that I would like you to remember is this formula $k_B T_c$ equal to $1.14 \hbar \omega_D$; do not remember the formula, but remember that there is an $\hbar \omega_D$ sitting here, even in Δ there was this term; minus 1 by V_0 into some the original density of states. So, D of E_F ; the normal state density of states at E_F .

Now, this of course, is a small quantity because $V_0 \Delta$ is a $V_0 D E_F$ is very very small. So, that is on that that is used here and ok. So, that is what we that is now that is of course, that gives me the weak this is called the weak coupling, very weak V_0 and that is what happens in old BCS superconductors. The coupling is extremely weak and it is retarded over this frequency range $\hbar \omega_D$. Defined only there across the fermi surface, above the fermi surface. So, but the thing that is interesting is this $\hbar \omega_D$ by in front and times there is a numerical factor that; if you remember your ω_D by what is it?

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The slide on the left contains the following text and graph:

$$1 = 1.6 Z \left(\frac{\hbar}{k_B T_c} \right) \ln \frac{1.14 \hbar \omega_D}{k_B T_c} \quad \text{or}$$

$$k_B T_c = 1.14 \hbar \omega_D \left(\frac{1}{1.6 Z} \right)^{1/2}$$

This formula for the transition temperature T_c is, to within a constant factor, identical to that for the gap energy $\Delta(0)$ at $T = 0$ K. BCS relationship between the gap energy $\Delta(0)$ and the transition temperature $\Delta(0)/k_B T_c = 2/1.14 = 1.764$.

Thus $k_B T_c$ corresponds to about half of the gap energy at $T = 0$ K.

The graph shows $\frac{\Delta(T)}{\Delta(0)}$ on the y-axis and $\frac{T}{T_c}$ on the x-axis. The curve starts at 1.0 when $T/T_c = 0$ and decreases to 0 at $T/T_c = 1.0$. A label $\Delta(0) = 1.764 T_c$ is present.

Temperature dependence of the energy gap in the BCS theory

The whiteboard on the right has handwritten notes:

$$\omega_D \sim \sqrt{\frac{K}{M}}$$

$$T_c \sim \frac{1}{\sqrt{M}} \sim M^{-1/2}$$
 Isotope effect

It is the frequency of the phonons typical Debye frequency. So, so frequency of phonons and that we know is generally its root over K by M right. The spring constant divided by the mass; that means, if you change the mass of the atoms that are responsible in forming the super conductivity, I mean the material atoms of the material where super conductivity is happening. Then of course, you will find that if you change the atoms by a different mass, its Isotope for example, then your superconducting T_c should be proportional to also root over 1 by M and that is something. So, that is m to the power minus half. That is something called the Isotope effect.

So, as you change your mass of the isotope change your atom to its isotope, then the T_c also changes as M to the power of minus half and this is called the famous Isotope effect. This was checked of course, this was this was already known and people found this out from BCS theory easily. You can see that how it see how it appears ok. So, these are some issues that I would have liked to mention that remember that there are strong experimental verifications that exist. We will show you further experimental verification of BCS theory.

So, there are there are extremely important issues in super conductivity like for example, this isotope effect whenever a superconductor appears, people try to check if there is an Isotope effect; why? Because if there is an isotope effect, then you know that the lattice vibrations are

involved in the superconductivity somehow and so, that is why isotope effect is an extremely important experimental fact that one checks ok.

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Specific Heat:
 With $\Delta(T)$ determined, the temperature-dependent set of fermion excitation energies $\epsilon_k = [\xi_k^2 + \Delta(T)^2]^{1/2}$ is fixed. These energies determine the quasi-particle occupation numbers $f_k = (1 + e^{\beta \epsilon_k})^{-1}$, which in turn determine the electronic entropy in the usual way for a fermion gas, namely,

$$S_{el} = -2k_B \sum_k [(1 - f_k) \ln(1 - f_k) + f_k \ln f_k]$$

Given $S_{el}(T)$, the specific heat can be written as $C_v = T \frac{dS_{el}}{dT} = -\beta \frac{dS_{el}}{d\beta}$

Using expression for S_{el} , $C_v = 2k_B \sum_k \frac{\partial f_k}{\partial \beta} \frac{\epsilon_k}{1 - f_k} = -2k_B \sum_k f_k \frac{\partial \epsilon_k}{\partial \beta}$

$$= -2k_B \sum_k f_k \frac{\partial \epsilon_k}{\partial \beta} \left(\epsilon_k + \beta \frac{d\Delta}{d\beta} \right)$$

$$= 2k_B \sum_k \frac{\partial f_k}{\partial \beta} \left(\epsilon_k^2 + \beta \frac{d\Delta}{d\beta} \right)$$

The first term is the usual one coming from the redistribution of quasi-particles among the various energy states as the temperature changes. The second term is more unusual and describes the effect of the temperature-dependent gap in changing the energy levels themselves.
 Evidently, both terms in C_v will be exponentially small at $T \ll T_c$, where the minimum excitation energy Δ is much greater than kT .

So, let us go ahead and then of course, we did this specific heat and as we showed that the specific heat for example, has this behavior right the see the at low temperatures, there has to be a there is a gap in the spectrum, the excitations. After all you have to excite these quasi particles to get a specific heat right. You are supplying some energy thermal energy and then, these quasi particles are excited. But then of course, we know that these quasi particles are gapped right.

So, then, then the specific heat must be equal to the change in energy for example, $\frac{dE}{dT}$ of the superconducting energy change. I mean this we are already calculated and the ΔE is this is written as $\frac{dE}{dT} = 2 \frac{\Delta E}{T_c} \frac{dT_c}{dT}$ right. And then, you can immediately see that you can convert it into $\frac{dE}{dT} = 2 \frac{\Delta E}{T_c} \frac{dT_c}{dT}$ and that gives you some term which is $\frac{\Delta^2}{T_c^2} e^{-\frac{2\Delta}{k_B T}}$.

So, that is the form that this specific heat should take at temperatures below T_c and above T_c of course, it becomes like a normal metal γT . So, $T < T_c$ and detailed calculation is doable and done and this calculation we had done.

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Then, as $\Delta(T) \rightarrow 0$, one can replace ξ_k by ξ_k^N . The first term then reduces to the usual normal-state electronic specific heat

$$C_n = \gamma T = \frac{2\pi^2}{3} N(0)k^2 T$$

which is continuous at T_c . The second term is finite below T_c , where $d\Delta^2/dT$ is large, but it is zero above T_c , giving rise to a discontinuity ΔC in the electronic specific heat at T_c . The size of the discontinuity is readily evaluated by changing the sum to an integral, as follows:

$$\Delta C = C_n - C_n^N = N(0)k^2 \int_{-\infty}^{\infty} \left(\frac{-\partial f}{\partial T} \right) d\xi$$

$$= N(0) \left(\frac{-d\Delta^2}{dT} \right)_{T_c}$$

where we have used the fact that $\partial f / \partial \xi = \partial f / \partial \xi$ since $\partial f / \partial \xi$ is an even function of ξ . Using the approximate form for $\Delta(T)$, with $\Delta(0) = 1.76 T_c$, we obtain $\Delta C = 9.4 N(0)k^2 T_c$. Comparing we find that the normalized magnitude of the discontinuity is

$$\frac{\Delta C}{C_n} = \frac{9.4}{2\pi^2/3} = 1.43$$

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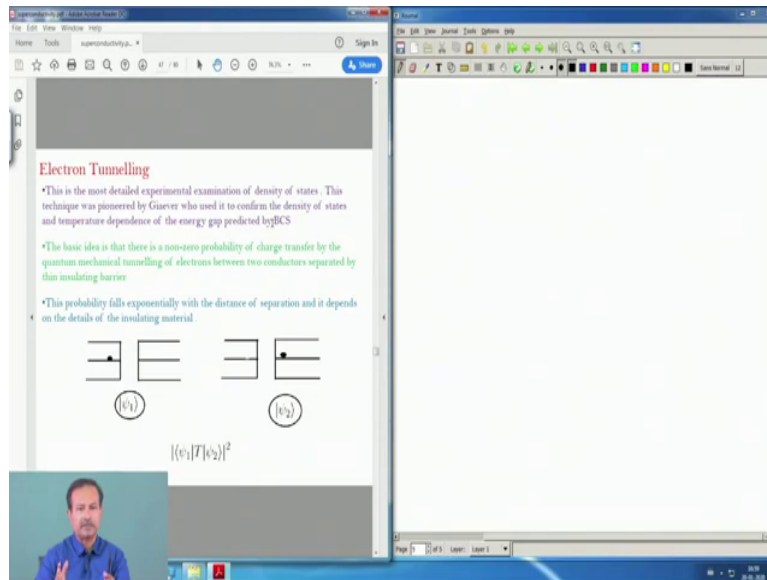
(a) $\frac{C}{\gamma T_c}$ vs T/T_c showing a jump at T_c .
 (b) $\frac{C}{\gamma T_c}$ vs T/T_c showing the normal state specific heat.
 (c) $\frac{U}{\gamma T_c}$ vs T/T_c showing the entropy jump at T_c .
 (d) $\frac{\partial U}{\partial T}$ vs T/T_c showing the temperature derivative of the entropy.

And we what we found was indeed a specific heat which is exponentially down at low temperatures and then of course, we also calculated this jump, this discontinuity at T_c which we called ΔC and that turns out to be ΔC by C_n is 1.43 and these are also checked experimentally.

So, the entropy also gets depressed before below T_c because now you are in the in the condensate and there is a gap in the excitation spectrum. So, entropy goes down and finally,

of course, it reaches 0 as does this one, the normal one, so that is reflected in this specific heat ok.

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Now, there is another experimental technique that was extremely useful and that was done to directly look at the gap; whether there is a gap in the spectrum and how much is the gap, how does the gap behave and so on. So, that is kind of a direct experimental verification of BCS theory and that is the electron tunneling. Obviously, in a semiconductor for example, how do we know something is semiconductor, we can do an optical experiment and shine a light and try to excite an electron from the valence to the conduction band and see if there is a gap to it. So, up to a certain energy of the photon, there will be nothing and then act this twice the gap, the photon will start getting absorbed.

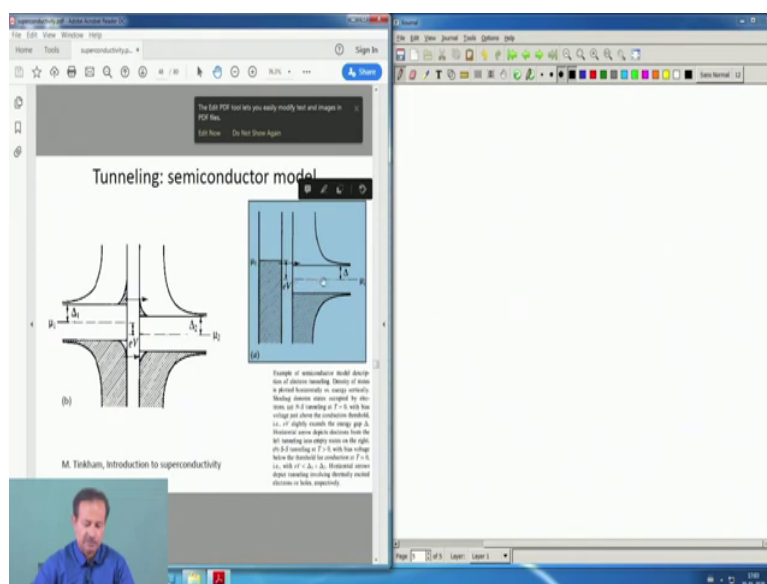
So, So, similar things can be done here, but here what is done is one looks at the tunneling of an electron from a normal state to a superconductor, from a superconductor to another superconductor and so on and that will actually see as I will show that c is the gap, that kind of processes actually samples the not only the gap the density of states after that also. So, here also you will expect that an electron when it goes from a metal to a superconductor, superconductor has a gap. So, when it enters into the gap region; then, it will not enter right. If you have to supply an energy sufficient to go over the gap to form from part of the

excitation spectrum. So, that gap will come up here also, in some way or other and that is what we are going to show.

So, the way this experiment is done is it was pioneered by Giaever, who also got a Nobel Prize for this experiments temperature. So, he looked at the gap the density of states and the temperature dependence of it. So, the basic idea is that a non-zero probability of charge transfer by the quantum mechanical tunneling of electrons between two conductors separated by thin insulating barrier. This was the original idea. This was being done in metals for some time. Now, this was brought into a superconducting situation. So, what one does is that to a metal and a superconductor brought into proximity for example, or a metal and a metal or a metal or two superconductors, they are brought in to their proximity, separated by a very thin layer of insulator.

So, let us assume that the electron was in states ψ_1 , it was here. This ψ_1 states ψ_2 is where, it has tunneled into the other material on the right hand material. So, that matrix element is this thing $\psi_1^\dagger T \psi_2$ and (Refer Time: 20:09) and generally, that is taken to be a temperature independent k independent and so on and so forth. Just treated as a constant; corrections can be done and those are details but for the time being we will just follow this procedure. So, that matrix element will be taken out from the integrals that come ok.

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Now, the model that one follows is a typical model of a semi for example, semiconductor from a semiconductor to a semiconductor or a metal to a semiconductor. So, for example, in this right hand picture, if you see that the chemical potential in an insulator is on the and in the middle and that is what is. So, that is the amount of energy you have to supply from the left hand side to get into a tunnel into the tunnel.

So, μ_1 has to be sufficient that it has this eV energy, it can supply this eV energy and only then, can you go to the top band. Because this band, the lower band is completely filled up. These lower states are filled up you have to go here. So, the energy required from the fermi level or the chemical potential is eV. So, that is a typical model from metal to semiconductor for example.

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Within the independent-particle approximation, the tunneling current from metal 1 to metal 2 can be written as

$$I_{1 \rightarrow 2} = A \int_{-\infty}^{\infty} |T|^2 N_1(E) N_2(E + eV) [f(E) - f(E + eV)] dE$$

Where, V is the applied voltage, eV is the resulting differences in the chemical potential across the junction and $N(E)$ is the appropriate normal or superconducting density of states.

The factors Nf and $N_2[1 - f]$ Give the numbers of occupied initial states and of available final states in unit of energy interval. The expression assumes a constant tunnelling matrix element T . A is the constant of proportionality.

Subtracting the reverse current gives the net current.

$$I = A|T|^2 \int_{-\infty}^{\infty} N_1(E) N_2(E + eV) [f(E) - f(E + eV)] dE$$

The diagram shows two energy bands, N_1 and N_2 , with a tunneling barrier between them. The chemical potentials μ_1 and μ_2 are shown, with μ_1 higher than μ_2 . The voltage V is applied across the junction.

Handwritten derivations on the whiteboard:

$$I_{1 \rightarrow 2} = A |T|^2 \int N_1(E) f(E) N_2(E + eV) [1 - f(E + eV)]$$

$$I_{2 \rightarrow 1} = A |T|^2 \int N_1(E) N_2(E + eV) [1 - f(E)] f(E + eV)$$

$$I = I_{1 \rightarrow 2} - I_{2 \rightarrow 1}$$

$$= A |T|^2 \int N_1(E) N_2(E + eV) [f(E) - f(E + eV)]$$

$$= eV f'(E)$$

So, ok; let us go ahead and do it. So, this independent particle; this is basically independent particles. This left hand and right hand particles do not see each other and do not interfere with each other. Besides there is no interaction when it is it enters the second system except for whatever is already. Suppose, the second one is superconductor; then of course, there will be a superconducting pairing interaction and all that.

But so, the in this kind of tunneling picture like a single electron tunneling, there is this is trivial. This result is straightforward because what does it say that it is says what is the

current from 1 to 2 and that current is some constant times this mod T square, which I will take out, times the density of states in material 1. So, there has to be states there. So, that at that energy to tunnel into where the electron is and then, tunnel to the right.

And of course, the electron has to be there that probability is $f(E)$ into you have to have again states on the right hand side at an energy E plus eV if the potential you are maintaining is V . So, the potential difference being V so, the energy of the electron that enters is E plus eV . And then therefore, and there has to be no electron already present at that energy. So, this is $f(E + eV)$.

So, that is the basic equation that you have. Of course, there is also a current that is going from 2 to 1 right and that will be again this a times T square into $N_1(E) - N_2(E + eV)$. There is a reverse current, back current also. But now, you have to have a state absent at $1 - f(E)$ and there has to be a state at $f(E + eV)$ right. So, this so, the net current I is $I_{1 \rightarrow 2} - I_{2 \rightarrow 1}$ ok.

Now, you can just do the algebra straightforward, what you will find is that this is A times this mod T square into $N_1(E) - N_2(E + eV)$ into $f(E) - f(E + eV)$ plus small e into capital V that is it. And now if of course, these E is that we are talking about are closed at the fermi level at the chemical potential. So, compared to that this eV could be smaller and one can just make an expansion and again this will give me eV times if I just subtract $f(E)$ from this thing, then this will be minus f' time e . Just do a Taylor expansion and this will be the term here ok.

Now, minus f' of E , we know is a delta function at low temperatures. So, we can replace it by that and then, the result is what is shown on the on here. This is the result.

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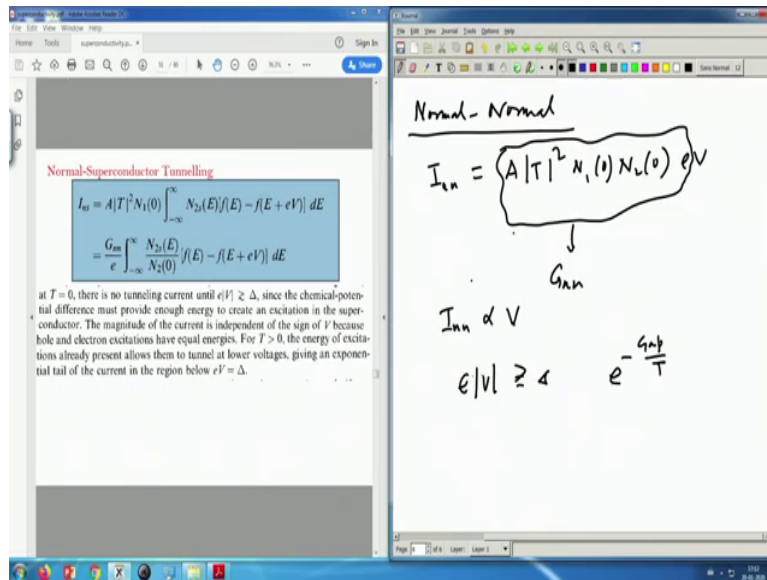
So, for example, for electron so, this is the main result and then, for a metal-metal where this result doing a tunneling between metal to metal. So, that is called normal to normal.

Student: (Refer Time: 26:03) so that we are (Refer Time: 26:05).

So, there see this remember when I use this $f'(\text{prime } E)$ as a delta function, remember I am using it for the normal state that is where the delta function the fermi function of course, appears here and then, I will replace this by the their densities of states at the; fermi function derivative is a delta function at low temperatures and that will convert all both this E 's at the fermi level. So, that is what I am going to do for normal-normal tunneling. So, normal-normal will then be just a $T^2 N_1(0) N_2(0)$ into $e V$.

Now, this whole thing up to this is called $G_{\text{normal-normal}}$ and this is then, you can see that this thing is the junction he is now an Ohmic junction because the current is proportional to the voltage. So, this is this relation tells you that I_{nn} is proportional to voltage and all these quantities here are more or less temperature independent, voltage independent and all that. So, G_{nn} is not temperature dependent, not voltage dependent right. To first approximation that is correct ok. So, this can also be derived from a normal, from another argument which is shown which gives the same argument; so, the same result.

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Then, we can go for a normal to superconductor tunneling. So, in normal superconducting case of course, N_2 has to be replaced by the superconducting density of states and that is exactly what is done and so, then it is just this relation $f(E) - f(E + eV)$ and N_2 is by $N_2(0)$. See this, $N_2(0)$ is divided just to convert it into G_{nn} . So, that you get another $N_2(0)$ at here in front. So, that that is what is done and there is E additional E that comes in because you had eV and here you do not have it. So, that is this relation. There is no direct proportionality to D to V .

So, at T equal to 0, there is no tunneling current until eV equal to Δ of course. As you can see from here there is no tunneling current until you hit until your eV hits Δ becomes Δ or more than equal to Δ . So, So, that is that is the result that we were and we were anticipating that is what will happen. Since, the chemical potential difference must provide this much of energy to create an excitation in the superconductor. So, the magnitude of the current is independent of the sign of V which is because hole and electron excitations have equal energies. The both are plus both are E of k ; capital E k .

Of course, for T greater than 0, there are thermally excited quasi particles and these of course, will lead to. So, the electron can go to can tunnel into and thermally excited into this. There

are already present thermally excitations at that energy. So, it can tunnel through and there is a tunneling which is of course, down by the factors thermally again by gap divided by T.

So, so that means, there is at fine at 0 temperature, tunneling starts only at eV equal to delta. At finite temperature, there is a very small tunneling that happens which is exponentially down and then finally, takes off at delta.

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The image shows a presentation slide on the left and a handwritten whiteboard on the right.

Slide Content:

A more direct comparison of theory and experiment can be made if one considers the differential conductance dI/dV as a function of V .

$$G_{nn} = \frac{dI_{nn}}{dV} = G_{nn} \int_{-\infty}^{\infty} \frac{N_2(E)}{N_1(0)} \left[-\frac{\partial f(E+eV)}{\partial (eV)} \right] dE$$

Since $-\partial f(E+eV)/\partial (eV)$ is a bell-shaped weighting function peaked at $E = -eV$, with width $\sim 4kT$ and unit area under the curve, it is clear that as $kT \rightarrow 0$, this approaches

$$G_{nn} \Big|_{T \rightarrow 0} = \frac{dI_{nn}}{dV} \Big|_{T \rightarrow 0} = G_{nn} \frac{N_2(E[V])}{N_1(0)}$$

Thus, in the low-temperature limit, the differential conductance measures directly the density of states. At finite temperatures, as shown in Fig. the conductance measures a density of states smeared by $\sim 2.2kT$ in energy, due to the width of the weighting function. Because this function has exponential "skirts," it turns out that the differential conductance at $V = 0$ is related exponentially to the width of the gap. In the limit $kT \ll \Delta$, this relation reduces to

$$\frac{G_{nn}}{G_{nn} \Big|_{T \rightarrow 0}} = \left(\frac{2\pi\Delta}{kT} \right)^{1/2} e^{-\Delta/kT}$$

Whiteboard Content:

Normal-Normal

$$I_{nn} = (A|T|^2 N_1(0) N_2(0) eV)$$

↓
 G_{nn}

$I_{nn} \propto V$

$e|V| \geq \Delta \quad e^{-\frac{\Delta}{kT}}$

The. So, there is this other thing that one can see which will I will discuss in the next class where what one does is that what instead of looking at directly the tunneling spectrum which you can of course, one has actually looks at the derivative of the current with respect to the field and these are called differential conductance and this gives you a directive look at the density of states that you are seeing on the side into which you are tunneling. So, that is what I will do next.